Buckling and Post-buckling of Composite Plates and Shells Subjected to Elevated Temperature

Effects of temperature on buckling and post-buckling behavior of reinforced and unstiffened composite plates or cylindrical shells are considered. First, equilibrium equations are formulated for a shell subjected to the simultaneous action of a thermal field and an axial loading. These equations are used to predict a general form of the algebraic equations describing the post-buckling response of a shell. Conditions for the snap-through of a shell subjected to thermomechanical loading are formulated. As an example, the theory is applied to prediction of post-buckling response of flat large-aspect-ratio panels reinforced in the direction of their short edges.

Introduction

The paper considers post-buckling response of composite cylindrical shells subjected to compressive axial loads and elevated temperature. It is shown that a quasi-static increase of temperature at a constant compressive load can result in a snap-through of the shell to a new equilibrium position. A shell can also exhibit a snap-through if it is subjected to an increasing compression and a steady-state thermal field. In the latter case, temperature reduces the upper and lower critical loads. Therefore, a concept of temperature sensitivity associated with a snap-through of a heated shell can be introduced by analogy to the well-known imperfection sensitivity concept in mechanically loaded structures.

The research on imperfection sensitivity of shells was initiated in Koiter's thesis (1945) and in his subsequent studies (1967). This theory was later extended and applied to both static and dynamic buckling problems by Budiansky and Hutchinson (1964), Budiansky (1967), and Elishakoff (1980). Imperfection sensitivity of composite material shells was studied by Simises, et al. (1985), Hui and Du (1987), and others. The problem of post-buckling behavior of composite cylindrical shells reinforced in the axial and circumferential directions was considered by Birman (1990a).

Effects of temperature on behavior of composite cylindrical shells have been investigated by Ambartsumyan (1974), Kalam and Tauchert (1978), Hyer and Rousseau (1987), and Hyer and Cooper (1986) among others. The recent papers dealt with transient thermal stresses in an orthotropic cylinder (Kordateas, 1989) and interfacial stresses in bi-annular assemblies subjected to a uniform temperature (Suhir and Sullivan, 1989).

Thermoelastic problems of reinforced composite cylindrical shells received relatively little attention. Birman (1991) considered axisymmetric thermal bending of shear deformable shells reinforced by ring stiffeners. The problems of thermal shock of reinforced cylindrical shells were also studied (Birman, 1990). Dynamic stability of reinforced composite cylinders was analyzed by Birman and Bert (1991).

An important difference between the studies of imperfection sensitivity and temperature sensitivity is the nature of imperfections versus that of temperature. Initial imperfections which are usually inherited from a technological process are probabilistic in the sense that neither their shapes nor their magnitude can be predicted a priori with certainty. This leads to a probabilistic approach to shell buckling reflected in the works of Roorda and Hansen (1976), Elishakoff and Arbozcz (1981), and others. In contrast to imperfections, temperature is usually due to working equipment, solar or aerodynamic heating. Therefore, the distribution of a thermal field is usually more predictable and thus the problem is more deterministic.

Analysis: Governing Equations

Consider a thin circular cylindrical shell or panel reinforced by axial and ring stiffeners and subjected to the simultaneous action of axial compressive loads of intensity $N_1$ and thermal field $T(x,y,z)$ where $x$ and $y$ are the coordinates in the axial and circumferential directions, respectively, and $z$ is the radial coordinate. A nonlinear thermoelastic version of Love's first approximation theory is used in the analysis.

Nonlinear strain-displacement relationships for an arbitrary layer of such a shell are

$$
\varepsilon_x = u_x + (1/2)\omega_x^2 + \alpha_x T
$$

$$
\varepsilon_y = u_y - \frac{w}{R} + (1/2)\omega_y^2 + \alpha_y T
$$

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\[ y_{xy}^0 = u_{,y} + v_{,y} + w_{,w} w_{,y} + \alpha_{xy} T \]
\[ k_x = -w_{,xx} \]
\[ k_y = -w_{,yy} + \frac{u_{,y}}{R} \]
\[ k_{xy} = -2w_{,xy} + \frac{u_{,y}}{R}. \]  
(1)

All notations in (1) are standard in the theory of shells. The superscript "0" refers to the strains in the middle surface. The positive directions of the radial coordinate and the radial deflection \( w \) are inward on the shell.

The constitutive relations for the shell are written here by assuming that the material is symmetrically laminated about the middle surface and the torsional stiffenings of reinforcements is negligible:
\[ N_x = A_{11} \delta_x + A_{12} \delta_y + \sum_s \delta(x-x_s)E_r A_r (z_s + \alpha_{zr}) - N_x^T \]
\[ N_y = A_{12} \delta_x + A_{22} \delta_y + \sum_s \delta(x-x_s)E_r A_r (z_s + \alpha_{zr}) - N_y^T \]
\[ N_{xy} = A_{66} \delta_{xy} - N_{xy}^T \]
\[ M_x = D_{11} \delta_x + D_{12} \delta_y + \sum_s \delta(x-x_s)E_r A_r (z_s + \alpha_{zr}^2) + I_{66} \alpha_x - M_x^T \]
\[ M_y = D_{12} \delta_x + D_{22} \delta_y + \sum_s \delta(x-x_s)E_r A_r (z_s + \alpha_{zr}^2) + I_{66} \alpha_y - M_y^T \]
\[ M_{xy} = D_{66} \delta_{xy} - M_{xy}^T. \]  
(2)

In (2), \( I_{66} \) and \( I_{66} \) are moments of inertia of circumferential and axial stiffeners with respect to the middle surface, \( E_r \) and \( E_r \) are moduli of elasticity of circumferential and axial stiffeners, and \( A_r \) and \( A_r \) are cross-sectional areas of the respective stiffeners. The distances from centroids of cross-sections of circumferential and axial stiffeners to shell middle surface are denoted by \( z_s \) and \( z_s \) respectively. The coordinates of the stiffeners are \( x_s \) and \( y_s \). Thermal terms in (2) are given by
\[ \{N'_{xy}, M'_{xy}\} = \int_{-h/2}^{h/2} \{A_{11} \alpha_x [1, z] T dz + \frac{A_{12} \alpha_y [1, z] T dz}{R} + \sum \delta(x-x_s)E_r A_r \alpha_x^2 [1, z] T(z_s) \}
+ \sum \delta(x-x_s)E_r A_r \alpha_y^2 [1, z] T(z_s) \}
\[ \{N'_{xx}, M'_{xx}\} = \int_{-h/2}^{h/2} \{A_{12} \alpha_x [1, z] T dz + \frac{A_{22} \alpha_y [1, z] T dz}{R} + \sum \delta(x-x_s)E_r A_r \alpha_y^2 [1, z] T(z_s) \}
+ \sum \delta(x-x_s)E_r A_r \alpha_x^2 [1, z] T(z_s) \}
\[ \{N'_{yy}, M'_{yy}\} = \int_{-h/2}^{h/2} \{A_{66} \alpha_{xy} [1, z] T dz \}
\]  
(3)

where \( \alpha_x^2 \) and \( \alpha_y^2 \) are the coefficients of thermal expansion of the ring and stringer materials. Equations of equilibrium of a shell written within the framework of Love's first approximation theory are
\[ N_{xx} + N_{yy} = 0 \]
\[ N_{xy} = \frac{M_{xx} + M_{yy}}{R} + \frac{N_{xx} + N_{yy} + \alpha_z}{R} \]
\[ M_{xx} = 2M_{xy} + M_{yy} - N_{xy} \]
\[ \{N'_{xx}, M'_{xx}, N'_{yy}, M'_{yy}\} = \{0, 0, 0, 0\}. \]  
(4)

Where \( N > 0 \) implies compression. The substitution of (2) into (4) yields equations of equilibrium in terms of displacements:
\[ \left[ A_{11} + \sum_s \delta(x-x_s)E_r A_r \right] u_{,xx} + A_{66} u_{,xy} + (A_{12} + A_{66}) u_{,xy} 
+ \left[ A_{11} + \sum_s \delta(x-x_s)E_r A_r \right] u_{,yy} + A_{66} u_{,xy} + (A_{12} + A_{66}) u_{,xy} 
= \left[ A_{11} + \sum_s \delta(x-x_s)E_r A_r \right] \frac{u_{,y} + \frac{1}{2} w_{,y}^2}{R} \]
\[ (A_{12} + A_{66}) u_{,yx} + \left[ A_{66} + \frac{D_{66}}{R^2} \right] u_{,xx} 
+ \left[ A_{12} + A_{66} \right] u_{,yx} + \left[ A_{66} + \frac{D_{66}}{R^2} \right] u_{,xx} \]
\[ \frac{1}{R} \left[ D_{22} + \sum_s \delta(x-x_s)E_r A_r \right] u_{,yy} \]
\[ \frac{1}{R} \left[ D_{22} + \sum_s \delta(x-x_s)E_r A_r \right] w_{,yy} \]
\[ \frac{1}{R} \left[ D_{22} + \sum_s \delta(x-x_s)E_r A_r \right] w_{,yy} \]
\[ = \left[ D_{11} + \sum_s \delta(x-x_s)E_r A_r \right] w_{,xxx} + (D_{12} + D_{66}) w_{,xx} 
+ \frac{D_{12}}{R} \left[ \left[ A_{12} + \sum_s \delta(x-x_s)E_r A_r \right] u_{,yx} + \left[ A_{66} + \frac{D_{66}}{R^2} \right] u_{,xx} \right] 
+ \sum \delta(x-x_s)E_r A_r \alpha_z^2 \left[ \frac{u_{,y} + \frac{1}{2} w_{,y}^2}{R} \right] \]
\[ \left[ D_{11} + \sum_s \delta(x-x_s)E_r A_r \right] w_{,xxx} + (D_{12} + D_{66}) w_{,xx} \]
\[ \frac{1}{R} \left[ D_{22} + \sum_s \delta(x-x_s)E_r A_r \right] u_{,yx} \]
\[ \frac{1}{R} \left[ D_{22} + \sum_s \delta(x-x_s)E_r A_r \right] w_{,yy} \]
\[ + \sum \delta(x-x_s)E_r A_r \alpha_z^2 \left[ \frac{u_{,y} + \frac{1}{2} w_{,y}^2}{R} \right] \]
\[ \left[ D_{11} + \sum_s \delta(x-x_s)E_r A_r \right] \frac{u_{,y} + \frac{1}{2} w_{,y}^2}{R} \]
It is important to consider the form of equilibrium equations obtained if each of the shell displacements is modeled by a single assumed function of each coordinate, i.e.,

\[ u = UF(x)\phi_1(y) \]
\[ v = VF(x)\phi_2(y) \]
\[ w = WF(x)\phi_3(y) \]  

where \( F(x) \) and \( \phi_1(y) \) are shape functions that satisfy boundary and periodicity conditions. The Galerkin procedure applied to (5) yields

\[ U = k_1 W + n_1 W^2 + t_1 \]
\[ V = k_2 W + n_2 W^2 + t_2 \]
\[ a_1 W^3 + a_2 W^2 + (a_1 + b_1 T) W + b_0 N_1 W + a_0 T = 0. \]  

Here Eqs. (7) are obtained from the first two Eqs. (5) that are solved for \( U \) and \( V \). Then these expressions are substituted into the third Eq. (5) yielding (8).

In (7) and (8), \( k_i, n_i, t_i, a_i, \) and \( b_i \) are constant coefficients and \( T \) is a parameter that characterizes a steady thermal field. An example of these coefficients for the case of a simply supported, flat, large-aspect-ratio panel reinforced along the short edges is shown later in the paper.

Response of Flat Composite Panels to a Uniform Thermal Field (Constant Compression)

If a panel is flat and the temperature is uniform, the analysis of the coefficients of equation (8) indicates that \( a_0 = 0 \). Accordingly,

\[ T = -(a_2 W^2 + a_1 W + a_1 + b_0 N_1)/b_1 \]  

where \( W \neq 0 \). The curve \( T(W) \) has an extremum at \( dT/dW = 0 \), i.e., at \( W_0 = -a_2/2a_1 \) as shown in Fig. 1. As follows from this figure, the plate subjected to elevated temperature can exhibit a snap-through to a new equilibrium position ("immediate snap-through") cases a, c. Note that the curves corresponding to Figs. 1(a) and 1(c) are similar to those describing a behavior of imperfection-sensitive shells. Therefore, buckling is possible at \( T < T_1 \) if initial imperfections are present.

On the other hand, a behavior corresponding to the cases \( b \) and \( d \) could be called a "delayed snap-through." In these cases plate deformations increase with an increase of temperature beyond \( T = T_1 \). However, at \( W = W_0 \) the response becomes unstable and a snap-through occurs. Obviously, if \( a_1 \) and \( a_2 \) have different signs, \( W_0 \) is positive while, if the signs of \( a_1 \) and \( a_2 \) coincide, the root of \( dT/dW = 0 \) is negative. The character of the response, i.e., the temperature of a snap-through can be examined by an analysis of the sign of \( dT/dW \) at \( W = 0 \), \( T = -(a_1 + b_0 N_1)/b_1 \). A snap-through occurs at \( T = T_1 \), if \( dT/dW < 0 \) and \( a_1 a_2 < 0 \) (Fig. 1(a)) and if \( dT/dW > 0 \) and \( a_1 a_2 > 0 \) (Fig. 1(c)).

In the cases shown in Figs. 1(b) and 1(d), a snap-through occurs at a higher temperature calculated by (9) where \( W = W_0 \). Note here that although the structure exhibits a snap-through in all four cases shown in Fig. 1, it has higher stability resources and prebuckling deformations in the cases corresponding to Figs. 1(b), 1(d).

An immediate snap-through occurs, if

\[ a_1 b_1 > 0, \quad a_2 a_3 < 0 \]  

which correspond to Figs. 1(a) and 1(c), respectively. Delayed snap-through conditions are

\[ a_1 b_1 < 0, \quad a_2 a_3 > 0 \]  

and

\[ a_1 b_1 > 0, \quad a_2 a_3 > 0 \]  

Similar conclusions are obtained by examining the sign of \( d^2 T/W^2 = -2a_1/b_1 \). The relationships between the coefficients of (8) correspond to a behavior reflected in Figs. 1(a) - 1(d) and shown in Table 1. In a particular case of an unstiffened panel, the coefficient \( a_2 = 0 \). In this case

\[ d^2 T \]
\[ dW^2 = -2a_1/b_1 W, \]  

i.e., an extremum of the curve \( T(W) \) corresponds to \( W = 0 \) (Fig. 2). By examining the sign \( d^2 T/W^2 \) one obtains the conclusion that a panel does not exhibit a snap-through, i.e., it behaves similarly to an imperfection-insensitive structure, if \( a_1 b_1 < 0 \) (Fig. 2(a)). On the other hand, if \( a_1 b_1 > 0 \), any increase of temperature beyond \( T_1 \) will result in a snap-through (Fig. 2(b)).

It can be noted here that all unstiffened panels known to the authors satisfy the condition \( a_1 b_1 < 0 \). However, an addition of stiffeners results in either immediate or delayed snap-through
Table 1  Interrelationships among coefficients corresponding to responses shown in Figs. 1 and 3

<table>
<thead>
<tr>
<th>Case</th>
<th>Relationship among coefficients of (8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figs. 1(a), 3(a) or</td>
<td>[ a_1 &lt; 0 \quad a_2 &gt; 0 \quad b_1 &gt; 0 ]</td>
</tr>
<tr>
<td>Figs. 1(b), 3(b) or</td>
<td>[ a_1 &gt; 0 \quad a_2 &lt; 0 \quad b_1 &lt; 0 ]</td>
</tr>
<tr>
<td>Figs. 1(c), 3(c) or</td>
<td>[ a_1 &lt; 0 \quad a_2 &lt; 0 \quad b_1 &gt; 0 ]</td>
</tr>
<tr>
<td>Figs. 1(d), 3(d) or</td>
<td>[ a_1 &gt; 0 \quad a_2 &gt; 0 \quad b_1 &lt; 0 ]</td>
</tr>
</tbody>
</table>

Note: subscript \( i = 1 \) and 0 for Figs 1 and 3, respectively.

![Figure 2: Response of flat unstiffened plates to a uniform temperature; Case a: \( b_1 b_0 < 0 \); Case b: \( a_1 b_0 > 0 \)]

of a panel in a uniform thermal field. Physically this can be associated with structure asymmetry due to stiffeners.

Response of Flat Composite Panels to Increasing Compression in the Presence of a Constant Uniform Temperature

In this problem, \( a_0 = 0 \) as in the previous case and the load-deflection curve is given by

\[ N_1 = \frac{\alpha_1 W^2 + \alpha_2 W + (a_1 + b_1 T)}{b_0} \]

where \( W \neq 0 \). Obviously, temperature changes can only shift \( N_1(W) \) curves along the \( N_1 \) - axis by an amount \( \Delta N_1 = - b_1 T/b_0 \).

Possible responses of the structure are shown in Fig. 3. To establish the kind of response for a particular structure, three conditions must be examined (similar to those used to obtain relationships between the coefficients for Fig. 1).

(a) Extremum, i.e., \( dN_1/dW = 0 \), occurs at \( W_0 = -a_2/2a_1 \). Obviously, \( a_2 < 0 \) corresponds to Figs. 3(a) and 3(b).

(b) Maximum or minimum, i.e., \( \text{sign}(d^2N_1/dW^2) = \text{sign}(-a_2/b_0) \). Maximum occurs if \( a_2 b_0 > 0 \), minimum corresponds to \( a_2 b_0 < 0 \).

(c) The slope of the curve at \( W = 0 \) is \( dN_1/dW = -a_2/b_0 \).

Hence, Figs. 3(a) and 3(d) correspond to \( a_2 b_0 > 0 \) and Figs. 3(b) and 3(c) require that \( a_2 b_0 < 0 \). The analysis of these three conditions yields the criteria of panel behavior summarized in Table 1.

Note that coefficients \( a_1 b_1 \) do not affect the character of the response. However, they determine the value of upper and lower critical loads. Obviously, Figs. 3(a) and 3(c) correspond to a snap-through behavior (temperature-sensitive panel) while in Figs. 3(b) and 3(d) deflections increase gradually with load (temperature-insensitive panel).

If the panel is unstiffened, its behavior can be characterized by Fig. 2 where \( T \) and \( T_1 \) are replaced by \( N_1 \) and \( N_2 \), respectively, the latter being the buckling load in the presence of temperature. It is easy to show that Fig. 2(a) corresponds to \( a_1 b_0 < 0 \) while Fig. 2(b) represents the case \( a_1 b_0 > 0 \).

Snap-Through of a Shell Subjected to a Constant Compression and Increasing Temperature

In a general case, none of the coefficients of (8) is equal to zero. The condition \( dT/dW = 0 \) yields

\[ 2a_2 b_1 W^2 + (3a_1 a_3 + a_2 b_1) W + 2a_1 a_2 W + a_1 + b_1 N_1 = 0 \]

The situation shown in Figs. 4(a) and 4(b) occurs if \( dT/dW > 0 \) at \( W = 0 \). This condition implies

\[ a_1(1 + b_0 N_1) < 0 \]

The condition (16) can be satisfied, i.e., the shell bends in the inward \( (W > 0) \) direction, if

\[ \left\{ \begin{array}{l} a_0 > 0 \\ N_1 < -a_1/b_0 \end{array} \right\} \text{ or } \left\{ \begin{array}{l} a_0 < 0 \\ N_1 > -a_1/b_0 \end{array} \right\} \]

As follows from the Descartes rule of signs, the shell exhibits a snap-through if the condition (17) is satisfied and there is at least one change of sign of the coefficients of (15). Then Eq. (15) has at least one positive root and the behavior of the shell reflects that shown in Fig. 4(a). Since the last term in (15) is negative, this requirement yields

\[ a_1 b_0 > 0 \quad \text{or} \quad 3a_1 a_3 + a_2 b_1 > 0 \quad \text{or} \quad a_2 b_1 > 0 \]

If none of the conditions (18) is satisfied, the behavior of the shell corresponds to Fig. 4(b). The situation shown in Figs. 4(c) and 4(d) occurs if

\[ a_0(1 + b_0 N_1) > 0 \]

Then the condition of a snap-through reduces to that of Eq. (15) having at least one negative real root. This occurs if there is at least one change of sign of the coefficients of the equation obtained from (15) by replacement of \( W \) with \( -W \). The
corresponding mathematical formulation can be obtained easily.

Snap-Through of a Shell Subjected to a Constant Temperature and Increasing Compression

The equilibrium of a shell subjected to an action of a steady-state thermal field can be obtained from (8). If \( N_1 = 0 \) and temperature is relatively low so that deflections remain small, a linear counterpart of (8) yields

\[
W_T = -a_0 T/(a_1 + b_1 T).
\]

An extremum of the curve \( N_1 (W) \) corresponds to \( dN_1/dW = 0 \), i.e.,

\[
f(W) = 2a_1 W + a_0 W^2 - a_0 T = 0.
\]

(21)

The direction of the curve \( N_1 (W) \) at \( W = W_T \), i.e., the sign of its slope is determined as sign \( f(W_T) \) where

\[
F(W_T) = -f(W_T)b_0.
\]

(22)

If \( F(W_T) > 0 \) and Eq. (21) has a positive root, the shell has a snap-through (Fig. 5(a)). Note that a possibility of a maximum corresponding to a negative value of \( W \) (curve 3) is not discussed here. The case where \( F(W_T) > 0 \) but none of the roots of (21) is positive is shown in Fig. 5(b): the shell does not have a snap-through as temperature increases. Accordingly, it can be characterized as temperature insensitive. The conditions of temperature insensitivity follow from the Routh-Hurwitz criterion:

\[
a_2/a_1 > 0 \quad a_0/a_0 > 0
\]

These conditions yield

\[
a_1 > 0 \quad a_2 > 0 \quad a_0 > 0
\]

(23)

or

\[
a_3 < 0 \quad a_0 < 0 \quad a_0 > 0
\]

(24)

which must be satisfied together with \( F(W_T) > 0 \).

If \( F(W_T) < 0 \) and one of the roots of equation (21) is negative, the shell behavior is of the type shown in Fig. 5(c) (the case shown by curve 3 is not considered here). The number of negative roots is equal to the number of sign changes of coefficients of the equation

\[
f(-W) = 0, \text{ i.e.,}
\]

\[
2a_3 W^3 - a_2 W^2 + a_0 T = 0.
\]

(25)

On the other hand, if all roots of Eq. (25) are positive, i.e.,

\[
a_2/a_1 < 0 \quad a_0/a_0 < 0
\]

(26)

the shell behaves according to Fig. 4(d). Conditions (26) can also be written as

\[
a_1 > 0 \quad a_2 < 0 \quad a_0 < 0
\]

or

\[
a_3 < 0 \quad a_0 > 0 \quad a_0 > 0
\]

(27)

which must be satisfied together with \( F(W_T) < 0 \). Obviously, if \( T = 0 \), the extremum of the curve \( N_1 (W) \) corresponds to \( W_0 = -a_2/a_0 \). The case \( W_0 > 0 \) is obtained if \( a_2/a_0 < 0 \). This condition contradicts conditions of temperature insensitivity given by (24). If \( W_0 < 0 \) and \( a_2/a_0 > 0 \), conditions of temperature insensitivity (27) cannot be satisfied either. Notably, the extremum of \( N_1 (W) \) in the absence of temperature does not have to be a minimum as shown in Figs. 5(a) and 5(c). Even if this extremum corresponds to a maximum (not shown in Fig. 5), the conclusions related to temperature sensitivity remain valid.

Example: Cylindrical Thermal Buckling of Reinforced Flat Panels.

Consider a flat, large-aspect-ratio panel reinforced by equally spaced stiffeners in the x-direction, i.e., along the short edges (Fig. 6). Such a panel subjected to a uniform temperature can buckle forming a cylindrical surface. Equilibrium Eq. (5) are uncoupled so that the displacement \( V \) does not affect the solution in the planes \( y = \text{constant} \). If the long edges cannot move in the x-direction, displacements of the panel can be represented by

\[
u = \frac{U \sin 2\alpha x}{a_0}
\]

(28)

where \( \alpha = \frac{m \pi}{L} \).

The substitution of (28) into (5) and use of the Galerkin procedure yield Eq. (8) where
mechanical compressive loading is kept at a constant pre-
buckling level while the uniform temperature is increased steadily. In the second case, a uniform prebuckling thermal field is complemented by quasi-statically increasing mechanical compressive loading. In both cases, conditions of shell snap-through are analyzed for stiffened plates as well as for reinforced cylindrical shells and criteria for various types of behavior are established.

Cylindrical thermal buckling of a large-aspect-ratio symmetrically laminated flat panel reinforced in the direction of its short edges by closely spaced identical stiffeners is considered as an example. It is shown that such plates always exhibit snap-through as soon as a buckling temperature is reached. On the other hand, if reinforcements are absent, the plate deformations increase gradually when temperature exceeds the buckling value and no snap-through occurs.

References

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