

Tritium has a half-life of 12.5 y against beta decay. What fraction of a sample will remain undecayed after 25 y?

Simple solution:

time (y)	# of half-lives	fraction left
0	0	100%
12.5	1	50%
25	2	25%

25% remains after 25 years.

More general mathematical solution:

$$N_0 := 1$$

$$T_{\text{half}} := 12.5$$

$$\lambda := \frac{.693}{T_{\text{half}}}$$

$$N(t) := N_0 \cdot \exp(-\lambda \cdot t)$$

$$N(25) = 0.25 \quad \text{This is the fraction left after 25 years.}$$

The most probable energy of a thermal neutron is 0.025 eV at room temperature. In what distance will half of a beam of 0.025-eV neutrons have decayed? The half-life of the neutron is 10.3 min.

In a time of 10.3 min, half of the neutrons in the beam will have decayed. We simply need to calculate how far 0.025 eV neutrons travel in this time.

0.025 eV is a small energy, so a nonrelativistic calculation is sufficient.

$$E := 0.025 \cdot 1.6 \cdot 10^{-19} \quad \text{neutron energy in Joules}$$

$$m_{\text{neutron}} := 1.675 \cdot 10^{-27}$$

$$v := \sqrt{\frac{2 \cdot E}{m_{\text{neutron}}}}$$

$$v = 2.185 \cdot 10^3 \quad \text{meters/second}$$

$$\text{distance} := v \cdot 10.3 \cdot 60$$

$$\text{distance} = 1.351 \cdot 10^6 \quad \text{meters}$$

Find the probability that any particular nucleus of Cl-38 will undergo beta decay in any 1.00-s period. The half-life of Cl-38 is 37.2 min.

$$T_{\text{half}} := 37.2 \cdot 60 \quad \lambda := \frac{.693}{T_{\text{half}}} \quad \delta t := 1$$

$$\text{The probability is } \lambda \delta t: \quad \text{probability} := \lambda \cdot \delta t \quad \text{probability} = 3.105 \cdot 10^{-4}$$

The activity of a certain radionuclide decreases to 15 percent of its original value in 10 d. Find its half-life.

$$\text{The decay law is } R := R_0 \cdot e^{-(\lambda \cdot t)}$$

Because λ and T_{half} are related, we can use the decay law to find the half-life.

$$\frac{R}{R_0} := e^{-(\lambda \cdot t)} \quad \frac{R_0}{R} := e^{\lambda \cdot t} \quad \ln\left(\frac{R_0}{R}\right) := (\lambda \cdot t) \quad \lambda := \left(\frac{1}{t}\right) \cdot \ln\left(\frac{R_0}{R}\right)$$

$$\frac{.693}{T_{\text{half}}} := \left(\frac{1}{t}\right) \cdot \ln\left(\frac{R_0}{R}\right) \quad T_{\text{half}} := \frac{.693}{\left(\frac{1}{t}\right) \cdot \ln\left(\frac{R_0}{R}\right)}$$

After 10 days, R is 15% of R_0 .

$$t := 10 \quad R_0 := 1 \quad R := .15$$

$$T_{\text{half}} := \frac{.693}{\left(\frac{1}{t}\right) \cdot \ln\left(\frac{R_0}{R}\right)} \quad T_{\text{half}} = 3.653 \quad \text{days}$$

The half-life of Na-24 is 15.0 h. How long does it take for 80 percent of a sample of this nuclide to decay.

$$T_{\text{half}} := 15 \quad \lambda := \frac{\ln(2)}{T_{\text{half}}}$$

$$N_0 := 1 \quad \text{Start with 100\%} \quad N := 0.2 \quad 20\% \text{ is left after 80\% decayed}$$

$$N := N_0 \cdot \exp(-\lambda \cdot t)$$

Solving for t as in example 12.2 gives

$$t := \frac{1}{\lambda} \cdot \ln\left(\frac{N_0}{N}\right) \quad t = 34.829 \quad \text{hours}$$

The radionuclide Na-24 beta-decays with a half-life of 15.0 h. A solution that contains 0.0500 μCi of Na-24 is injected into a person's bloodstream. After 4.50 h the activity of a sample of the person's blood is found to be 8.00 pCi/cm³. How many liters of blood does the person's body contain?

$$T_{\text{half}} := 15.0 \quad \text{hours} \quad \lambda := \frac{.693}{T_{\text{half}}} \quad \text{The initial activity was } R_0 := 0.05 \cdot 10^{-6}$$

The first step is to find the activity after 4.5 hours:

$$t := 4.5$$

$$R := R_0 \cdot \exp(-\lambda \cdot t)$$

$$R = 4.061 \cdot 10^{-8} \quad \text{or } 40.6 \text{ nCi (n means nano)}$$

The total activity remaining in the person is 40.61 nCi, or 40.61×10^3 pCi. One cubic centimeter of blood contains 8.00 pCi. We assume the Na-24 is uniformly dispersed throughout the person's blood after 4.5 hours. A simple ratio gives the total volume of blood.

$$\frac{V_{\text{total}}}{1_{\text{cc}}} := \frac{40.61 \cdot 10^3}{8.00} \quad V_{\text{total}} := \frac{40.61 \cdot 10^3}{8}$$

$$V_{\text{total}} = 5.076 \cdot 10^3 \quad \text{cubic centimeters} \quad \text{or a total volume of 5.08 liters.}$$

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Problem 12.7

O. A. Pringle

One g of Ra-226 has an activity of nearly 1 Ci. Determine the half-life of Ra-226.

This looks difficult at first, but let's think about what we know.

We can calculate how many Ra-226 atoms there are in a gram of Ra.

The activity and the number of radioactive nuclei present are related by $R := \lambda \cdot N$

So if we know R (which we do) and N (which we can calculate), then we can calculate λ , and from λ we can calculate T_{half} .

First, let's calculate N. In words,

$$N := (1_{\text{gram}}) \cdot \left(\frac{\text{kg}}{1000_{\text{gram}}} \right) \cdot \left(\frac{\text{Ra}_{226_{\text{atom}}}}{226.025_{\text{u}}} \right) \cdot \left(\frac{\text{u}}{1.66054 \cdot 10^{-27} \cdot \text{kg}} \right)$$

In numbers,

$$N := \left(\frac{1}{10^3} \right) \cdot \left(\frac{1}{226.025} \right) \cdot \left(\frac{1}{1.66054 \cdot 10^{-27}} \right)$$

$$N = 2.664 \cdot 10^{21} \quad \text{looks reasonable}$$

Now we use the activity to find λ

$$R := 3.7 \cdot 10^{10} \quad \text{That's 1 Ci of activity.} \quad R := \lambda \cdot N \quad \lambda := \frac{R}{N} \quad T_{\text{half}} := \frac{.693}{\lambda}$$

$$T_{\text{half}} = 4.99 \cdot 10^{10} \quad \text{Because the activity was given in events/s, this is the time in seconds.}$$

The half-life in years is

$$T_{\text{half}} := \frac{T_{\text{half}}}{60 \cdot 60 \cdot 24 \cdot 365.25} \quad T_{\text{half}} = 1.581 \cdot 10^3$$

The mass of a millicurie of Pb-214 is 3.0×10^{-14} kg. Find the decay constant of Pb-214.
This is similar to problem 12.7.

The number of atoms in the given mass of lead is

$$N := 3 \cdot 10^{-14} \cdot \frac{1}{213.999} \cdot \frac{1}{1.66054 \cdot 10^{-27}} \quad N = 8.442 \cdot 10^{10}$$

We are given the activity: $R := 3.7 \cdot 10^{10} \cdot 10^{-3}$ one millicurie

$$\lambda := \frac{R}{N} \quad \lambda = 4.383 \cdot 10^{-4}$$

The half-life of ${}_{92}\text{U}^{238}$ against alpha decay is 4.5×10^9 y. Find the activity of 1.0 g of U-238.

The number of atoms in the given mass of uranium is

$$N := 1 \cdot 10^{-3} \cdot \frac{1}{238.051} \cdot \frac{1}{1.66054 \cdot 10^{-27}} \quad N = 2.53 \cdot 10^{21}$$

From the half-life we get λ :

$$T_{\text{half}} := 4.5 \cdot 10^9 \cdot 60 \cdot 60 \cdot 24 \cdot 365 \quad \text{Don't forget to convert years to seconds.} \quad \lambda := \frac{.693}{T_{\text{half}}}$$

The activity is then

$$R := \lambda \cdot N \quad R = 1.235 \cdot 10^4 \text{ disintegrations/second (or Bq)}$$

Or, in terms of Curies,

$$R := \frac{R}{3.7 \cdot 10^{10}} \quad R = 3.339 \cdot 10^{-7} \text{ About } 0.34 \mu\text{Ci}$$

The potassium isotope K-40 undergoes beta decay with a half-life of 1.83×10^9 y. Find the number of beta decays that occur per second in 1.00 g of pure K-40.

$T_{\text{half}} := 1.83 \cdot 10^9 \cdot 60 \cdot 60 \cdot 24 \cdot 365$ Half-life in seconds, using 365 days per year.

$$\lambda := \frac{0.693}{T_{\text{half}}}$$

$$\text{The number of atoms in a gram of K-40: } N := 1 \cdot 10^{-3} \cdot \frac{1}{39.964} \cdot \frac{1}{1.66054 \cdot 10^{-27}} \quad N = 1.507 \cdot 10^{22}$$

$$R := \lambda \cdot N$$

$$R = 1.809 \cdot 10^5 \text{ events/s, or Bq}$$

The half-life of the alpha-emitter Po-210 is 138 d. What mass of Po-210 is needed for a 10 mCi source?

The equation to use is $R = \lambda N$. Once we calculate N , the number of Po-210 atoms, we can calculate M .

$$R := 10 \cdot 3.7 \cdot 10^{10} \cdot 10^{-3} \text{10 milli Curies}$$

$$T_{\text{half}} := 138 \cdot 24 \cdot 3600 \text{ half-life in seconds}$$

$$\lambda := \frac{\ln(2)}{T_{\text{half}}} \quad N := \frac{R}{\lambda}$$

$N = 6.365 \cdot 10^{15}$ This is the number of Po-210 atoms which gives the desired activity.

$$M := N \cdot 209.98 \cdot 1.66054 \cdot 10^{-27} \text{mass in kg}$$

$$M = 2.219 \cdot 10^{-9} \text{ kg}$$

In Example 12.5 it is noted that the present radiocarbon activity of living things is 16 disintegrations per minute per gram of carbon content. From this figure find the ratio of C-14 to C-12 atoms in the CO_2 of the atmosphere.

$$R := \lambda \cdot N$$

We can use the above equation to find the number of radioactive carbon-14 atoms in a gram of carbon:

$$N := \frac{R}{\lambda}$$

$$\frac{N}{m} := \frac{1}{\lambda} \cdot \frac{R}{m}$$

$$\frac{N}{m} := \frac{T_{\text{half}}}{.693} \cdot \frac{16}{\text{min} \cdot \text{gm}}$$

$$\frac{N}{m} := \frac{5760 \cdot \text{years} \cdot 365 \cdot \frac{\text{days}}{\text{year}} \cdot 24 \cdot \frac{\text{hours}}{\text{day}} \cdot 60 \cdot \frac{\text{minutes}}{\text{hour}}}{.693} \cdot \frac{16}{\text{min} \cdot \text{gm}}$$

Working out the numbers:

$$N := \frac{5760 \cdot 365 \cdot 24 \cdot 60 \cdot 16}{.693}$$

$N = 6.99 \cdot 10^{10}$ This is how many radioactive atoms there are in a gram of carbon.

There are 6.02×10^{23} atoms in 1 mole, or 12 grams of carbon, so the fraction of carbon-14 atoms is

$$f := N \cdot \frac{12}{6.02 \cdot 10^{23}}$$

$$f = 1.393 \cdot 10^{-12}$$

The relative radiocarbon activity in a piece of charcoal from the remains of an ancient campfire is 0.18 that of a contemporary specimen. How long ago did the fire occur?

$$\lambda := \frac{\ln(2)}{5760} \quad t := \frac{1}{\lambda} \cdot \ln\left(\frac{1}{.18}\right) \quad t = 1.425 \cdot 10^4 \text{ years}$$

Natural thorium consists entirely of the alpha-radioactive isotope Th-232 which has a half-life of 1.4×10^{10} y. If a rock sample known to have solidified 3.5 billion years ago contains 0.100 percent of Th-232 today, what was the percentage of this nuclide it contained when the rock solidified?

The equation for geological dating is

$$t := \frac{1}{\lambda} \cdot \ln\left(\frac{N_0}{N}\right) \quad T_{\text{half}} := 1.4 \cdot 10^{10} \quad \lambda := \frac{\ln(2)}{T_{\text{half}}} \quad t := 3.5 \cdot 10^9$$

Let M = the number of atoms in the rock sample. Then N is the present number of Th-232 atoms, and $N = 0.100 \cdot 10^{-2} \cdot M$ (the 10^{-2} comes from "percentage"). The initial number of Th-232 atoms was $N = f \cdot M$, where f is the initial fraction.

$$t := \frac{1}{\lambda} \cdot \ln\left(\frac{f \cdot x}{.001 \cdot x}\right) \quad \lambda \cdot t := \ln\left(\frac{f}{.001}\right) \quad \frac{f}{.001} := \exp(\lambda \cdot t)$$

$$f := .001 \cdot \exp(\lambda \cdot t) \quad f = 0.001189$$

The radionuclide ${}_{92}\text{U}^{238}$ decays into a lead isotope through the successive emissions of eight alpha particles and six electrons. What is the symbol of the lead isotope? What is the total energy released?

Each alpha particle changes A by four and Z by two. Each emission of an electron signifies the decay of a neutron to a proton, increasing Z by one, leaving A constant. Each emission of a positron signifies the decay of a proton to a neutron, decreasing Z by one, leaving A constant.

The emission of eight alpha particles thus reduced Z by sixteen and A by thirty two. The emission of six electrons increased Z by six. The final A should be $238 - 32 = 206$. The final Z should be $92 - 16 + 6 = 82$.

A check of the Appendix gives: ${}_{82}\text{Pb}^{206}$, a stable isotope of 24.1% relative abundance.

The total energy released is proportional to the mass difference between the original U-238 less eight helium atoms, six electrons, and lead 206.

$$m_{\text{He}} := 4.002603 \quad m_{\text{Pb}} := 205.974455$$

$$m_{\text{e}} := 0.0005486 \quad m_{\text{U}} := 238.050786$$

$$E := -(8 \cdot m_{\text{He}} + 6 \cdot m_{\text{e}} + m_{\text{Pb}} - m_{\text{U}}) \cdot (931.5)$$

$$\text{The total energy released: } E = 48.639 \text{ MeV}$$

The radionuclide U-232 alpha-decays into Th-228.

(a) Find the energy released in the decay.

$$m_{\text{U}} := 232.037168 \quad m_{\text{Th}} := 228.028750 \quad m_{\text{He}} := 4.002603$$

$$Q := (m_{\text{U}} - m_{\text{Th}} - m_{\text{He}}) \cdot 931.5$$

$$Q = 5.417 \text{ MeV}$$

(b) Is it possible for U-232 to decay into U-231 by emitting a neutron?

$$m_{\text{n}} := 1.008665 \quad m_{\text{U231}} := 231.036270$$

$$Q := (m_{\text{U}} - m_{\text{U231}} - m_{\text{n}}) \cdot 931.5$$

$$Q = -7.235 \quad \text{A negative } Q \text{ means energy must be input; this reaction will not go spontaneously.}$$

(c) Is it possible for U-232 to decay into Pa-231 by emitting a proton?

$$m_{\text{H}} := 1.007825 \quad \text{Use H mass to account for electron.}$$

$$m_{\text{Pa}} := 231.035880$$

$$Q := (m_{\text{U}} - m_{\text{Pa}} - m_{\text{H}}) \cdot 931.5$$

$$Q = -6.089 \quad \text{A negative } Q \text{ means energy must be input; this reaction is therefore not possible.}$$

The energy liberated in the alpha decay of Ra-226 is 4.87 MeV.

(a) Identify the daughter nuclide



(b) Find the energy of the alpha particle and the recoil energy of the daughter atom.

$$m_{\text{Ra}} := 226.025406 \quad m_{\text{Rn}} := 222.017574 \quad m_{\text{He}} := 4.002603$$

$$Q := (m_{\text{Ra}} - m_{\text{Rn}} - m_{\text{He}}) \cdot 931.5 \quad A := 222$$

$$K_{\alpha} := \frac{A-4}{A} \cdot Q \quad K_{\alpha} = 4.783 \text{ MeV}$$

$$K_{\text{Rn}} := Q - K_{\alpha} \quad K_{\text{Rn}} = 0.088 \text{ MeV}$$

(c) If the alpha particle has the energy in (b) within the nucleus, how many of its de Broglie wavelengths fit within the nucleus.

The nuclear diameter is

$$D := 2 \cdot 1.2 \cdot 10^{-15} \cdot 226^{\frac{1}{3}} \quad D = 1.462 \cdot 10^{-14}$$

The de Broglie wavelength is $\lambda = h/mv$

$$\text{mass} := m_{\text{He}} \cdot 1.67 \cdot 10^{-27}$$

$$v := \sqrt{\frac{2 \cdot K_{\alpha} \cdot 10^6 \cdot 1.6 \cdot 10^{-19}}{\text{mass}}} \quad \text{Don't forget } 10^6 \text{ for MeV}$$

$$v = 1.513 \cdot 10^7 \quad \text{looks OK}$$

$$\lambda := \frac{6.63 \cdot 10^{-34}}{\text{mass} \cdot v} \quad \lambda = 6.555 \cdot 10^{-15} \quad \frac{D}{\lambda} = 2.23 \quad \text{About 2.2 wavelengths fit inside the nucleus.}$$

(d) How many times per second does the alpha particle strike the nuclear boundary.

The number of times per second is just the inverse of the time it takes the alpha particle to travel one nuclear diameter.

$$\text{time} := \frac{D}{v} \quad \text{time} = 9.661 \cdot 10^{-22} \quad \text{frequency} := \frac{1}{\text{time}}$$

$$\text{frequency} = 1.035 \cdot 10^{21} \quad \text{The alpha particle strikes } 10^{21} \text{ times per second}$$

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Problem 12.25

O. A. Pringle

Positron emission resembles electron emission in all respects except that the shapes of their respective energy spectra are different: there are many low energy electrons emitted, but few low energy positrons. Thus, the average electron energy in beta decay is about $0.3 K_{\text{max}}$, whereas the average positron energy is about $0.4 K_{\text{max}}$. What is the reason for this difference?

Because the electron has negative charge, its kinetic energy is slightly reduced by the Coulomb attraction to the nucleus. The positron, on the other hand, is repelled by the like charge of the nucleus and accelerated outward, shifting the energy distribution to higher energies.

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Problem 12.30

O. A. Pringle

Calculate the maximum energy of the electrons emitted in the beta decay of ${}^5_5\text{B}^{12}$.

Because the mass difference between the emitted particle (electron) and the recoiling nuclide is very large, we assume that the total energy released is carried by the electron. In practice appreciable energy is carried by the antineutrino so that the observed energy of the electron is rarely equal to the total energy released.

The energy released is the mass difference between ${}^5_5\text{B}^{12}$ and ${}^6_6\text{C}^{12}$. This is because the ${}^6_6\text{C}^{12}$ appears as an ION, with the electron flying free. To find the energy, we take the difference between the ATOMIC reaction products and the ATOMIC parent. The mass of the electron need not be calculated separately.

In other words, we can use ${}^5_5\text{B}^{12} \rightarrow {}^6_6\text{C}^{12+} + e^-$ or ${}^5_5\text{B}^{12} \rightarrow {}^6_6\text{C}^{12}$ whichever is more convenient.

$$m_{\text{B}} := 12.014353 \quad m_{\text{C}} := 12.000000 \quad m_{\text{e}} := 0.0005486$$

$$E_{\text{max}} := (m_{\text{B}} - m_{\text{C}}) \cdot (931.5) \quad E_{\text{max}} = 13.37 \quad \text{MeV}$$

The cross section for comparable neutron and proton induced nuclear reactions vary with energy in approximately the manner shown in Fig. 12.30. Why does the neutron cross section decrease with increasing energy whereas the proton cross section increases?

The probability of capture of a neutron depends upon the time δt the neutron spends above the potential well of the nucleus. The transit time of the neutron depends inversely upon the velocity and directly upon the width. The velocity depends upon the square root of the kinetic energy. Therefore, increasing kinetic energy leads to decreasing capture probability for neutrons.

The probability for proton capture at lower energies is lower because of the coulomb repulsion of the proton by the positively charged nucleus.

A slab of absorber is exactly one mean free path thick for a beam of certain incident particles. What percentage of the particles will emerge from the slab?

According to equation 12.21, the mean free path is $\lambda=1/n\sigma$. Plugging λ into equation 12.20 gives

$$N := N_0 \cdot \exp(-n \cdot \sigma \cdot \lambda) \quad N := N_0 \cdot \exp\left[\frac{-(n \cdot \sigma)}{n \cdot \sigma}\right] \quad \frac{N}{N_0} := \exp(-1)$$

$$\text{Fraction} := \frac{N}{N_0} \quad \text{Fraction} := \exp(-1) \quad \text{Fraction} = 0.368$$

The capture cross section of Co-59 for thermal neutrons is 37b. (a) What percentage of a beam of thermal neutrons will penetrate a 1 mm sheet of Co-59? The density of Co-59 is $8.9 \times 10^3 \text{ kg/m}^3$. (b) What is the mean free path of thermal neutrons in Co-59?

The intensity of a beam of particles varies as: $N(x) := N_0 \cdot e^{-n \cdot \sigma \cdot x}$

Where n is the number of target nuclei per unit volume, and σ is the interaction cross section.

The number of target nuclei per unit volume is:

$$n := \rho \cdot \frac{A}{M} \quad n := 8.9 \cdot 10^3 \cdot \frac{(6.02 \cdot 10^{23})}{(59 \cdot 10^{-3})} \quad n = 9.081 \cdot 10^{28}$$

The cross section is:

$$b := 10^{-28} \quad \sigma := 37 \cdot b$$

The fraction passing through 1 mm (thickness= x) is: $x := 10^{-3}$

$$f := e^{-n \cdot \sigma \cdot x}$$

$$f = 0.715 \quad \text{about } 71.5\%$$

The mean free path is $1/n\sigma$ and equals:

$$l := \frac{1}{n \cdot \sigma} \quad l = 2.976211 \cdot 10^{-1} \text{ meters}$$

The cross section for the interaction of a neutrino with matter is 10^{-47} m^2 . Find the mean free path of neutrinos in solid iron, whose density is $7.8 \times 10^3 \text{ kg/m}^3$ and whose average atomic mass is 55.9 u. Express the answer in light years, the distance light travels in free space in a year.

$$\text{lyr} := 9.46 \cdot 10^{15} \quad \mu := 1.660566 \cdot 10^{-27} \quad \rho := 7.8 \cdot 10^3 \quad M := 55.9$$

$$\sigma := 1 \cdot 10^{-47} \quad n := \frac{\rho}{M \cdot \mu} \quad n = 8.403 \cdot 10^{28} \quad \text{Just checking } n.$$

$$\lambda := \frac{1}{n \cdot \sigma}$$

$$\lambda = 125.8 \cdot \text{lyr}$$

$$\lambda = 1.19 \cdot 10^{18} \text{ meters}$$

This distance is beyond any reasonable experimentally achievable value. Any experiments must find atoms with larger neutrino cross sections.

The boron isotope B-10 captures neutrons in an (n, α) - neutron in alpha particle out - reaction whose cross section for thermal neutrons is $4 \times 10^3 \text{ b}$. The density of B-10 is $2.2 \times 10^3 \text{ kg/m}^3$. What thickness of B-10 is needed to absorb 99 percent of an incident beam of thermal neutrons?

$$M := 10 \cdot 10^{-3} \quad N_A := 6.02 \cdot 10^{23} \quad b := 10^{-28} \quad \sigma := 4 \cdot 10^3 \cdot b \quad \rho := 2.2 \cdot 10^3$$

$$n := \rho \cdot \frac{N_A}{M} \quad n = 1.324 \cdot 10^{29}$$

The fraction absorbed in distance x with density n and cross section σ is:

$$f := e^{-n \cdot \sigma \cdot x} \quad n \cdot \sigma = 5.298 \cdot 10^4$$

Solving for x as a function of the fraction to be absorbed gives the following (note that if 99 percent is absorbed, 0.01 percent goes through)

$$x(f) := \frac{-\ln(f)}{n \cdot \sigma} \quad \sigma = 4 \cdot 10^{-25}$$

$$x(0.01) = 8.693 \cdot 10^{-7} \text{ meters}$$

There are approximately 6×10^{28} atoms/m³ in solid aluminum. A beam of 0.5 MeV neutrons is directed at an aluminum foil 0.1 mm thick. If the capture cross section for neutrons of this energy in aluminum is 2×10^{-31} m², find the fraction of incident neutrons that are captured.

$$N_A = 6.02 \cdot 10^{23} \quad b := 10^{-28} \quad \sigma := 2 \cdot 10^{-31} \quad n := 6 \cdot 10^{28} \quad \sigma = 2 \cdot 10^{-31} \cdot b$$

The fraction absorbed in distance x with density n and cross section σ is:

$$f := e^{-n \cdot \sigma \cdot 0.1 \cdot 10^{-3}} \quad n \cdot \sigma \cdot 0.1 \cdot 10^{-3} = 1.2 \cdot 10^{-6}$$

$$f = 0.9999988$$

Solving for the fraction absorbed gives the following (note that if 99 percent pass through, 0.01 percent is absorbed.)

$$\text{Fraction absorbed: } 1 - f = 1.199999 \cdot 10^{-6}$$

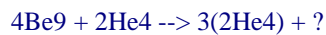
Complete these nuclear reactions:



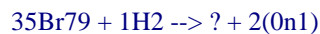
$$\text{Answer: } {}^3\text{Li}_6 + {}^1\text{H}_2 \rightarrow {}^4\text{Be}_7 + {}^0\text{n}_1$$



$$\text{Answer: } {}^{17}\text{Cl}_{135} + ? \rightarrow {}^{16}\text{S}_{32} + 2{}^4\text{He}_4$$



$$\text{Answer: } {}^4\text{Be}_9 + 2{}^4\text{He}_4 \rightarrow 3({}^4\text{He}_4) + {}^0\text{n}_1$$



$$\text{Answer: } {}^{35}\text{Br}_{79} + {}^1\text{H}_2 \rightarrow {}^{36}\text{Kr}_{79} + 2({}^0\text{n}_1)$$

Make sure that the sum of nucleons (the upper numbers) is conserved. Neutrons and protons may interchange, but their number is conserved. Charge conservation is the key to the sum of the lower numbers (number of protons).

U-235 loses about 0.1 percent of its mass when it undergoes fission.

$$c := 3 \cdot 10^8$$

(a) How much energy is released when 1 kg of U-235 undergoes fission?

$$E := 0.001 \cdot 1 \cdot c^2 \quad \text{The fraction "burned" times the mass times } c \text{ squared.}$$

$$E = 9 \cdot 10^{13} \quad \text{The energy released in Joules.}$$

(b) One ton of TNT releases about 4GJ when it is detonated. How many tons of TNT are equivalent in destructive power to a bomb that contains 1 kg of U-235?

$$\text{Tons} := \frac{E}{(4 \cdot 10^9)}$$

$$\text{Tons} = 2.25 \cdot 10^4 \quad \text{Twenty two kilotons of TNT.}$$

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Problem 12.60

O. A. Pringle

In their old age, heavy stars obtain part of their energy by the reaction



How much energy does each such event give off?

$$m_{\text{He}} := 4.002603 \quad m_{\text{C}} := 12.000000 \quad m_{\text{O}} := 15.994915$$

$$Q := (m_{\text{He}} + m_{\text{C}} - m_{\text{O}}) \cdot 931.5$$

$$Q = 7.161 \text{ MeV}$$

Physics 107

Problem 12.64

O. A. Pringle

Show that the fusion energy that could be liberated in $1\text{H}2 + 1\text{H}2$ from the deuterium in 1.0 kg of seawater is about 600 times greater than the 47 MJ/kg heat of combustion of gasoline. About 0.015 percent by mass of the hydrogen content of seawater is deuterium.

$$1 \text{ kg of seawater contains } 1 \cdot 0.015 \cdot 0.01 \cdot \left(\frac{2}{18}\right) = 1.667 \cdot 10^{-5} \text{ kg of deuterium}$$

The factor 2/18 is needed because 2/18 of the mass of water is hydrogen.

Beiser already calculated the energy released in the fusion of deuterium in equations 12.28 and 12.29 on page 462. This calculation is just like problem 12.60. I won't repeat it here.

Because Beiser asks for the amount that "could" be liberated, let's take the more energetic reaction, equation 12.28. The mass of a deuterium is

$$m_{\text{H}2} := 2.014102 \cdot 1.66054 \cdot 10^{-27} \text{ kg} \quad \text{It takes 2 of these to liberate 4 MeV of energy.}$$

A kg of water contains

$$\text{number} := \frac{1 \cdot 0.015 \cdot 0.01 \cdot \left(\frac{2}{18}\right)}{m_{\text{H}2}} \text{ pairs of deuteriums} \quad \text{number} = 4.983 \cdot 10^{21}$$

$$\text{Energy_released} := \text{number} \cdot 4 \cdot 10^6 \cdot 1.6 \cdot 10^{-19} \quad (\text{converted to Joules})$$

$$\text{Energy_released} = 3.189 \cdot 10^9 \text{ Joules}$$

$$\text{Compared with dynamite: } \text{ratio} := \frac{\text{Energy_released}}{47 \cdot 10^6} \quad \text{ratio} = 67.858 \text{ Beiser meant 60 instead of 600}$$