VIBRATION ANALYSIS OF DISCRETE ONE AND TWO DEGREE OF FREEDOM SYSTEMS

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9.1 OBJECTIVES

1. To determine the natural frequencies of a one and a two degree of freedom systems under forced vibration.
2. To find the damping parameter of the two systems and understand how that parameter affects the system.
3. To determine the effects of external damping by a dashpot in the two degree of freedom system.

9.2 BACKGROUND

The two systems studied in this lab represent examples of the simplest dynamical systems which might be encountered in engineering practice. Although each system is composed of continuous elements (spring steel strips, and aluminum blocks), the nature in which theses elements are combined allows the combination to be modeled as a *discrete* system[1]. That is, a system whose motion may be accurately described in terms of a finite number of discrete degrees of freedom. In spite of the simplicity of these systems, however, much useful information can be gained from their study. In fact, the behavior of continuous (infinite degree of freedom systems) can usually be well modeled by the behavior of a finite number of discrete one-degree-of-freedom (DOF) systems. This is the principle behind modal analysis, which is used constantly in industry to minimize noise emitted by, or prevent fatigue failure in structures.

The principal experimental tool used to study the vibration of structures is the *spectrum analyzer*. The spectrum analyzer decomposes the temporal response of a system into its harmonic components. In other words, a system's time response may be composed of many different frequency components, and a spectrum analyzer separates each of these, and displays the magnitude of each with respect to its individual frequency. This decomposition is accomplished using an algorithm called the *Fast Fourier Transform* or FFT. This algorithm will not be discussed here, but there are many reference books available on the subject for those who are interested in learning more [2]. There are many benefits in representing the response of a system in the frequency domain, but one of the most important is that the frequency components which contribute the greatest amplitude to the overall response may be determined. These frequency components may then be avoided, or the system modified such that they no longer cause a problem.

The motion of a vibrating structure is conveniently measured by a device called an *accelerometer* [3] which transduces the local acceleration of the structure. This signal may then be integrated twice to produce the local displacement of the object.

9.3 MODELING OF A FORCED ONE DEGREE OF FREEDOM SYSTEM

A one-DOF system can be represented by a simple system consisting of a mass, a spring and a damper (see Figure 1). One practical example which may be modeled as shown in Figure 1 is the so-called *vibration isolation* system. Such a system consists of a package, the motion of which is denoted x_2, often a machine or an electronic system, coupled to some vibrating substrate denoted by x_1. The damping and stiffness parameters are chosen such that the package will receive the lowest possible level of vibration for the given

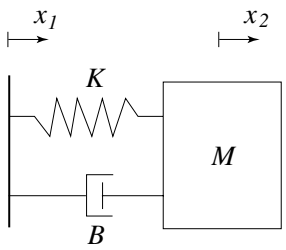

Figure 1. Idealized model of a single degree of freedom system.

frequency.

In the following analysis, a means to determine the amount of damping present in a vibration isolation system will be established for the simple model shown in Figure 1. It should be noted that this is only an approximate model even for the simple laboratory system it represents.

Applying Newton's second law to the simple system in Figure 1, we obtain the equation of motion

$$M\ddot{x}_2 = -B(\dot{x}_2 - \dot{x}_1) - K(x_2 - x_1). \tag{1}$$

Letting $z = x_2 - x_1$ allows the following substitutions into Equation (1):

$$\begin{aligned} x_2 = z + x_1 \quad &\Rightarrow \quad z = x_2 - x_1 \\ \dot{x}_2 = \dot{z} + \dot{x}_1 \quad &\Rightarrow \quad \dot{z} = \dot{x}_2 - \dot{x}_1 \\ \ddot{x}_2 = \ddot{z} + \ddot{x}_1 \quad &\Rightarrow \quad \ddot{z} = \ddot{x}_2 - \ddot{x}_1, \end{aligned}$$

where x_0 is the amplitude of input displacement. Thus, Equation (1) becomes

$$M\ddot{z} + B\dot{z} + Kz = -M\ddot{x}_1. \tag{2}$$

Since we are using a sine wave generator to drive the motion of the first body, x_1, we have

$$\begin{aligned} x_1 &= x_0 \sin \omega t \\ \dot{x}_1 &= x_0 \omega \cos \omega t \\ \ddot{x}_1 &= -x_0 \omega^2 \sin \omega t. \end{aligned} \tag{3}$$

Hence, Equation (2) becomes

$$\ddot{z} + \frac{B}{M}\dot{z} + \frac{K}{M}z = -x_0 \omega^2 \sin \omega t. \tag{4}$$

Equation (5) may be recast into a canonical form in terms of the dimensionless damping constant, ζ, and the undamped natural frequency, ω_n

$$\ddot{z} + 2\zeta\omega_n \dot{z} + \omega_n^2 z = -x_0 \omega^2 \sin \omega t, \tag{5}$$

where $2\zeta\omega_n = \frac{B}{M}$, and $\omega_n = \sqrt{\frac{K}{M}}$.

The steady-state solution to Equation (5) may readily found using the so-called phasor method. In the phasor method, we note, from Euler's relationship, that $\sin \omega t = Im\left\{e^{j\omega t}\right\}$, where $j = \sqrt{-1}$. Hence, substitution of $\sin \omega t = Im\left\{e^{j\omega t}\right\}$, and thus

$$z(t) = ZIm\left\{e^{j\omega t}\right\} \tag{6}$$

into Equation (5), and solving for Z yields

$$Z = \frac{x_0\omega^2 e^{-j\phi}}{\sqrt{(\omega_n^2-\omega^2)^2+4\zeta^2\omega_n^2\omega^2}} = \frac{x_0 r^2 e^{-j\phi}}{\sqrt{(1-r^2)^2+4\zeta^2 r^2}}, \tag{7}$$

where, $r = \frac{\omega}{\omega_n}$ is the ratio between the forcing and natural frequency, and

$$\phi = \begin{cases} \arctan\left(\frac{2\zeta r}{1-r^2}\right) & \text{if } r \leq 1 \\ 180^\circ + \arctan\left(\frac{2\zeta r}{1-r^2}\right) & \text{if } r > 1 \end{cases}, \tag{8}$$

The steady-state solution may thus be written as

$$z(t) = |Z|\sin(\omega t - \phi). \tag{9}$$

Note in Equation (7) that Z is a complex constant which is here cast in polar form. Note also that this method readily extends to systems of harmonically forced ordinary differential equations. For more information on the phasor method, see *D. S. Stutts, "The Phasor Method for Analysis of Harmonically Forced Systems"* which may be found at http://www.umr.edu/˜stutts/Phasor.pdf [4].

A transfer function between the output amplitude, $|Z|$, and the input amplitude, x_0, may be defined as

$$\Lambda(r) = \frac{|Z|}{x_0} = \frac{r^2}{\sqrt{(1-r^2)^2+4\zeta^2 r^2}}. \tag{10}$$

The transfer function, $\Lambda(r)$ is called the *transmission ratio*, since it provides a measure of the displacement transmitted between the vibrating substrate, or base, and the mass.

The value of r where $\Lambda(r)$ is maximum may be found by taking the derivative of Equation (10) with respect to r, setting the result to zero, and solving for r_{max}. This is a rather tedious operation by hand but easily done using Maple™. The result may be found to be

$$r_{max} = \frac{1}{\sqrt{1-2\,\zeta^2}}. \tag{11}$$

The reader should note that Equation (11) is valid only for $0 \leq \zeta < 1/\sqrt{2}$.

Substitution of r_{max} into Equation (10) yields the maximum value of Λ

$$\Lambda_{max} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}, \tag{12}$$

which may be inverted to obtain the dimensionless damping constant

$$\zeta = \frac{\sqrt{\Lambda_{\max}\left(\Lambda_{\max} - \sqrt{{\Lambda_{\max}}^2 - 1}\right)}}{\sqrt{2}\Lambda_{\max}}. \tag{13}$$

The reader should note that Equation (13) is one of four roots of a quartic equation, but is the only valid one, since the others are either negative, or result in $\zeta \geq 1/\sqrt{2}$.

Equation (13) provides a means to measure the dimensionless damping constant for systems modeled as shown in Figure 1. The maximum value of the transfer function, Λ_{max}, may be computed from the measured maximum displacements, x_2, and $x_1 = x_0$

$$\Lambda_{max} = \left| \frac{x_2 - x_1}{x_1} \right| = \left| \frac{x_2 - x_0}{x_0} \right|, \tag{14}$$

where x_2, and x_0 are obtained by integrating the respective accelerometer signals twice (using equipment supplied with the accelerometers), and connecting the two signals to the spectrum analyzer. The spectrum analyzer will show the spectral content of the two signals, which consist primarily of the single forcing frequency, so the respective amplitudes of the two signals may be easily obtained and used to compute Λ_{max}, which may then be substituted into Equation (13) to obtain ζ. This must be done for a range of frequencies, as described in the Data Acquisition Procedure, in order to obtain the maximum transmission ratio.

9.4 MODELING OF A FORCED TWO DEGREE OF FREEDOM (2DOF) SYSTEM

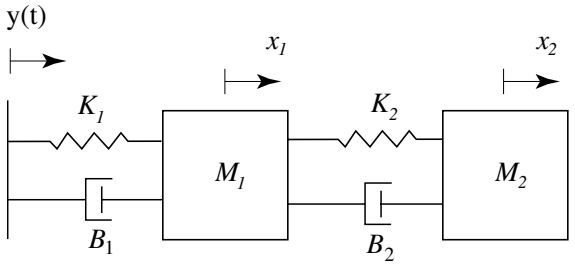

Figure 2. Idealized model of a two degree of freedom system.

The two degree of freedom system with displacement input shown in Figure 2 may be modeled as (you should convince yourself of this)

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{bmatrix} B_1 + B_2 & -B_2 \\ -B_2 & B_2 \end{bmatrix} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} + \begin{bmatrix} K_1 + K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} K_1 y(t) + B_1 \dot{y}(t) \\ 0 \end{pmatrix} \tag{15}$$

The system of equations described in Equation (15) is coupled in both the stiffness and damping terms. This is usually described as static and dynamic coupling respectively. While it is usually possible to measure the mass and stiffness parameters, the equivalent viscous damping parameters are much harder to measure. This is because a 2DOF system can move in more than one way, and the measured damping is a function of which mode or combination of modes the system is vibrating in. Intuitively, it stands to reason that the two dampers (in Figure 2) would supply greater damping force if M_1, and M_2 were moving in the opposite direction than if they were moving in the same direction. Why? Because the relative velocity between the two masses would be greater, and since the damping force is proportional to velocity, it too must increase. Note as well that the restoring force supplied by the springs would also be greater, and hence, the absolute speed at which the masses pass through their equilibrium positions would be increased due to the greater resultant acceleration. When you examine (play with) the 2DOF system pictured in Figure 3, you will find that this rather seat-of-the-pants engineering description is fairly accurate.

Since the damping parameters are difficult to measure, and we don't know them a priori, lets figure out what the system undamped natural frequencies should be. We will do this by assuming that B1 = B2 = 0. We saw in the section on single degree of freedom systems that for light damping, the undamped natural frequency is not too far from the frequency of maximum response (see Equation (11), so it is reasonable to expect a similar result in 2DOF systems [5,6]. Assuming free, undamped vibration, Equation (16) becomes

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{bmatrix} K_1 + K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} K_1 y(t) \\ 0 \end{pmatrix} \tag{16}$$

Assuming a harmonic solution of the form

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} e^{j\omega t}, \tag{17}$$

where A_1 and A_2 are constants, yields

$$\begin{bmatrix} K_1 + K_2 - M_1\omega^2 & -K_2 \\ -K_2 & K_2 - M_2\omega^2 \end{bmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{18}$$

From linear algebra, we know that Equation (18) can only be valid for nontrivial A_1 and A_2, if the determinant of the coefficient matrix vanishes

$$det \begin{bmatrix} K_1 + K_2 - M_1\omega^2 & -K_2 \\ -K_2 & K_2 - M_2\omega^2 \end{bmatrix} = 0. \tag{19}$$

Equation (19) results in the *characteristic*, or *frequency equation*

$$M_1 M_2 \omega^4 - (M_1 K_2 + M_2 (K_1 + K_2)) \omega^2 + K_1 K_2 = 0. \tag{20}$$

Solving Equation (20) yields

$$\omega_{1,2} = \sqrt{\frac{(M_1 K_2 + M_2(K_1 + K_2)) \pm \sqrt{(M_1 K_2 + M_2(K_1 + K_2))^2 - 4M_1 M_2 K_1 K_2}}{2M_1 M_2}}, \tag{21}$$

where choosing the minus sign yields the first natural frequency and choosing the plus sign yields the second natural frequency.

9.5 EXPERIMENTAL DETERMINATION OF THE DIMENSIONLESS VISCOUS DAMPING PARAMETER

Equation (13) provides one way to determine the dimensionless viscous damping parameter. The drawback to this method is that you must provide a sinusoidal excitation force, so you need a signal generator, you must be able to accurately measure the amplitude of the response and determine the natural frequencies, so you probably need a spectrum analyzer or something similar. This is a lot of equipment! Another way to measure damping is the so-called *logarithmic decrement* method. With this method, all you require is a strip-chart recorder. Of course, you still need a transducer to measure the response, so hang on to the accelerometers.

A typical free response of an underdamped second-order system is shown in Figure 3. In this hypothetical single degree of freedom system, whose total free response is given by $x(t) = e^{-0.2t}\cos(10t)$, we note that there are seven peaks (labeled on the graph) before $t = 4$ seconds. This corresponds to seven complete cycles or periods of motion. As you can see from the figure, each successive peak is lower than the previous one. This decay in amplitude is governed by the indicated exponential envelope, $e^{-0.2t}$. Since this hypothetical system is a second-order, single degree of freedom system, we know that its equation of motion may be written in the usual canonical form:

$$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega^2 x = 0. \tag{22}$$

From our knowledge of linear systems, we know that Figure 3 represents a system where $\zeta\omega_n = 0.2$, and $\omega_d = \omega_n\sqrt{1 - \zeta^2} = 10$, where ω_d is the so-called *damped natural frequency*. Hence, our hypothetical system has an undamped natural frequency of $\omega_n = 9.998$ radians per second, and a dimensionless damping

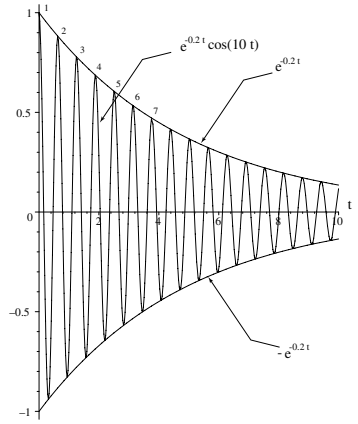

Figure 3. Typical free response of an underdamped system.

parameter of $\zeta = 0.02$, or 2% of critical ($\zeta = 1$) damping.

In general, the amplitude of a second-order system at the kth peak at time t_k is given by

$$x(t_k) = x_k = x_1 e^{-\zeta\omega t_k}, \tag{23}$$

where x_1 is the initial amplitude. Thus, at the first peak, $t_k = t_1 = 0$, and $x(0) = x_1$. Similarly, at the third peak,

$$x(t_3) = x_3 = x_1 e^{-\zeta\omega t_3} = x_1 e^{-\zeta\omega 2T_d}, \tag{24}$$

since there are two damped natural periods (T_d) between the first and third peaks. Hence, the ratio of the first peak to the kth peak is given by

$$\frac{x_1}{x_k} = e^{\zeta\omega(k-1)T_d}, \tag{25}$$

but

$$T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}}. \tag{26}$$

Hence,

$$\frac{x_1}{x_k} = e^{\zeta\omega(k-1)T_d} = e^{\frac{2(k-1)\pi}{\sqrt{1-\zeta^2}}\zeta}. \tag{27}$$

Taking the natural log of Equation (27), we obtain

$$\ln\left(\frac{x_1}{x_k}\right) = \frac{2(k-1)\pi}{\sqrt{1-\zeta^2}}\zeta = \delta, \tag{28}$$

where δ is called the *logarithmic decrement*. Solving equation (28) for ζ yields

$$\zeta = \frac{\delta}{\sqrt{\delta^2 + 4(k-1)^2\pi^2}}. \tag{29}$$

Thus, by measuring the amplitude of any two sequential peaks, and computing the logarithmic decrement, we may use Equation (29) to obtain the system dimensionless damping parameter. Note also that we may easily measure the damped natural frequency by measuring the distance along the time axis between any two peaks on the strip-chart. Knowing the speed of the strip-chart recorder, we can calculate ω_d, and given ζ, we can solve Equation (26) for ω_n.

9.6 EXPERIMENTAL SETUP

A schematic of the one degree of freedom apparatus is provided in Figure 4. The forced vibration will be provided by a shaker, which is driven by a sine wave function generator. The system is attached to the shaker plate and then forced into motion. The motion is sensed using two accelerometers, one attached to the base and one attached to the upper portion of the system.

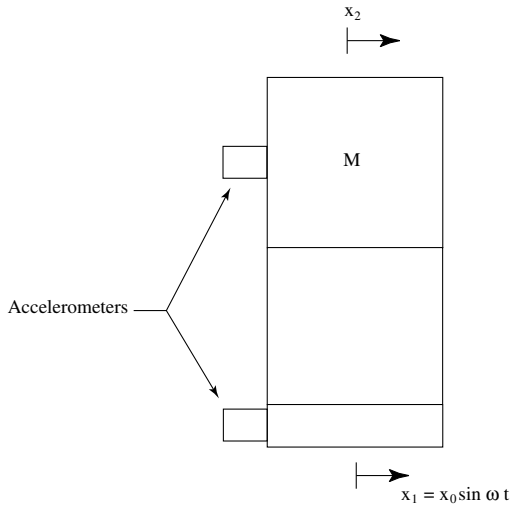

Figure 4. Schematic of single degree of freedom system.

The accelerometers pass their signals through conditioners which can be set to select displacement, velocity, or acceleration. The signals then enter a spectrum analyzer, where the frequency response in voltage output from the accelerometers may be read.

Comparison of the signal from the accelerometer attached to the excited support with that from the accelerometer attached to the mass reveals the location of the system natural frequency. Examination of Equation (11) reveals that at $\omega \approx \omega_n$ (for small ζ), the ratio of the output displacement to the input displacement is maximum. Hence, the natural frequency may be detected by sweeping through the excitation frequencies until the maximum difference between the input amplitude, and the output amplitude is encountered. However, if there is significant damping present, r_{max} is no longer approximately equal to unity. In fact, when there is significant damping, $r_{max} > 1$, so $\omega_{max} > \omega_n$. A similar setup is used for the two degree of freedom system as shown in Figure 5.

9.7 DATA ACQUISITION PROCEDURE

9.7.1 One Degreee of Freedom System

1. Familiarize yourself with the equipment.
2. Place the one degree of freedom system on the shaker plate and insure that the accelerometers are in place and all cables are connected. If there are any problems ask your lab instructor for assistance.
3. Set the frequency on the sine wave generator to 25.0 Hz.
4. Set the frequency range to x1.
5. Set the amplitude on the signal generator to its minimum setting, and make sure the amplitude knob on the power amplifier is no higher than 2.
6. Turn the equipment on. Slowly increase the amplitude setting on the frequency generator until x_2 accelerometer is indicating about 2 volts.

7. Run the experiment at this low setting for approximately 5 min. to warm up the grease.

8. Adjust the frequency center point on the spectrum analyzer so that the waves are centered on the screen.

9. Adjust the trigger level one spectrum analyzer (if required).

10. Slowly increase the frequency of excitation until you observe the maximum voltage output from the x_2 accelerometer. Make sure that this value is not greater than 4.5 volts to obtain the most accurate readings.

11. Read x_1 and x_2.

12. Increase the frequency from 30 Hz to 40 Hz in increments of 1.0 (11 data points) and from 45 Hz to 60 Hz in increments of 5.0 (4 data points) noting x_1 and x_2 at each step.

13. Remove the accelerometers and the system from the shaker plate.

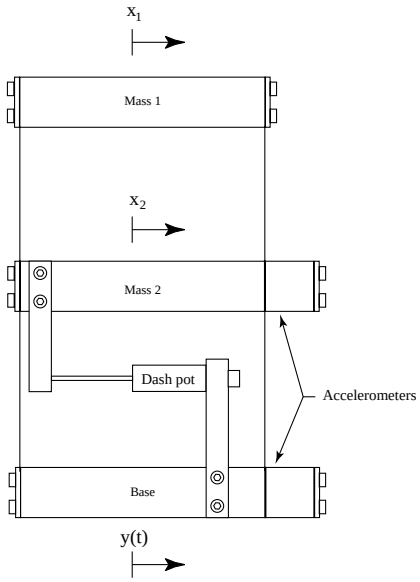

Figure 5. Schematic of two degree of freedom system.

9.7.2 Two Degree of Freedom System

1. Attach the two degree of freedom system (see Figure 5 to the shaker plate and insure that the accelerometers and cables are in place.

2. Set the frequency range to x1.

3. Turn the frequency generator on and set it to 25 Hz.

4. Set the amplitude on the signal generator to its minimum setting, and make sure the amplitude knob on the power amplifier is no higher than 2.

5. Slowly increase the amplitude setting on the frequency generator until the x_1 accelerometer is indicating about 2 volts.

6. Run the experiment at this low setting for approximately 5 min. to warm up the grease, then set the excitation frequency to 5 Hz.

7. Slowly increase the excitation frequency from 5 to 10 Hz, and estimate the frequency of greatest transmission ratio. This is approximately the first natural frequency.

8. Repeat step 3 from 10 to 25 Hz to estimate the second natural frequency.

9. Turn off the frequency generator, the amplifier and the Spectrum Analyzer.

10. Attach the cable from the upper accelerometer (x_1) of the two degree system to the strip chart recorder and then turn the recorder on.

11. Set the recorder for 20 V full scale and 60cm/min. speed.

12. Align the pen with the center of the strip.

13. Set the strip chart recorder to RECORD and then excite the system by pulling the top out toward you, deflecting the top no more than about one-half inch, and then releasing it. Hold the base (x_1) while doing this to stabalize it.

14. Allow the system to oscillate unhindered until it stops.

15. Stop the strip chart recorder.

16. Turn everything off, remove the accelerometers from the system, and then remove the system from the shaker plate.

9.8 DISCUSSION

9.8.1 One Degree of Freedom System

1. Plot the transmission ratio verses frequency and determine the frequency of maximum transmission ratio.

2. Use Equation (13) to compute ζ.

3. Use Equation (11) and the frequency of maximum transmission ratio determined in step 1. above to determine the undamped natural frequency, ω_n.

4. What is the physical significance of the limitation of $0 < \zeta < \frac{1}{\sqrt{2}}$ in Equation (11)? To discover this, it may be helpful to plot Equation (10) with respect to r for various values of ζ within the above range.

9.8.2 Two Degree of Freedom System

1. Calculate the damping ratio for one dashpot setting using the log decrement method.

2. Find the fundamental and secondary natural frequencies of the system and then compare them to the theoretical values obtained using Equation (21) and the mass and stiffness parameters provided by your instructor.

3. List as many sources of energy loss which are applicable to this system that you can think of. Are any of them truly viscous in nature?

4. How does the air-pot differ from a viscous damper?

5. Using the model of the two degree of freedom system given by Equation (15), and assuming negligible damping ($B_1 = B_2 = 0$) and harmonic forcing of the form $y(t) = Y_0 \sin \omega t$, determine the ratio of K_2/M_2 which minimizes x_1. Hint: apply the phasor method by setting

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \mathrm{Im}\left\{e^{j\omega t}\right\}, \tag{30}$$

substituting into Equation (15) with $B_1 = B_2 = 0$, and solving for $x_1(t)$. Recall that $\sin \omega t = Im\left\{e^{j\omega t}\right\}$, so the system of ordinary differential equations given in Equation (15) reduces to an algebraic equation with unknowns X_1 and X_2. Discuss at least one practical application of this result.

6. Sketch the two mode shapes that this system is capable of, and label each according to its frequency. You may use the simple approach described in the lecture.

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9.10 INSTRUCTORS NOTES

Required Items:

1. Two Accelerometers
2. Two Signal Integrators with cables for accelerometers
3. Frequency Generator
4. Power Amplifier
5. Shaker
6. Spectrum Analyzer
7. Strip Chart recorder (must be able to be set for 60cm/min. speed)
8. One and Two degree of freedom systems
9. Wrenches
    - 3/16 Allen for 2 degree system
    - 9/64 Allen for 1 degree system

- 1/4 in. Wrench for attaching dashpot to two DOF system and mounting both systems on the shaker plate

10. Flat-headed screwdriver

Useful Settings:

1. Spectrum Analyzer
   - Set the trigger level so that the pointer is between the **L** and **E** which are nearest the plus sign. This allows the best sampling.
   - Use the Time Average function, with intervals of four seconds to sample data.
   - If the trigger hangs-up, adjust the sensitivity on channel **A** and restart until it begins sampling shaker plate.
   - If an Overload occurs, adjust the sensitivity (HIGHER) until the overload warning stops. Then resample the data – Do not use overload data as it will be incorrect.
2. Power Amplifier
   - Make sure the amplitude knob is set at the mid point of its range.
3. Strip Chart Recorder
   - Set speed for 60cm/min.
   - Set range for 20 V full-scale.
   - Center the pen in the middle of the strip on the 50 line.