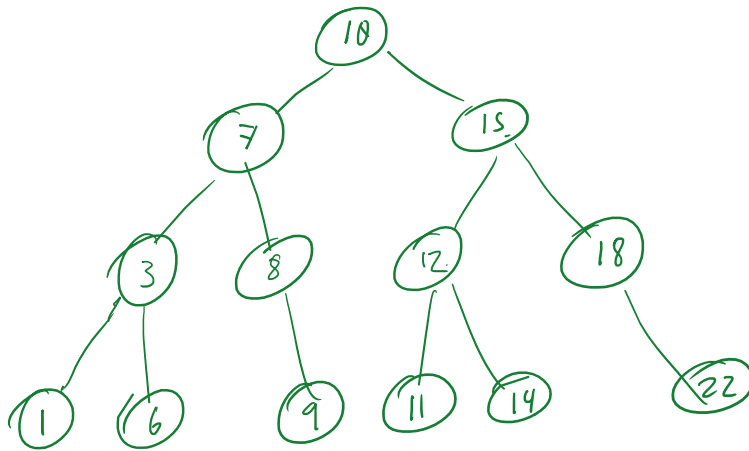


The ADT Binary Search Tree and its implementations.

Monday, November 4, 2019 5:17 PM

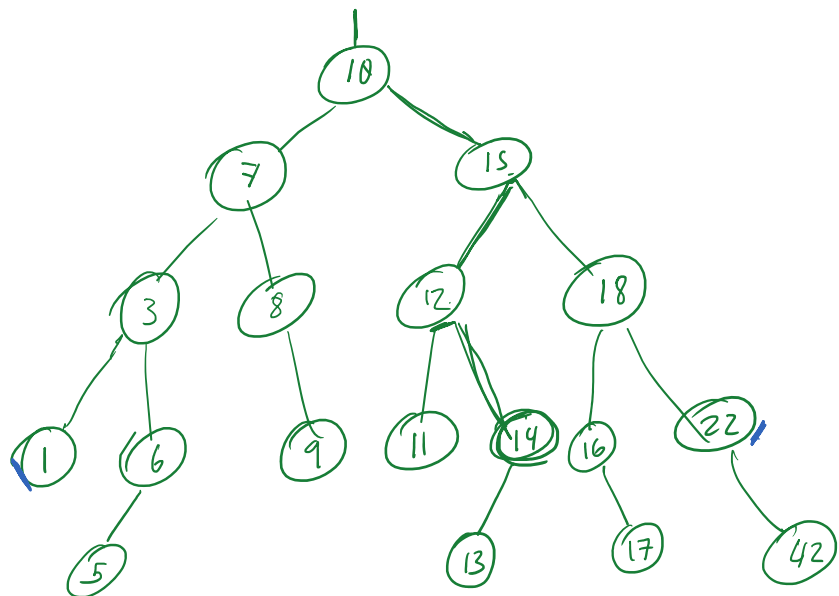
- Binary.- The tree is of degree 2.
- Distinguish Left from Right subtree
- The elements in the tree are comparable.
- Search property.-

for every node n in the tree
- n is greater than any ~~any~~ node in n 's left subtree
- n is lesser than any node in n 's right subtree (duplicates not allowed.)



Operations :

- * find (x, T)
- get Min (T)
- get Max (T)
- insert (x, T)
- remove (x, T)



$find(x, t)$: req as many comparisons as height of tree.

Average height is $\log_2(n)$: n number of elements in tree

	n	$\log_2(n)$
5000.	5000	13
5,000,000	5,000,000	23

remove(x, T)

Case 1: X has no children

Example 13

Case 2: X has one child.

Example 16

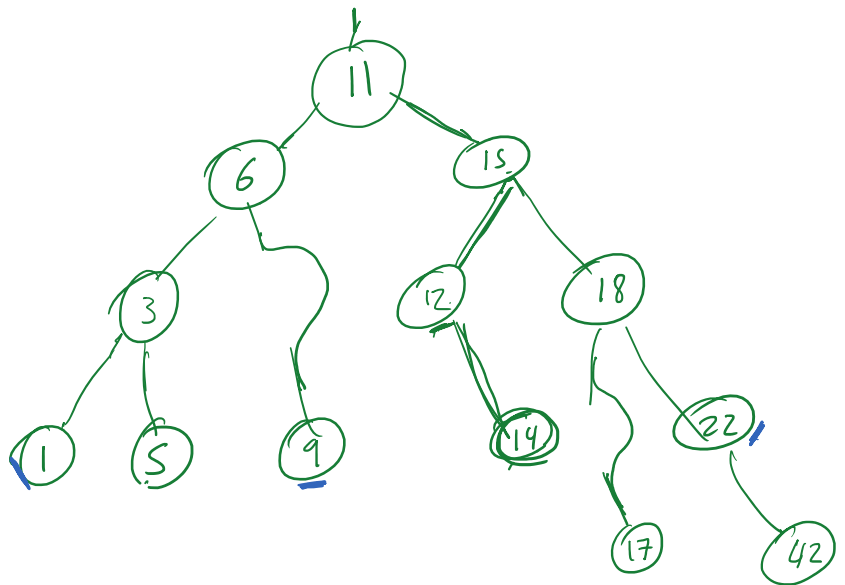
Example 8

put child under parent of X

Case 3: X has 2 children.

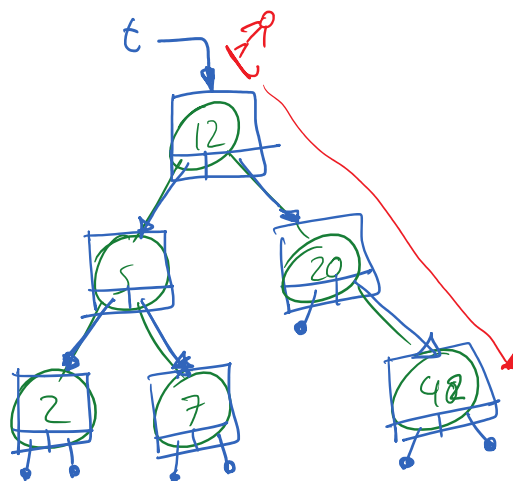
Example 10.

Example 7.



The Data Structure TreeNode

```
template <typename T>
class TreeNode{
public:
    T m_data;
    TreeNode<T>* m_left;
    TreeNode<T>* m_right;
};
```



• find

```
bool find( TreeNode* t, T& x)
{
    if (t==NULL) return false;
    if (t->m_data == x ) return true;
    if (x < t->m_data )
        return find( t->m_left, x );
    else // ( x > t->m_data )
        return find( t->m_right, x );
}
```

• get Min [Recursive]

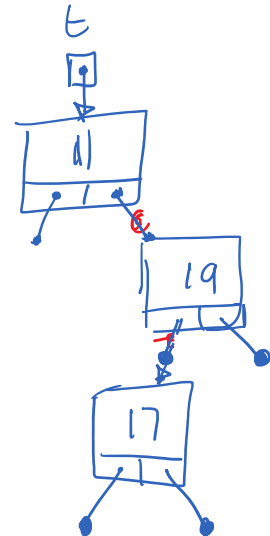
```
TreeNode* getMin( TreeNode* t )
{
    if (t==NULL) return NULL;
    if (t->m_left == NULL)
        return t;
    else
        return getMin( t->m_left )
}
```

• get Max [Iterative]

```
TreeNode* getMax( TreeNode* t )
{
    if (t==NULL) return NULL;
    TreeNode* tmp = t;
    while (tmp->m_right != NULL)
        tmp = tmp->m_right;
    return tmp;
}
```

• Insert

```
void insert( TreeNode* &t, T& x )
{
    if ( t == NULL )
        t = new TreeNode(x, NULL, NULL);
    else
        if ( x < t->m_data )
            insert( t->m_left , x)
        else if ( x > t->m_data )
            insert( t->m_right, x)
        else // x == t->m_data
            return;
}
```



Remove

```
void remove( TreeNode* &t, T& x )
{
    if( t==NULL ) return;
    if( x < t->m_data )
```



remove
(15)

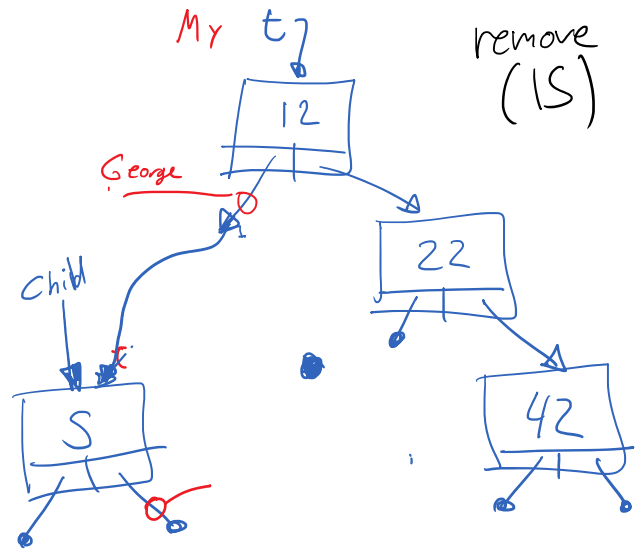
```

void remove( TreeNode* &t, T& x )
{
    if( t==NULL ) return;
    if( x < t->m_data)
        remove( t->m_left, x)
    else if ( x > t->m_data)
        remove( t->m_right, x)
    else // x == t->m_data
    {
        // no children
        if (t->mleft == NULL &&
            t->m_right == NULL){
            delete t;
            t = NULL;
        }
        // one-child
        else if (t->mleft == NULL ||
                 t->m_right == NULL){

            TreeNode* child = t->m_left;
            if ( child == NULL )
                child = t->m_right;

            delete t;
            t = child;
        }
        else // two children
        {
            t->m_data = getMax( t->m_left );
            remove( t->m_left, t->m_data );
        }
    }
}

```

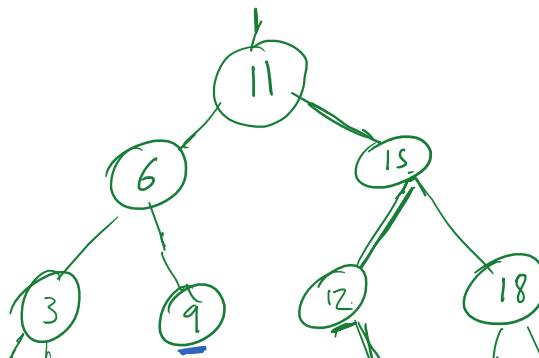


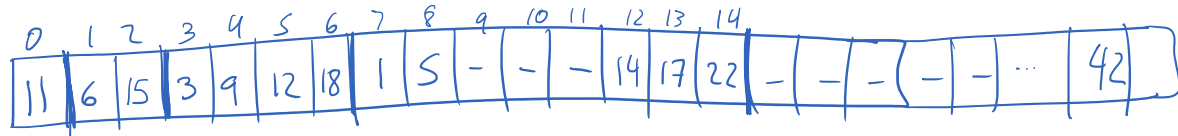
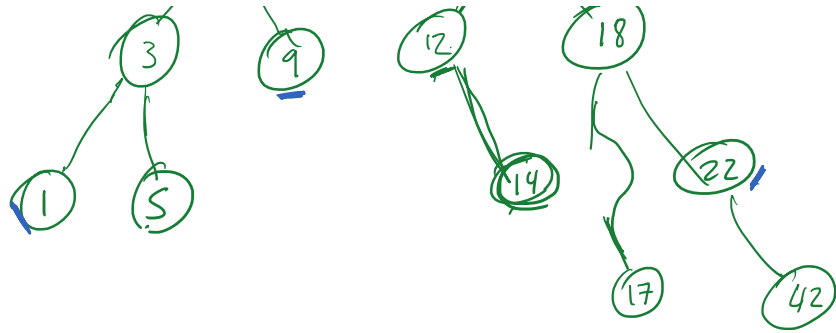
Motivation

find	$\log_2 n$
Insert	$\log_2 n$
remove.	$\log_2 n$

$$\text{height} \approx \log_2 n$$

The Data Structure ArrayTree





- cons: space may be wasted

- pro: $left(i) = 2 \cdot i + 1$
 $right(i) = 2 \cdot i + 2$

$$parent(i) = \left\lfloor \frac{i-1}{2} \right\rfloor$$