Application of the Intermittency Transition Function
to the Wray-Agarwal Turbulence Model in OpenFOAM

Hakop J. Nagapetyan
Department of Mechanical Engineering and Material Science
Washington University in St. Louis

Advisor: Ramesh K. Agarwal

Abstract
This paper integrates an intermittency transport equation $\gamma$ into the Wall-Distance-Free Wray-Agarwal (WA) one-equation turbulence model to create a two-equation WA-$\gamma$ model for computing transitional flows. The model is validated by computing transitional flows past flat plates in zero and slowly varying pressure gradients for which experimental data and computations using other transition models are available. Computational results for benchmark ERCOFTAC flat plate transitional flow cases namely the T3A, T3B, and T3A under zero pressure gradients as well as the T3C2-5 under slowly varying pressure gradients are obtained and compared with the experimental data and the computations from other transitional models. For all transitional flow test cases considered, good agreement between the computations and the experimental data is obtained. The WA-$\gamma$ is found to be accurate and very efficient in computing transitional flows.

Introduction
Transitional flows occur in many industrial applications ranging from airplanes and turbomachinery to wind turbines. Simulations of relatively low to moderate Reynolds number flows can be challenging since lower velocities can result in the flow undergoing laminar to transitional before becoming fully turbulent. The use of empirical methods to predict transition has been shown to introduce large uncertainty in the transition location, which necessitates a more conservative design. Accurate simulation of the transition effects is therefore very important since it will directly aid in the design of more efficient airfoils for airplane wings, UAVs, turbomachinery and wind turbine blades among other applications.

Currently, the most widely known and used model for computing transition is the four-equation SST-Transition model developed by Menter et al. [1]. This model is based on three correlations which are functions of the local transition onset momentum thickness Reynolds number obtained from a transport equation. More recently, a three-equation model was developed based on the local correlation-based transition-modelling concept, removing the need for a transport equation for the transition onset momentum thickness [2]. The goal of this paper is to improve upon the accuracy of the previously developed transition models while reducing the number of transport equations to one equation for intermittency factor $\gamma$. The local correlation-based transition-modelling concept is applied to the Wray-Agarwal (WA) wall-distance-free model to create a two-equation Wray-Agarwal transition model WA-$\gamma$. This newly developed Wray-Agarwal transition model WA-$\gamma$ is validated by computing the benchmark transition flow fields over the T3 series of flat plates. The model is then applied to compute the transitional flow past airfoils namely the S809 and Aerospatiale-A for which the experimental data and computations from other transitional models are available.
Wray-Agarwal Transition Model (WA-γ)
The Wray-Agarwal (WA) model is a one-equation eddy-viscosity model derived from the k-ω closure. An important distinction between the WA model and previous one-equation models based on k-ω closure is the inclusion of the cross diffusion term in the $R = k/ω$ equation and a blending function, which allows smooth switching between the two destruction terms. The model determines $R = k/ω$ by the following transport equation. This model alone cannot model the transition and is therefore modified to include the correlation based intermittency equation $γ$ employing the local correlation-based transition-modelling concept. In this aspect, the modeling philosophy behind the two equations WA-γ model is similar to that of the four equations Shear-Stress-Transport (SST) transition model of Menter et al. [1].

In WA model, the eddy-viscosity is given by:

$$\nu_T = f_\mu R$$  \hspace{1cm} (1)

The transport equation for $R$ which includes the effect of $γ$ in the production term is formulated as:

$$\frac{\partial \rho R}{\partial t} + \frac{\partial \rho u_j R}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ (\mu + \sigma R_T \mu_T) \frac{\partial R}{\partial x_j} \right] + \gamma \rho C_1 RS + \rho f_1 C_{2k \omega} \frac{\partial R}{\partial x_j} \frac{\partial S}{\partial x_j} S + p_{Rlim}^{lim}$$

$$- (1 - f_1) \rho C_{2ke} \min \left( \frac{R^2}{S^2} \frac{\partial S}{\partial x_j} \frac{\partial S}{\partial x_j}, Cm \frac{\partial R}{\partial x_j} \frac{\partial R}{\partial x_j} \right)$$  \hspace{1cm} (2)

In Eq. (2), $p_{Rlim}^{lim}$ is used to ensure proper generation of $R$ for very low values of turbulent intensity $Tu$. This term is formulated as:

$$p_{Rlim}^{lim} = 1.5W \max(\gamma - 0.2, 0) (1.0 - \gamma) \min \left( \max \left( \frac{Re_v}{2420} - 1, 0 \right), 3 \right) \max(3v - v_t),$$  \hspace{1cm} (3)

Following [2], the intermittency transport equation can be written as:

$$\frac{\partial \rho y}{\partial t} + \frac{\partial \rho u_j y}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \frac{\mu + \mu_T}{\sigma_y} \right) \frac{\partial y}{\partial x_j} \right] + F_{\text{length}} \rho S y (1 - \gamma) F_{\text{onset}}$$

$$- \rho c_{a2} \Omega y F_{\text{turb}} (c_{e2} y - 1)$$  \hspace{1cm} (4)

In Eq. (3), $F_{\text{onset}}$ triggers the intermittency production and is a function of $R_T, Re_v,$ and $Re_{\theta c}$ as shown in the following equations:

$$F_{\text{onset1}} = \frac{Re_v}{2.2 Re_{\theta c}} , \quad F_{\text{onset2}} = \min(F_{\text{onset1}}, 2.0), \quad F_{\text{onset3}} = \max \left( 1 - \left( \frac{R_T}{3.5} \right)^3, 0 \right)$$  \hspace{1cm} (5)

$$F_{\text{onset}} = \max(F_{\text{onset2}} - F_{\text{onset3}}, 0)$$  \hspace{1cm} (6)
\[ F_{\text{turb}} = e^{-\left(\frac{R_T}{2}\right)}, \quad R_T = \frac{\mu_t}{\mu}, \quad Re_v = \frac{\rho d_w^2 S}{\mu} \]  

(7)

The model constants for the intermittency equation are the following [2]:

\[ F_{\text{length}} = 100, \quad c_{e2} = 50, \quad c_{a2} = 0.06, \quad \sigma_y = 1.0 \]  

(8)

The local turbulence intensity \( T_u_L \) is given by [2]:

\[ T_u_L = \min \left( 100 \frac{\sqrt{\frac{2R}{3}}}{\sqrt{\frac{S}{0.3} \cdot d_w}}, 100 \right) \]  

(9)

where \( d_w \) is the wall distance. In the original formulation of \( T_u_L \) obtained from Ref. [2], \( R \) replaces turbulent kinetic energy \( k \) (note that \( R = k/\omega \)) and \( \omega \) from the original formulation is replaced by \( \omega \approx S/0.3 \).

The formula for the pressure gradient parameter can be written as [2]:

\[ \lambda_{\theta L} = -7.57 \cdot 10^{-3} \frac{dV}{dy} \frac{d_w^2}{d_w} + 0.0128 \]  

(10)

This term is bounded by \(-1.0 \leq \lambda_{\theta L} \leq 1.0\) for numerical robustness.

The \( Re_{\theta c} \) correlation is given by [2]:

\[ Re_{\theta c} = 100.0 + 1000.0 \exp[-1.0 \cdot T_u_L \cdot F_{PG}] \]  

(11)

where \( F_{PG} \) is a correlation function of \( \lambda_{\theta L} \):

\[ F_{PG} = \begin{cases} 
\min(1 + C_{PG1} \lambda_{\theta L}, C_{PG1}^{\text{lim}}), & \lambda_{\theta L} \geq 0 \\
\min(1 + C_{PG2} \lambda_{\theta L} + C_{PG3} \min[\lambda_{\theta L} + 0.0681, 0], C_{PG2}^{\text{lim}}), & \lambda_{\theta L} < 0 
\end{cases} \]  

(12)

\[ C_{PG1} = 14.68, \quad C_{PG2} = -7.34, \quad C_{PG3} = 0.0 \]  

(13)

\[ C_{PG1}^{\text{lim}} = 1.5, \quad C_{PG2}^{\text{lim}} = 3.0 \]  

(14)

\( F_{PG} \) is limited in order to avoid negative values:

\[ F_{PG} = \max(F_{PG}, 0) \]  

(15)

The wall blocking effect is accounted for by the damping function \( f_\mu \).

\[ f_\mu = \frac{\chi^3}{\chi^3 + C_w^3}, \quad \chi = \frac{R}{v} \]  

(16)
$S$ is the mean strain given below.

$$S = \sqrt{2S_{ij}S_{ij}}, \quad S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$  \hspace{1cm} (17)

$W$ is the mean vorticity given below.

$$W = \sqrt{2W_{ij}W_{ij}}, \quad W_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)$$  \hspace{1cm} (18)

While the $C2k\omega$ term is active, Eq. (2) behaves as a one-equation model based on the standard $k$-$\omega$ equations. The inclusion of the cross diffusion term in the derivation causes the additional $C2k\varepsilon$ term to appear. This term corresponds to the destruction term in one equation models derived from the standard $k$-$\varepsilon$ closure. The presence of both terms allows the WA model to behave as either a one-equation $k$-$\omega$ or one equation $k$-$\varepsilon$ model based on the switching function $f_1$. The blending function was designed so that the $k$-$\omega$ destruction term is active near solid boundaries and the $k$-$\varepsilon$ destruction term becomes active away from the wall near the end of the log-layer. This function was modified from the original Wray-Agarwal model to remove its dependence on the wall distance. The following equations describe the formulation of $f_1$ for wall distance free WA model.

$$f_1 = \tanh(\text{ar} \ g_1^4)$$  \hspace{1cm} (19)

$$\text{ar} \ g_1 = \frac{R + n}{2} \frac{\eta^2}{C_\mu k\omega}$$  \hspace{1cm} (20)

$$k = \frac{v_T S}{\sqrt{C_\mu}}, \quad \omega = \frac{S}{\sqrt{C_\mu}}, \quad \eta = S \cdot \max \left(1, \frac{W}{S} \right)$$  \hspace{1cm} (21)

The model constants and equations for $C1$, $C2k\omega$, $C2k\varepsilon$, and $\sigma_R$ terms are described by the following:

$$C_1 = f_1(C_{1k\omega} - C_{1k\varepsilon}) + C_{1k\omega}$$  \hspace{1cm} (22)

$$C_{2k\varepsilon} = \frac{C_{1k\varepsilon}}{\kappa^2} + \sigma_{k\varepsilon}$$  \hspace{1cm} (23)

$$C_{2k\omega} = \frac{C_{1k\omega}}{\kappa^2} + \sigma_{k\omega}$$  \hspace{1cm} (24)

$$C_{1k\omega} = 0.0829, \quad C_{1k\varepsilon} = 0.1284$$  \hspace{1cm} (25)

$$\sigma_R = f_1(\sigma_{k\omega} - \sigma_{k\varepsilon}) + \sigma_{k\varepsilon}$$  \hspace{1cm} (26)

$$\sigma_{k\omega} = 0.72, \sigma_{k\varepsilon} = 1.0$$  \hspace{1cm} (27)

$$\kappa = 0.41, \ C_\omega = 8.54, \ C_m = 8.0$$  \hspace{1cm} (28)
Software
The open-source computational fluid dynamics (CFD) code OpenFOAM was used to compute the flow fields. OpenFOAM has an extensive library of numerical algorithms and turbulence modes for solving incompressible and compressible RANS equations. In this paper, incompressible RANS equations are solved using the SIMPLE/PISO algorithm in conjunction with the WA-γ transition model.

Zero-Pressure Gradient Flat Plate Flow
Computations were performed for the three zero pressure gradient flat plate cases (T3A, T3B, T3A-), which employ different free-stream velocities $U_\infty$ and free-stream turbulence intensities $T_u_\infty$ as shown in Table 1. The mesh used in simulations of all three cases is the same shown in Fig. 1. The WA-γ model results are compared to the results from the four-equation SST-Transition model and the experimental data.

<table>
<thead>
<tr>
<th></th>
<th>$U_\infty$ (M/S)</th>
<th>$T_u_\infty$ (%)</th>
<th>$\mu_t/\mu$</th>
<th>$P$ (KG/M$^3$)</th>
<th>$M$ (KG/MS)</th>
</tr>
</thead>
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<tr>
<td>T3A</td>
<td>5.4</td>
<td>3.5</td>
<td>13.3</td>
<td>1.2</td>
<td>1.8e-5</td>
</tr>
<tr>
<td>T3B</td>
<td>9.4</td>
<td>6.5</td>
<td>100</td>
<td>1.2</td>
<td>1.8e-5</td>
</tr>
<tr>
<td>T3A-</td>
<td>19.8</td>
<td>0.874</td>
<td>8.72</td>
<td>1.2</td>
<td>1.8e-5</td>
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</table>

Fig. 1 Grid in the computational domain for flow over T3 series flat plates.

Skin friction results of the three T3A, T3B and T3- flat plate cases are in shown in Figs. 2(a) – 2(c). As shown in Figs. 2(a)-(2c), the computed results for the WA-γ model for all three flat plate cases are in good agreement with the experimental data and outperform the results from the SST-Transition model.
Fig. 2(a) T3A Flat Plate results

Fig. 2(b) T3B Flat Plate results
Non-Zero-Pressure Gradient Flat Plate Flow

The ERCOFTAC cases T3C2-5 test take into account the effect of pressure gradient and free-stream turbulence decay on transition prediction. A non-zero pressure gradient effect in the simulations was achieved by using the polynomial expressions from Suluskna et al [3] to modify the shape of the duct upper boundary. T3C4 was the only case that required a different shape. The polynomial expression for the domain width profile for the T3C cases is expressed by Eq.(29), and the profile for T3C4 is expressed by Eq.(30) as follows:

\[
\frac{h}{D} = \min(1.231x^6 - 6.705x^5 + 14.061x^4 - 14.113x^3 + 7.109x^2 - 1.9x + 0.95, 1.0) \tag{29}
\]

\[
\frac{h}{D} = \min(1356x^6 - 7.591x^5 + 16.513x^4 - 17.510x^3 + 9.486x^2 - 2.657x + 0.991, 1.0) \tag{30}
\]

where \(h\) is the upper boundary height, \(D\) is the inlet height (0.3m), and \(x\) is the plate distance from the leading edge. Each T3C case uses different free-stream velocity \(U_\infty\) and free-stream turbulence intensity \(\tau_{u_\infty}\) as shown in Table 2.

<table>
<thead>
<tr>
<th>(U_\infty) (M/S)</th>
<th>(\tau_{u_\infty}) (%)</th>
<th>(\mu_T/\mu)</th>
<th>(P) (KG/M³)</th>
<th>(M) (KG/MS)</th>
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<tr>
<td>T3C2</td>
<td>5.0</td>
<td>3.10</td>
<td>9.0</td>
<td>1.2</td>
</tr>
<tr>
<td>T3C3</td>
<td>3.7</td>
<td>3.10</td>
<td>6.0</td>
<td>1.2</td>
</tr>
<tr>
<td>T3C4</td>
<td>1.28</td>
<td>3.10</td>
<td>2.5</td>
<td>1.2</td>
</tr>
<tr>
<td>T3C5</td>
<td>8.4</td>
<td>3.70</td>
<td>15</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Skin friction results of the T3C flat plate cases given in Table 2 are in shown in Figs. 3(a)-3(d). The computed results from the WA-\(\gamma\) model for all four flat plate cases are in reasonably good agreement with the experimental values.
Fig. 3(a) T3C2 Flat Plate results

Fig. 3(b) T3C3 Flat Plate results
Conclusions and continued Work
In this paper, the local correlation-based transition-modelling concept of Menter et al. [2] was successfully implemented in OpenFOAM with the Wray-Agarwal (WA) turbulence model. The accuracy of the Wray-Agarwal-transition model WA-\( \gamma \) was evaluated by computing several known ERCOFTAC benchmark flat plate transition cases in zero and mild varying pressure gradients and was compared with the experimental data and the numerical results from four-equation SST transition model. The two-equation WA-\( \gamma \) Transition model outperformed the four-equation SST-Transition model in all of three zero-pressure gradient T3 transitional flat plate cases. The model also performed very well in accurately predicting the experimental results for
the non-zero-pressure gradient T3C transitional flat plate cases. More transitional flow test cases are currently being run to assess the accuracy and efficiency of the WA-γ transition model by comparing the results with experimental data and from computations using other transition models.

Acknowledgments
The author wishes to recognize and thank the financial support of the NASA Missouri Space Grant. He is also grateful to his advisor Dr. Ramesh Agarwal for his continued mentoring and patience. The author is also grateful to his colleagues within the CFD lab for their help.

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