

## ME307 Extra Credit Problem 2

[10 – 70 points]

Assigned: October 23, 2013

Due: Friday, November 8, 12:00AM Midnight, via email attachment.

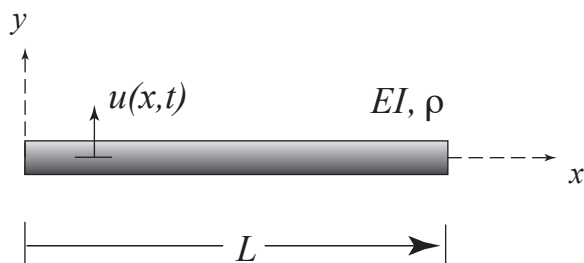


Figure 1. Beam with free-free boundary conditions.

**Given:** The beam shown in Figure 1 has free-free boundary conditions, length,  $L$ , density,  $\rho$  kg/ m<sup>3</sup>, Young's modulus,  $E$ , area moment of inertia with respect to the  $z$ -axis,  $I$ , and cross-sectional area,  $A$ , and has an equation of motion given by

$$\ddot{u} + \alpha^2 \frac{\partial^4 u}{\partial x^4} = 0 \quad (1)$$

**Do the following:**

1. Apply the boundary conditions and show that the eigenvalue equation may be expressed as [10 pts]

$$\cosh \beta L \cos \beta L - 1 = 0 \quad (2)$$

where

$$\beta^4 = \frac{\omega^2}{\alpha^2} \quad (3)$$

and

$$\alpha = \sqrt{\frac{EI}{\rho A}} \quad (4)$$

2. Derive an expression for the mode shapes,  $U_n(x)$ . [5 pts]
3. Use any method you wish to solve for the first five numerical values of  $\beta L$ .<sup>1</sup> [5 pts]
4. Plot the components of the eigenvalue equation. You must include your name on the plot. You can customize the Maple code given at the end of this document to generate and export a plot similar to the one shown in Figure 2. [5 pts]
5. Use the values of  $\beta$  computed above to plot the first five mode shapes of the free-free beam. [10 pts]
6. For even more extra credit, typeset your solution, derived in 1. and 2. using L<sup>A</sup>T<sub>E</sub>X.<sup>2</sup> [35 pts]

### Instructions and Deliverables

1. You may submit as much of this assignment as you want for partial credit, but each part must be completed sequentially – i.e. parts 1, 2, 3, etc. For example, you cannot skip to part 4. and only submit the plot.

<sup>1</sup>You might consider using Maple and James Friend's multiroot.mpl interval root solver: <http://web.mst.edu/~stutts/MapleExamples/multiroot.mpl>

<sup>2</sup>See: <http://web.mst.edu/~stutts/LATEX/LaTeXVault.htm>

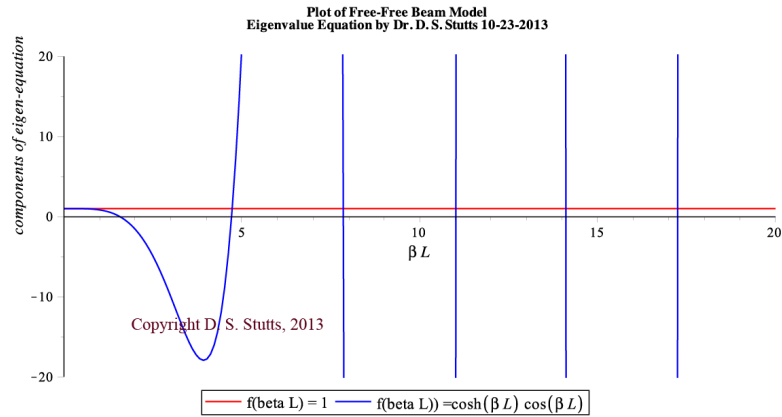


Figure 2. Plot of the components of the eigenvalue equation given in Equation (2).

2. You may use any code that you like, but I must be able to run your code to test it, so if you're planning on using anything besides MATLAB or Maple, please check with me first.
3. You must include a header in your code with your name, the date, and a description of your code – i.e. what it does.
4. You must also include comments within your code to explain what's going on – see the example I've posted.
5. You must bundle your figures and L<sup>A</sup>T<sub>E</sub>X code in a zipped directory and send it to me as a email attachment.<sup>3</sup>

### Maple Code<sup>4</sup>

Note that the line breaks in the following are only to allow the code to fit on the page in the `verbatim` environment.

```
restart:
with(plots):
read "multiroot.mpl";
fcn:=cosh(x)*cos(x)-1:
F:=unapply(fcn,x):
plot([1, cosh(x)*cos(x)],x=0..20, color=[red, blue],view=[0..20,-20..20],
legend=[typeset("f(beta L) = ",1), typeset("f(beta L) = ",cosh(beta*L)*cos(beta*L))],
labels=[typeset(beta*L),"components of eigen-equation"],
labeldirections=[horizontal, vertical], title="Plot of Free-Free Beam Model
Eigenvalue Equation by Dr. D. S. Stutts 10-23-2013",
titlefont=[Times, bold, 10],labelfont=[Times, italic, 12]);
```

<sup>3</sup>You can find all of the L<sup>A</sup>T<sub>E</sub>X source and figures for this assignment at: <http://web.mst.edu/~stutts/LATEX/LaTeXExamples/ExtraCreditProb2.htm>

<sup>4</sup>A number of Maple examples are already posted on BlackBoard, and more can be found here: <http://web.mst.edu/~stutts/MapleVault.htm>.