

This is a closed-book, closed-notes exam. The only items you are allowed to use are writing implements. Mark each sheet of paper you use with your name and clearly indicate the problem number.

The max number of points per question is indicated in square brackets after each question. The sum of the max points for all the questions is 53, but note that the max exam score will be capped at 50 (i.e., there are 3 bonus points, but you can't score more than 100%). Partial credit will be awarded, so show your work!

You have exactly 60 minutes to complete this exam. Keep your answers clear and concise while complete. Good luck!

1. (4 points) Is “This circle is both red and blue” a statement? Explain why or why not.
2. (8 points) Compute  $GCD(420, 72)$  using either unique prime factorization or Euclid's algorithm.
3. (8 points) Given  $D = \{2, 3, 4\}$ ,
  - (a) (4 points) List the elements of  $D \times D$ .
  - (b) (4 points) List the elements of  $\{(m, n) \in D \times D \mid m \bmod n = 1\}$ .
4. (10 points) Given the following premises, determine which, if any, of the conclusions listed make the argument valid. Explain your reasoning for your answer.

Premises:

$$p \rightarrow q \wedge \sim r$$

$$q \rightarrow p \vee \sim r$$

$$q \vee \sim r$$

Possible Conclusions:  $p$      $q \rightarrow p$      $\sim r$

5. (8 points) Given the statement “If I drink enough coffee, I will never need to sleep again”,
  - (a) (4 points) Write the negation of the statement.
  - (b) (4 points) Write the contrapositive of the statement.
6. (15 points) Prove that for every integer  $n$ ,  $n^2$  can be written as  $3k$  or  $3k + 1$ .  
Hint: divide  $n$  by 3 using the Quotient-Remainder Theorem.