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Modeling and Analysis of Functionally Graded Materials and Structures

This paper presents a review of the principal developments in functionally graded materials (FGMs) with an emphasis on the recent work published since 2000. Diverse areas relevant to various aspects of theory and applications of FGM are reflected in this paper. They include homogenization of particulate FGM, heat transfer issues, stress, stability and dynamic analyses, testing, manufacturing and design, applications, and fracture. The critical areas where further research is needed for a successful implementation of FGM in design are outlined in the conclusions. [DOI: 10.1115/1.2777164]

1 Introduction

Functionally graded materials (FGMs) are composite materials formed of two or more constituent phases with a continuously variable composition. FGMs possess a number of advantages that make them attractive in potential applications, including a potential reduction of in-plane and transverse through-the-thickness stresses, an improved residual stress distribution, enhanced thermal properties, higher fracture toughness, and reduced stress intensity factors. A number of reviews dealing with various aspects of FGM have been published in recent years [1–8]. Proceedings of the international symposiums on FGM also shed light on the most recent research in these materials, their manufacturing, mechanics, thermal properties, and applications [9].

At present, FGMs are usually associated with particulate composites where the volume fraction of particles varies in one or several directions. One of the advantages of a monotonous variation of volume fraction of constituent phases is the elimination of stress discontinuity that is often encountered in laminated composites and accordingly, avoiding delamination-related problems. FGM may also be developed using fiber-reinforced layers with a volume fraction of fibers that is coordinate dependent, rather than constant, producing the optimal set of properties or response [10,11]. In this review, our attention is concentrated on particulate-type FGM.

While particulate composite materials may be locally isotropic, they are also heterogeneous due to spatial variations of volume fractions of the phases. An example of such material is shown in Fig. 1 [12] where spherical or nearly spherical particles are embedded within an isotropic matrix. A FGM can also have a skeletal microstructure as depicted in Fig. 2 [13]. Besides the microstructure resembling typical particular composites shown in Figs. 1 and 2, FGM may have a different architecture that results in an orthotropic behavior. Typical examples of orthotropic FGM have lamellar and columnar microstructures obtained by plasma spray and electron beam physical vapor deposition manufacturing processes, respectively, the latter case being shown in Fig. 3 [14]. FGM may include more than two constituent phases. For example, Nemat–Alla analyzed a material consisting of a ceramic and two different metallic phases whose volume fraction varied in the thickness direction according to a power law [15]. Numerical examples in this paper illustrated that stresses in a representative silicon-aluminum-titanium plate subject to thermal loading may be reduced compared to conventional two-phase plates.

It is worth mentioning that the distribution of the material in functionally graded structures may be designed to various spatial specifications. A typical FGM represents a particulate composite with a prescribed distribution of volume fractions of constituent phases. A typical example considered in dozens of papers cited below is a ceramic-metal structure. A number of applications employ a piecewise variation of the volume fraction of ceramic particles, i.e., quasihomogeneous ceramic-metal layers. On the other hand, Reddy and his collaborators as well as numerous other researchers whose work is cited below assumed that the ceramic volume fraction can be represented as the following function of the thickness coordinate z:

$$V_c = \left(\frac{2z+h}{2h}\right)^n \quad \frac{-h}{2} \le z \le \frac{h}{2} \tag{1}$$

where h is the thickness of the structure, and n is a volume fraction exponent. Accordingly, the distribution of the modulus of elasticity of an isotropic FGM and its Poisson ratio can be defined in terms of the material constants of the constituent phases based on a selected homogenization approach.

The present review concentrates on several aspects that are important for the development, design, and applications of FGM. They include the following:

(1) approaches to homogenization of a particulate-type FGM
(2) heat transfer problems where only the temperature distribution is determined
(3) mechanical response to static and dynamic loads including thermal stress
(4) optimization of heterogeneous FGM
(5) manufacturing, design, and modeling aspects of FGM
(6) testing methods and results
(7) FGM applications
(8) fracture and crack propagation in FGM

The work reviewed in this paper is mostly confined to recent papers published since 2000, while earlier articles are omitted, unless their inclusion is necessary for the comprehension of the subject.

1.1 Homogenization of Functionally Graded Materials. In general, there are two possible approaches to homogenization of FGM. The choice of the approach should be based on the gradient of gradation relative to the size of a typical representative volume element (RVE). In the case where the variations of the material properties associated with gradation are relatively slow-changing functions of spatial coordinates, standard homogenization methods can be applied. Accordingly, the material is assumed locally homogeneous at the RVE scale (constant constituent phase volume fractions and homogeneous boundary conditions) but it is globally heterogeneous on the macroscopic structural scale. However, if the properties of the material vary rapidly with the coor-

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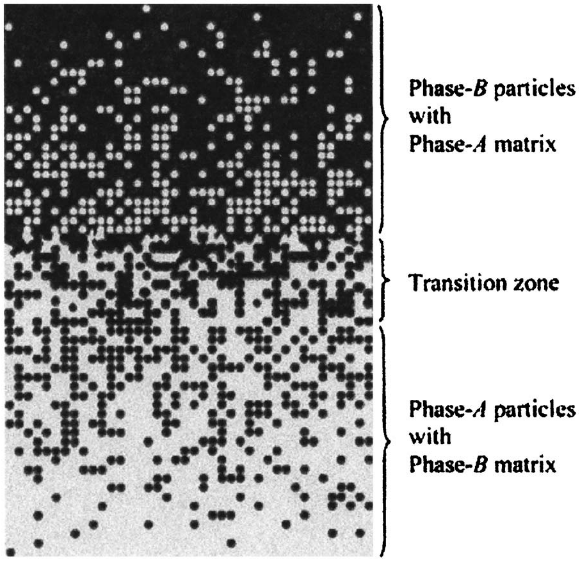

Fig. 1 A particulate FGM with the volume fractions of constituent phases graded in one (vertical) direction [12]

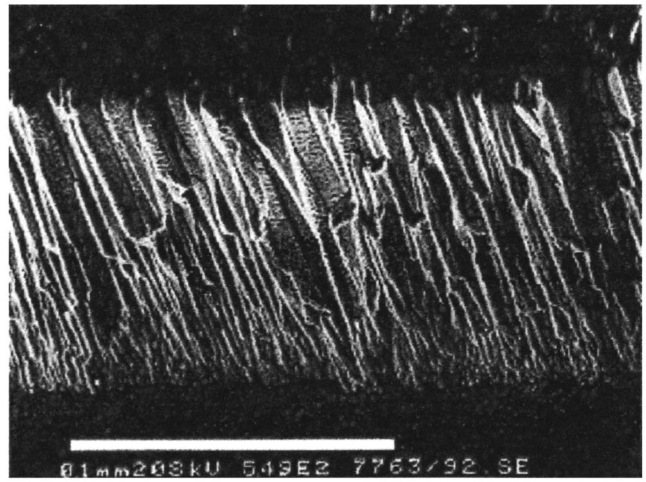

Fig. 3 Columnar FGM: TBC processed by electron beam physical vapor deposition technique ($ZrO_2-Y_2O_3$ with graded porosity) [14]

dinates, it is impossible to disregard the heterogeneous nature of RVE. In this case grading is reflected at both microscopic (RVE) as well as macroscopic (structural) scales.

The approach based on the assumption that the material remains homogeneous at the RVE scale and utilizing existing homogenization methods relies on their accuracy. The principal difference in the results available from various homogenization methods is related to a degree these methods account for the interactions of adjacent inclusions. The simplest approach, so-called dilute model, neglects this effect altogether, while more advanced averaging techniques, such as Mori–Tanaka and self-consistent methods, include the interaction through various mechanisms. Among comparisons of standard micromechanical techniques, Zuiker [16] considered the Mori–Tanaka, self-consistent and Tamura's models, and a fuzzy logic technique and recommended the self-consistent method for reliable first-order estimates over the entire range of volume fraction variations. A comparison between the Mori–Tanaka and self-consistent models and the finite element simulation of FGM was also presented in Refs. [17,18]. The Mori–Tanaka model was shown to yield accurate prediction of the properties with a "well-defined" continuous matrix and discontinuous inclusions, while the self-consistent model was better in skeletal microstructures characterized by a wide transition zone between the regions with predominance of one of the constituent phases. Based on their analysis, the authors concluded that the methods developed for homogeneous particulate materials may yield satisfactory results in FGM subjected to uniform and nonuniform global loads. More recently, Cho and Ha [19] compared three averaging techniques, i.e., the rule of mixtures, the modified rule of mixtures (Tamura's approach), and the Wakashima–Tsukamoto method to the results of a finite element analysis. Although the paper of Pal [20] does not refer to FGM, it represents an interest for the analysis of locally homogeneous but globally heterogeneous composites. Four models were developed in this paper using a differential approach to the solution for an infinitely dilute dispersion of spherical particles in an incompressible matrix.

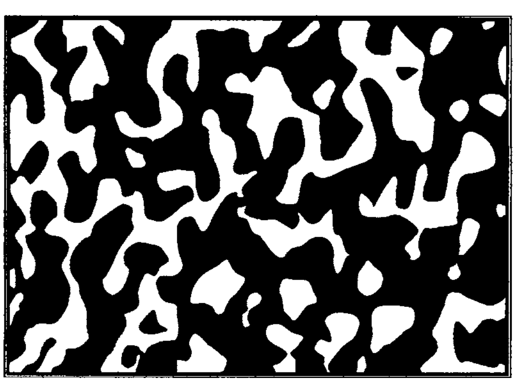

Fig. 2 Skeletal microstructure of FGM material [13]

The studies accounting for local material grading include the constitutive modeling theory based on the higher-order generalized method of cells applied to FGM [21] that was further extended to account for incremental plasticity, creep, and viscoplastic effects in Ref. [22]. The higher-order theory for FGM has been reformulated by Zhong and Pindera accounting for spatially variable thermal conductivity of the material at the local level, using the local/global conductivity matrix approach and reducing the number of equations necessary to solve boundary value problems (by 60% in some representative cases) [23]. An example of a finite element approach to the analysis of FGM where there is no need in assumptions regarding the physical and mechanical properties of the layers since such information evolves directly in the course of the solution is the paper by Biner utilizing so-called Voronoi elements [24]. The author of this paper emphasized a logical link between the microstructure of FGM and its micromechanical response as well as the direct evolution of the mechanical properties from the procedure adopted in the paper.

A micromechanical analysis of an elastic FGM accounting for the local interaction between the particles and the local gradation effect has recently been conducted [12]. The averaged strains were specified throughout the material using integrated contributions between each pair of particles. Subsequently, the effective elastic property distribution in the gradation direction was evaluated from the stress and strain field analysis. As is shown in Fig. 4 for a locally homogeneous material, the discrepancy between the model proposed in Ref. [12] and the Mori–Tanaka method becomes noticeable at the particle volume fractions exceeding 20%.

The subject of the effective thermal conductivity of FGM was also addressed in a recent paper [25]. The solution includes determining a pairwise thermal interaction followed with a derivation of the averaged heat flux fields of the phases in the particle-matrix zone. The effective thermal conductivity is derived from the relationship between the gradient of temperature and the heat flux

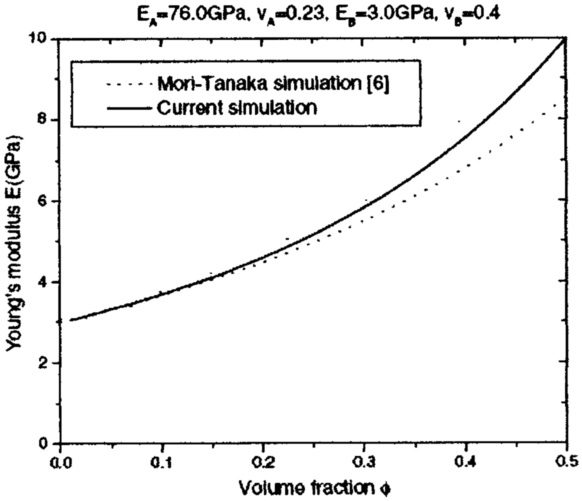

Fig. 4 Comparison of the effective modulus of glass/epoxy FGM obtained by the method developed in Ref. [12] and the Mori–Tanaka method

distribution.

In general, the homogenization models applicable to the analysis of FGM have been proven accurate. Although a comprehensive comparison between different models is outside the scope of this review, it is noted that the models most often used in the recent papers are the Mori–Tanaka method and the self-consistent method, i.e., the locally heterogeneous nature of FGM is usually disregarded.

Besides the analytical and numerical characterization of the property distribution in designed FGM components, it is important to be able to experimentally verify it. A related issue is also an experimental evaluation of the material properties in the already existing FGM to subsequently predict their response. Both tasks can be handled using such method as a neural network technique. An example of a recent study directed toward a development, training, and application of neural networks to determine a distribution of material properties in a FGM cylinder is found in Refs. [26,27]. In these papers, dynamic displacements at the surface were used as an input for the neural network, the output being material properties of the cylinder. Using the results of indentation tests in the framework of an inverse analysis represents another possible approach to validating or measuring the properties of FGM [28,29].

1.2 Heat Transfer in Functionally Graded Material. A typical FGM structure is affected by temperature both at the manufacturing phase and during its lifetime. Accordingly, it may be necessary to estimate postprocessing residual stresses due to thermal mismatch between the constituent materials, such as ceramic and metal phases in a ceramic-metal FGM. Such micromechanical stresses may cause initial damage, affect the lifetime stress distribution, and damage onset and propagation. For FGM in high-temperature environments, the temperature distribution in the material and associated thermal stresses at both macromechanical and micromechanical levels during its lifetime should also be considered.

Jin analyzed the solution of the problem of transient heat transfer in a FGM strip with the properties varying in the thickness direction whose surfaces are suddenly cooled to different temperatures [30]. The closed form asymptotic solution was obtained by subdividing the strip into a number of homogeneous layers. Transient heat transfer in a thick FGM subjected to a nonuniform volumetric heat source was also considered in Ref. [31] where both the temperature distribution and the stresses were found. The transient heat transfer problems for a FGM strip and for an infinitely long FGM cylinder subjected to stationary thermal loads and to thermal shock were solved by a local boundary integral method [32]. The transient heat transfer problem was also solved using the Galerkin boundary element method for a number of configurations such as a 3D FGM cube subjected to a prescribed heat flux regime and a cylinder with a constant surface temperature [33].

Chen and Tong used a graded finite element approach to analyze the sensitivity in the problems of steady state and transient heat conduction in FGM [34]. The problem of optimization of FGM can be formulated using this solution yielding the rate of changes in the response with respect to design variables, i.e., performing a sensitivity analysis. Sutradhar and Paulino developed a three-dimensional boundary element approach for the analysis of transient heat conduction in FGM and demonstrated that their solution was in excellent agreement with representative finite element and analytical solutions [35]. The quadratic, exponential, and trigonometric classes of material gradation can be used in conjunction with this solution. In a recent paper, a three-dimensional Galerkin boundary element formulation was successfully implemented in a FGM heat conduction problem and applied to the analysis of a compressor blade [36].

Numerous paper dealing with thermomechanical aspects of FGM include solutions of the heat transfer and/or temperature distribution problems. These papers are referred to and incorporated in the relevant sections of the review.

1.3 Stress and Deformation Analyses for Statically and Dynamically Loaded Functionally Graded Material Structures. The studies considered in this section are concerned with stress, deformation, stability, and vibration problems of FGM beams, plates, and shells accounting for various effects, such as geometric and physical nonlinearity and transverse shear deformability.

Sankar and Tzeng obtained exact solutions for thermal stress distributions in a FGM beam with an exponential variation of material properties through the thickness [37]. Sankar also considered a FGM beam subjected to a sinusoidal transverse load applied at one of the surfaces [38]. The exact elasticity solution for stresses and displacements was compared with the results obtained by the technical (Bernoulli–Euler) beam theory. It was shown that the latter theory yields acceptable results if the beam is slender and if the load is a slowly varying function of the axial coordinate. The stress concentration was found in short or deep beams that cannot be treated by the technical theory. The stresses depend on the manner the load is applied. The stress concentration that occurs on the loaded surface is higher in a FGM beam compared to a homogeneous counterpart if the load is applied at the "harder" surface and vice versa; it is smaller compared to a homogeneous beam if the load is applied at the "softer" surface of the beam. This reflects on the fact that grading may be beneficial or detrimental dependent on a specific design.

The impact of a sandwich beam with isotropic homogeneous facings and a functionally graded core was considered in a recent paper [39]. The modulus of elasticity of the core was represented by a polynomial function of the thickness coordinate, while its Poisson ratio was assumed constant. The solution was obtained by assumption of a semielliptical stress distribution under a rigid spherical impactor. Based on their analysis, the authors suggested that a FGM core can mitigate or even prevent impact damage in sandwich beam structures. Both thermal buckling and vibrations of a sandwich FGM beam with a viscoelastic core were considered in Ref. [40] using a finite element analysis and accounting for the effect of temperature on the material properties. The effect of a graded porosity in metal foam of metal-skin metal-foam sandwich beams has been considered in Ref. [41] where it was shown that such grading may yield a significant weight reduction under certain loading conditions.

The problems of free vibrations, wave propagation, and static deformations in FGM beams were solved using an especially developed finite element accounting for power law and other alternative variations of elastic and thermal properties in the thickness direction in Refs. [42,43]. The model employed a first-order shear deformation theory of beams. The transient thermoelastic problem for a FGM beam with the exponential variation of the moduli in the thickness direction and subject to a nonuniform convective heat supply was considered in Ref. [44] using a meshless local Petrov–Galerkin method.

Thermal stresses in a thin FGM plate subjected to heat flux in the thickness direction were analyzed by Tsukamoto [45]. The problems of transient thermal stresses in FGM rectangular plates due to a nonuniform (partial) heating have been considered in Ref. [46]. The exact solution for two-dimensional transient temperature and stress problems in a thick simply supported FGM strip subject to a heat flow as a result of suddenly applied nonuniform surface temperatures was recently published by Ootao and Tanigawa [31] by assumption that the strip is in the state of plane strain. Thermal stresses in a FGM cylinder where the modulus of elasticity and the coefficient of thermal expansion are linear functions of the radial coordinate were exactly determined by Zimmerman and Lutz [47].

The problem of the stress distribution and the effect of material grading in the FGM layer sandwiched between ceramic and metal layers of a three-layered plate subjected to a uniform thermal loading were considered accounting for the plastic effects in the metal phase in Ref. [48]. The constitutive law for the metallic phase incorporated plasticity through a power-law strain hardening model. The solution was validated through a comparison with the finite element results. The critical temperature corresponding to the onset of plasticity was determined as a part of the solution. The stress distribution was shown to be effectively controlled by an appropriate gradation in the FGM layer.

Static response of thick rectangular FGM plates was studied by Reddy using a third-order shear deformation theory [49]. The asymptotic expansion approach to the heat conduction problem was employed to address three-dimensional thermoelastic problems in simply supported FGM plates subject to thermal or mechanical loading at one of the surfaces in Ref. [50]. The intriguing conclusion from this benchmark solution was that while the standard assumption of a constant through-the-thickness deflection is acceptable in case of mechanical loading, it may become invalid if the load is thermal. Earlier investigations conducted by Reddy and co-workers on static and dynamic thermomechanical problems in FGM cylinders, rectangular, circular, and annular plates appear in Refs. [51–56].

The exact solution of the static thermomechanical three-dimensional problem of a simply supported rectangular FGM plate was presented by Vel and Batra [13] where the distribution of ceramic and metallic phases through the thickness was assumed to follow a power law for material volume fractions. The homogenization schemes employed in the paper included the Mori–Tanaka and self-consistent methods. The plates were modeled by first-order and third-order shear deformation theories. The extension of the exact analysis to the problem of free and forced vibrations was presented for plates with an arbitrary variation of properties in the thickness direction in Ref. [57]. The results obtained by the first-order shear deformation theory were in excellent agreement with the representative exact solution. An interesting conclusion from a three-dimensional analysis of transient thermally induced stresses in FGM plates was related to the effect of the mode of the thermal load [58]. While rapidly applied temperature boundary conditions could result in thermal stresses in an Al/SiC plate exceeding the steady-state counterparts by the factor of 8, the stresses produced by a transient heat flux were smaller than the steady-state stresses. Furthermore, it was found that inertia forces often have a negligible effect on deformations and stresses of thick FGM plates generated by transient thermal loads [59].

Three methods were used for the static and dynamic analyses of square thick FGM plates with simply supported edges [60]. The methods employed in the paper included a higher-order shear deformation theory and two novel solutions for FGM structures, i.e., the normal deformation theory accounting for the changes in the thickness and for normal stresses in the thickness direction and the meshless Petrov–Galerkin method. According to this paper, the application of the normal deformation theory may be justified if the in-plane size to thickness is equal to or smaller than 5. Among other findings, the authors indicated that the material property variations seem to have a small effect on the fundamental frequency of the plate.

A 3D elasticity bending solution for the stresses in a simply supported FGM plate with the exponentially varying through-the-thickness modulus of elasticity and a constant Poisson ratio was also presented by Kashtalyan [61] who used the Plevako method originally developed in 1971 to analyze an inhomogeneous isotropic media. Elishakoff and Gentilini have recently published a solution for bending of a clamped 3D FGM plate based on the linear theory of elasticity [62]. In particular, they showed that displacements and axial stresses in a FGM ceramic-metal plate do not necessarily lie between the values for purely ceramic and purely metal plate, emphasizing the significant effect of grading. An exact solution for an anisotropic three-dimensional functionally graded simply supported rectangular laminated plate was obtained by Pan using the pseudo-Stroh formalism [63].

Bending of shear-deformable heterogeneous (FGM) plates was analyzed using the Stroh complex potential formalism by Soldatos [64]. A finite element for the analysis of shear-deformable FGM Reissner–Mindlin plates with the properties varying through the thickness according to the power law was developed by Croce and Venini [65]. Shear strains in a nonhomogeneous rubberlike slab subjected to a thermal gradient in the thickness direction were considered in Ref. [66] where it was found that it is possible to design a functionally graded rubberlike material with a minimal effect of temperature on the magnitude of shear stresses. The paper of Cheng [67] contains the solution for nonlinear bending of transversely isotropic symmetric shear-deformable FGM plates. A three-dimensional static analysis of FGM plates was conducted by a discrete layer approach in Ref. [68] where the transition functions reflecting the effect of material gradation were incorporated into the governing equations. In the numerical examples for simply supported graphite/epoxy plates, a 25% decrease in deflections and a 20% reduction of in-plane normal stresses were achieved by the proper gradation of the material. Na and Kim presented a 3D finite element solution for FGM plates subjected to a uniform pressure and thermal loads that were assumed to be distributed according to a uniform, linear, or sinusoidal function of the thickness coordinate [69].

Chi and Chang recently published closed-form solutions for a rectangular simply supported thin FGM plate subjected to transverse loading [70,71] assuming that the Poisson ratio is not affected by grading, while the elastic modulus reflects the actual volume fraction of constituent phases. The latter volume fraction varied through the thickness according to a power law, sigmoid, or exponential function of the coordinate.

The effect of uncertainties in the material properties and loading on the bending response of thick FGM plates subject to lateral pressure and uniform temperature was considered in Ref. [72]. The analysis was conducted using the Reddy higher-order shear deformation theory combined with a first-order perturbation technique that enabled the authors to account for the random nature of the problem.

Functionally graded sandwich ceramic-metal panels have been considered in recent papers by Zenkour [73,74]. The panels had isotropic and homogeneous ceramic core and FGM facings where the distribution of ceramic and meal constituents varied through the thickness according to a power law. The results were obtained by the classical plate theory, first-order shear deformation theory, and a "sinusoidal" version of a shear deformation theory previously introduced by the author. The former paper presented the

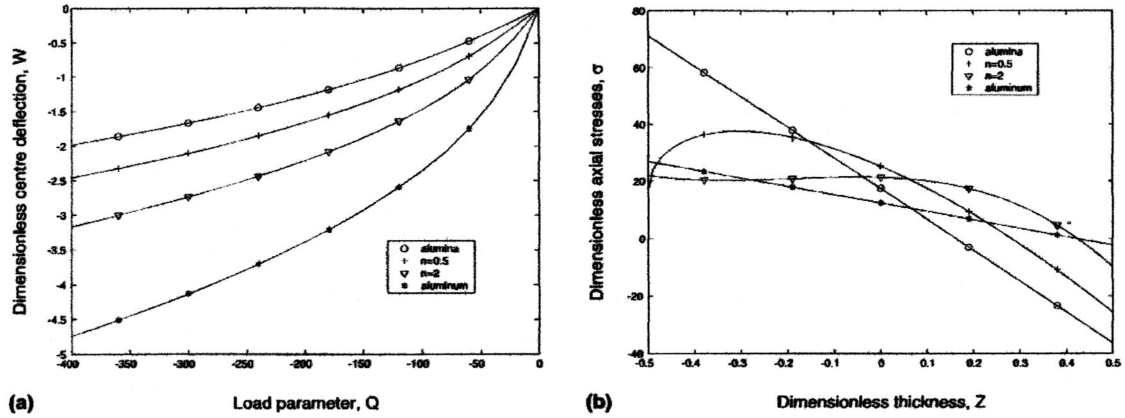

Fig. 5 Nonlinear behavior of FGM metal-ceramic (aluminum-alumina) plates subject to a transverse load [75]. The power-law exponent in the relationship for the volume fraction distribution through the thickness is denoted by *n*. Case (*a*): nondimensional central deflection; Case (*b*): nondimensional axial stress at the center of the plate.

solution of the bending problem for a simply supported panel, while the latter paper dealt with buckling and free vibrations of such panel.

Geometrically nonlinear deformations in FGM plates and in shallow shells that are functionally graded in the thickness direction and subject to transverse pressure and/or to different temperatures applied at the opposite surfaces were considered by Woo and Megiud [75]. The solution was obtained in double Fourier series for deflections and for the stress function, while modeling the structure by the von Karman theory. A representative comparison between stresses and displacements in purely ceramic, purely metallic, and functionally graded plates is shown in Fig. 5. As is obvious from this figure, deflections in a FGM plate even with a small volume fraction of alumina ($n=2$) are significantly smaller than those in the aluminum plate. Furthermore, while the stress distributions in isotropic metallic or ceramic plates are linear functions of the thickness coordinate, they become nonlinear in a functionally graded plate, reflecting a nonuniform property distribution through the thickness. This observation reflects the previously emphasized potential for a better "tailoring" of FGM structures compared to their homogeneous counterparts.

The problem of thermal stresses in FGM hollow cylindrical shells was analyzed in Ref. [76] where the shell was subdivided into a number of homogeneous sublayers. Subsequently, the stresses and displacements within each sublayer were determined from the homogeneous solution for the layer subject to the continuity and boundary conditions at the layer interfaces and at the surface of the shell (the latter conditions were used for the inner and outer layers).

Both the problem of temperature distribution as well as thermal stresses in thick hollow FGM cylinders were analyzed in Ref. [77] where the material was graded in the radial direction and different temperatures were applied at the inner and outer surfaces. The solution of the heat transfer problem yielded the distribution of temperature throughout the shell, and subsequently, displacements and thermal stresses were determined from the Navier equation.

The analysis of temperature, displacement, and stress distribution in hollow and solid cylindrically anisotropic cylinders subjected to various combinations of thermomechanical loads was conducted by Tarn [78]. The solution was obtained exactly for the case where the properties of the material vary in the thickness direction according to a power law and considering thermal loading represented by either prescribed surface temperature or a prescribed heat flux. In addition, the stress problem in a rotating cylinder was solved. Exact solutions for the stresses in internally pressurized FGM cylindrical and spherical shells with the modulus varying through the thickness according to the power law were suggested in Ref. [79].

An approximate analysis of stresses and displacements in a simply supported finite-length thick-walled FGM cylindrical shell subjected to internal pressure and a uniform temperature has recently been published [80]. The solution was obtained by subdividing the shell into subshell regions where the properties of FGM are assumed independent of the radial coordinate. A three-dimensional solution for a FGM cylindrical panel subject to thermomechanical loading was presented by Shao and Wang [81] who assumed that the material properties are temperature independent and graded in the thickness direction. However, it is noted that this solution obtained by assumption that the edges of the panel are simply supported mistakenly disregards thermal terms in the boundary conditions. Pelletier and Vel have recently considered thermoelastic response of a thick simply supported cylindrical FGM shell to thermomechanical loading by assumption that material properties are not affected by temperature [82]. This paper includes a solution of 3D heat conduction and thermoelasticity equations for a generalized plane strain in the axial direction. The analysis is conducted using the Flugge and Donnell shell theories.

The plane stress problem for a rotating solid disk and the plane strain problem for a rotating shaft were considered in Ref. [83]. In both problems, the modulus of elasticity of the material was graded in the radial direction according to either exponential or parabolic law.

Turning to stability issues, thermal and mechanical bucklings of rectangular plates were studied by Javaheri and Eslami [84–86]. Na and Kim used solid finite elements to investigate a three-dimensional thermal buckling problem, though it should be noted that the sinusoidal and linear through-the-thickness temperature distributions in their paper do not reflect the actual temperature distribution in a FGM plate [87]. The subsequent papers on the same subjects were recently published by these authors [88,89], postbuckling being also considered in the latter article. Thermal buckling of simply supported skew plates subjected to temperature varying in the thickness direction was considered in Ref. [90] using a first-order shear deformation theory and through-the-thickness temperature distribution obtained from the heat conduction equation. The problem of buckling of a FGM plate resting on a Pasternak-type elastic foundation was solved by Yang et al. [91] who considered material properties of the constituent phases and the parameters of the foundation as random independent variables. The solution utilized the first-order shear deformation theory to model the plate and the first-order perturbation procedure to account for the randomness of the problem.

Najafizadeh and Eslami considered axisymmetric buckling of simply supported and clamped circular FGM plates under a uni-

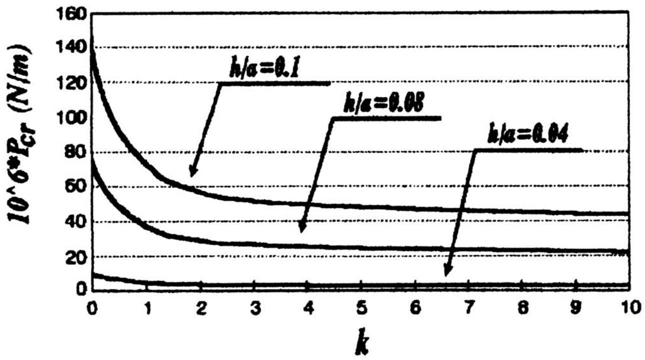

Fig. 6 Buckling radial stress resultant as a function of the volume fraction exponent and a thickness-to-radius ratio *h*/*a* of a circular simply supported plate [93]

form temperature or radial compression [92,93]. In general, it can be concluded that while grading can improve thermal properties and reduce stress concentration, the buckling resistance of FGM plates is inferior compared to the counterpart constructed of the stiffer phase. This is reflected in Fig. 6 where the buckling load of a circular ceramic-metal plate is shown as a function of the thickness-to-radius ratio and the volume fraction exponent k, $k=0$ corresponding to a fully ceramic plate. Both axisymmetric bending due to a uniform pressure and buckling due to radial compression applied to circular FGM plates were studied by the third-order and classical plate theories by Ma and Wang [94] who showed that the classical theory can adequately predict the response in FGM plate structures of typical dimensions found in applications. Buckling of FGM plates subjected to various nonuniform in-plane loads, including pin, partially uniform, and parabolic loads, was considered by Chen and Liew [95] using the first-order shear deformation theory.

Both nonlinear bending and buckling and postbuckling problems have been considered by Shen and his collaborators. In particular, nonlinear bending of thin FGM plates clamped along a pair of opposite edges and with various conditions on the other pair of edges was studied by von Karman geometrically nonlinear theory [96]. The solution was obtained by a perturbation technique combined with the one-dimensional differential quadrature approximation and the Galerkin procedure. An elastic foundation was included into consideration. The loads could include transverse pressure as well as in-plane compression and the authors emphasized that classical buckling can occur only in clamped plates, while plates with other boundary conditions receive transverse deflections, even if the applied load is small, as a result of the bending-stretching coupling. The analysis was further extended to geometrically nonlinear shear-deformable plates subject to thermal and/or mechanical loads using a nonlinear version of the higher-order Reddy theory [97]. An example of this analysis depicted in Fig. 7 where boundary conditions are shown to have a profound effect on deflections of FGM plates with a pair of in-plane movable and a pair of in-plane immovable edges subjected to a simultaneous effect of an elevated temperature and transverse pressure. Other papers of this research group dealing with various aspects of static and dynamic thermomechanical response of FGM plates include Refs. [98–100]. Postbuckling of simply supported and clamped cylindrical panels subject to axial compression and temperature was considered in Refs. [101,102], while the solution for a closed cylindrical shell was obtained in Ref. [103]. In these papers, geometric nonlinearity was incorporated in the analysis via the von Karman–Donnell formulation, while shear deformability of the shells was accounted for through the Reddy higher-order shear deformation theory in the former paper. The problem of postbuckling behavior of an imperfect FGM cylindrical shell subjected to external pressure and elevated temperature was also considered using the von Karman–Donnell theory and the perturbation technique [104]. Note that in all papers referred to in this paragraph the boundary conditions for in-plane stresses at movable edges were satisfied in the integral sense.

It is emphasized here that typical FGM plates subjected to thermal loading are asymmetric about the middle plane (the same observation applied to FGM beams and shells). Accordingly, they deflect even if temperature is small. This is due to thermal stress couples associated with nonuniform temperature and property distributions through the thickness and a nonuniform temperature-affected degradation of material properties. While the latter factor is often neglected, even the presence of thermal couples evaluated without accounting for the material degradation affects the response of all plates and other structures, except for those clamped along the boundaries where the reaction of the support prevents prebuckling deflections. Accordingly, classical bifurcation thermal buckling is observed only in clamped structures with the properties graded in the thickness direction that are subject to temperature that either varies only in this direction or remains uniform. In other situations, the plate develops bending deformations that increase with the load. Eventually, the plate fails due to the loss of strength or alternatively; the deformation-load path becomes unstable resulting in instability. In the contrary, isotropic plates with arbitrary boundary conditions always exhibit bifurcation buckling when subject to a uniform temperature. This observation that was disregarded in a number of studies of FGM structures subject to thermomechanical loading including some of the papers referred

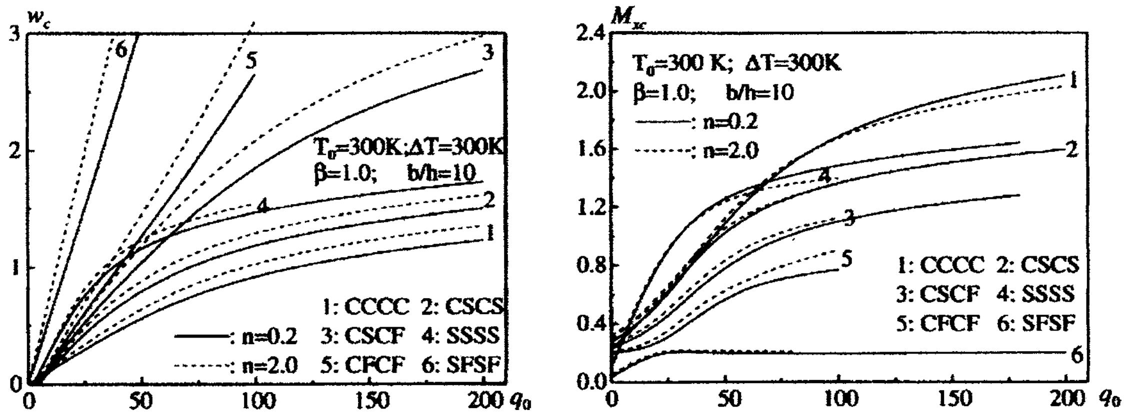

Fig. 7 Effect of boundary conditions on bending of FGM square plates subjected to a uniform pressure and temperature [97]. Case (*a*): nondimensional load-deflection behavior; Case (*b*): nondimensional load-bending couple relationships. Boundary conditions are C=clamped, S=simple support, and F=free edge.

to in this review is described in detail in the recent paper [105].

The comments in the previous paragraph are better understood considering that a structure (beam, plate, or shell) graded in the thickness direction is usually asymmetric, even though the material may be quasi-isotropic. In the latter case, even in rare applications where the material is symmetrically graded with respect to the middle surface, the thermoelastic constitutive equations assume the form analogous to that for a laminate composed of asymmetric isotropic layers. These relationships represent the vectors of stress resultants and stress couples in the form

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A}(T) & \mathbf{B}(T) \\ \mathbf{B}(T) & \mathbf{D}(T) \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} - \begin{Bmatrix} \mathbf{N}^T \\ \mathbf{M}^T \end{Bmatrix} \tag{2}$$

where A, B, and D are the matrices of extensional, coupling, and bending stiffnesses, respectively. These matrices reflect the effect of temperature on the material properties that should be evaluated as an outcome of the solution of the thermal problem accounting for the local volume fraction distribution and the effects of temperature T on the properties of the constituent materials. Furthermore, the vectors $\mathbf{N}^T$ and $\mathbf{M}^T$ are explicit thermally induced contributions to stress resultants and stress couples calculated accounting both for the temperature distribution throughout the structure and for thermal effects on material properties. The middle surface strain vector and the vector of changes of curvature and twist are denoted by ε^0 and κ. In quasi-isotropic FGMs, the elements of the stiffness matrices (in standard notation) $A_{16} = A_{26} = B_{16} = B_{26} = D_{16} = D_{26} = 0$.

The peculiarity of the constitutive equations (2) in the case of FGM materials subject to thermal loading is reflected in the definition of the stiffness and thermal terms in these equations:

$$\{\mathbf{A}(T),\mathbf{B}(T),\mathbf{D}(T)\} = \int_z \mathbf{Q}[T(\bar{\mathbf{r}}),\bar{\mathbf{r}}]\{1,z,z^2\}dz$$

$$\{\mathbf{N}^T,\mathbf{M}^T\} = \int_z \mathbf{Q}[T(\bar{\mathbf{r}}),\bar{\mathbf{r}}]\bar{\boldsymbol{\alpha}}[T(\bar{\mathbf{r}}),\bar{\mathbf{r}}]\{1,z\}dz \tag{3}$$

where the integration is conducted throughout the thickness of the structure (z), $\mathbf{Q}$ is a matrix of reduced stiffnesses, $\bar{\boldsymbol{\alpha}}$ is a vector of coefficients of thermal expansion, and $\bar{\mathbf{r}}$ is a position vector of the point within the domain occupied by the structure. As is reflected in these equations, the stiffness and thermal terms are affected by the following factors:

(1) solution of the heat transfer problem accounting for local properties and the effect of temperature on these properties
(2) location within the structure, accounting for the local mechanical properties and the coefficients of thermal expansion that are in turn affected both by the grading law and by the effect of local temperature.

The solution of the problem should ideally be coupled, incorporating micromechanics, heat transfer, and mechanical stress analysis into a unified formulation.

Note that the presence of nonzero coupling terms B_{11}, $B_{12} = B_{21}$, B_{22}, and B_{66} implies the presence of curvature and twist in a plate subject to compressive loading, even if thermal effects are absent. This is reflected in prebuckling deflections in the plate subjected to mechanical compression. However, in the absence of thermal effects, classical bifurcation buckling still occurs as is evident from the analysis of the variational equations of equilibrium and boundary conditions that are homogeneous with respect to displacements.

Thermal postbuckling of a thin FGM cylindrical shell with temperature-dependent properties subjected to a uniform temperature was considered by Shen using the nonlinear von Karman–Donnell theory [106] and accounting for possible initial imperfections. The end cross sections of the shell could be either clamped

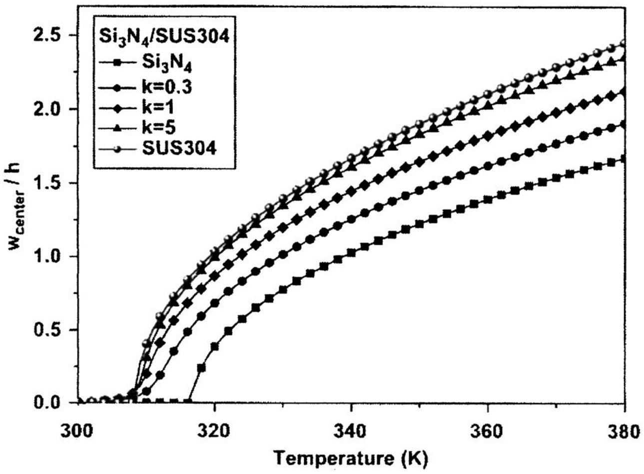

Fig. 8 Thermal postbuckling behavior of a Si_3N_4/SUS304 plate. The volume fraction index for the ceramic phase is denoted by k [110]

or simply supported, and their longitudinal displacements were prevented. The nonlinear prebuckling deformations were determined and subsequently, the perturbation technique was applied to evaluate the postbuckling behavior. The recent paper by Shen and Noda presents a postbuckling analysis of cylindrical shells subject to combined thermal and mechanical loads [107]. The solution is obtained using a higher-order shear deformation theory combined with the von Karman–Donnell geometrically nonlinear approach. The effect of temperature on material properties, possible initial imperfections, and nonlinear prebuckling deformations are incorporated in the analysis. Thermal buckling of shear-deformable FGM cylindrical shells was also considered using the first-order theory by Shahsiah and Eslami [108] and by Lanhe [109]. A recent paper by Park and Kim [110] contains a finite element analysis of thermal postbuckling behavior and free vibrations of geometrically nonlinear FGM plates. While geometric nonlinearity is accounted for through the von Karman theory, the first-order theory enables the authors to incorporate transverse shear deformations in the analysis. An example of a thermal postbuckling response of a simply supported FGM plate is shown in Fig. 8. The postbuckling behavior of FGM plates and shallow shells accounting for transverse shear deformations was also conducted in a recent paper [111] where the boundary conditions were shown to significantly affect the response. Both buckling and free vibrations of clamped FGM cylindrical shells subjected to a prescribed elevated temperature at the inner surface have been considered by Kadoli and Ganesan using the first-order shear deformation theory and accounting for the effect of temperature on material properties [112]. Geometrically nonlinear problems of bending and buckling of circular FGM plates have been considered for both simply supported and clamped boundaries [113] where the governing equations were solved numerically.

Both thermoelastic buckling and free vibrations of clamped truncated conical FGM shells subjected to a high temperature at the inner surface have recently been studied in Ref. [114]. The finite element solution was developed based on a first-order shear deformation theory and accounting for the effect of temperature on material properties. The FGM material was graded in the thickness direction according to the power-law volume fraction distribution.

A recent paper [115] presents an analysis of postbuckling of FGM cylindrical panels, accounting for their temperature-dependent properties. The panels considered in the paper were either clamped or simply supported. The material was graded in the thickness direction according to a power-law volume fraction distribution of the constituent phases. Geometric nonlinearity was accounted for through the von Karman–Donnell model. The loading scenario assumed an initial axial loading, followed with a uniform temperature increase.

The effect of temperature on the material properties was emphasized in the paper [116] concerned with thermal buckling and postbuckling of FGM plates subject to a uniform temperature. The analysis employed the first-order shear deformation theory and von Karman's geometrically nonlinear formulation. Both the buckling temperatures and postbuckling deformations were significantly overestimated by neglecting temperature dependence of the material properties. As a result, the authors concluded that such simplification may lead to an unsafe design.

The analytical solution of the problem of guided acoustic wave propagation in FGM plates that may be useful in nondestructive testing applications was published by Lefebvre et al. [117]. The problem of wave propagation in a FGM structure was also analyzed using a numerical technique based on confluent hypergeometric functions that was developed in Ref. [118]. Numerical solutions for two-dimensional problems of wave propagation in layered metal-ceramic FGM and in FGM with randomly distributed ceramic particles were presented in Ref. [119]. Problems of stress wave propagation in FGM cylindrical shells subject to radial line loads and to point loads were also considered [120,121].

The effect of initial imperfections on nonlinear vibrations of shear-deformable FGM plates consisting of the central homogeneous layer sandwiched between two FGM layers was considered in Ref. [122] using the third-order Reddy shear-deformable theory. Several boundary conditions were considered in this paper. In all cases, except for the case of plates with a pair of opposite free edges and large imperfection amplitude, the frequency-amplitude characteristic was hardening.

Dynamic response of a FGM initially prestressed rectangular plate subjected to an impulsive lateral load as well as free vibrations of such plate were considered in Ref. [123] for the cases where two opposite edges were clamped, while two other edges could be clamped, simply supported, or elastically clamped. It should be noted that in-plane static boundary conditions implying that the edges are free to move in the direction perpendicular to their axes were satisfied only in the integral sense. Supersonic flutter of flat FGM plates operating in a thermal environment was numerically investigated by Prakash and Ganapathi [124]. The analysis was conducted using a first-order shear deformation theory and accounting for the effect of temperature on material properties. Free vibrations and static deformations of magneto-electroelastic FGM plates exponentially graded in the thickness direction have recently been considered accounting for coupling between magnetic, elastic, and electric three-dimensional effects [125,126]. Random free vibrations of FGM plates modeled by a third-order shear deformation theory were also considered [127], accounting for an elevated temperature. The random nature of such material properties as the modulus of elasticity, the Poisson ratio and the thermal expansion coefficient have been reflected in this analysis.

The problem of a low-velocity impact of a simply supported cylindrical shell was solved in Ref. [128]. Dynamic stability of a FGM cylindrical shell subjected to a periodic-in-time axial load was analyzed in Ref. [129] where the problem was reduced to a system of Mathieu–Hill equations and it was shown that dynamic stability could be controlled through a choice of the volume fractions of the constituent phases. Dynamic stability of FGM truncated conical shells subjected to a time-dependent (power function of time) uniform external pressure was analyzed by Sofiyev [130] who illustrated that stability can be improved by an appropriate grading of the material through the thickness. In a recent paper, Tylikowski considered dynamic stability of functionally graded plates subject to time-dependent in-plane loads and accounting for geometrically nonlinear effects [131]. Dynamic stability of laminated FGM plates was also considered in Ref. [132] using a higher-order shear-deformable theory.

The problem of static indentation of a sandwich panel with a functionally graded core was considered by Anderson [133] using a three-dimensional theory of elasticity formulation. The layers of the panel were assumed orthotropic, while the core was isotropic with the elastic modulus exponentially graded in the thickness direction. Instead of the classical Hertz's approach to the evaluation of the contact pressure, the author used the requirement that the shape of the indenter must conform to the shape of the indented surface. The advantages of functional grading in the core of the panel were not explicit in the numerical examples presented in the paper.

The system of fundamental thermoelastic equations describing a three-dimensional distribution of temperature and elastic stresses in a FGM semi-infinite solid was derived in Ref. [134]. In this paper the material constants, such as the shear modulus, the coefficient of thermal expansion, and thermal conductivity, were assumed in the form of power products of the coordinate perpendicular to the surface.

Notably, the early studies of thermally loaded FGM structures usually considered only position-dependent properties neglecting the effect of temperature on the material. In later investigations, such as Refs. [98,99], the effect of the temperature on the material properties was incorporated in the analysis. The problem becomes even more complicated if thermal and mechanical aspects are coupled accounting for the effect of grading on heat transfer (see, for example, Ref. [100]). This is the case where both the properties and temperature are dependent on the position within the domain occupied by the body and grading affects both thermal and mechanical formulations. Obviously, the latter approach provides the most accurate solution.

Creep is a factor that may affect the behavior of FGM structures exposed to high temperature for long periods of time, such as thermal barrier coatings (TBCs). Pindera et al. investigated the effect of creep on the spallation under thermal loading in functionally graded TBC using the higher-order theory [135]. Numerical studies conducted by a finite element method accounting for creep in metal constituents of ceramic-metal TBC indicated that this phenomenon cannot be neglected [136]. As was shown by Zhai et al., creep can affect even ceramic-rich interlayers of FGM [137]. Creep in rotating metal-ceramic disks was also studied by Singh and Ray [138].

1.4 Optimization of Functionally Graded Material. The problems of optimization of FGM are natural for this class of multiphase materials that have been developed to enhance the properties and response of structures. Design parameters used in the optimization of FGM are usually related to an appropriate grading of the material. The objective functions vary dependent on the task and problem considered, but in general, they may include weight, maximum stresses or improved fracture resistance, and requirements to heat transfer and insulation. While the studies of optimization are important for the development of future materials, technological limitations may undercut the applicability of some of the conclusions from these studies.

Wang et al. [139] considered the optimization of thermal stresses in an infinite plate with a FGM coating subjected to a steady heat flux, while cooled on the opposite surface. The design variables considered in this paper were the thickness and the volume fraction composition of homogeneous FGM interlayers. The optimization of the properties of a transversely isotropic FGM layer that is inhomogeneous in the thickness direction was considered in Ref. [140]. Transient and steady-state thermal stresses in a ceramic-metal FGM were also optimized in Ref. [141] where the gradation was represented by a two-parameter curve with the coefficients serving as design variables. Cho and Shin employed a backpropagation artificial neural network to achieve an optimum material composition in a three-layered plate consisting of ceramic and metal layers and a FGM layer sandwiched between them [142].

Lipton suggested a numerical method that could yield a maximum torsional stiffness of a FGM shaft subject to a constraint on the mean square stress [143]. This paper analyzed a rather theoretical case where the radii of long fibers are varied to achieve the design requirement. Cho and Ha used a finite element method to optimize the distribution of constituent volume fractions in FGM with the purpose of reducing steady-state thermal stresses [144]. Numerical examples were presented for $Ni-Al_2O_3$ FGM with the properties assumed independent of temperature that was reduced from 1000 K to 300 K. In their other paper, the same authors illustrated the optimization approach to the minimization of thermal stresses in FGM based on the interior penalty-function and golden section methods [145]. The problem was formulated for a three-layered structure consisting of purely metallic (Ni) and ceramic (Al_2O_3) layers separated by the graded layer where the volume fraction distribution of constituent phases was optimized. The effect of the failure criterion, boundary conditions, and a distribution of layers in a ceramic-titanium FGM plate subjected to an elevated temperature on the surface of the ceramic layer was considered in Ref. [146] where the thickness of the ceramic layer was shown to be a dominant factor in preventing the failure of titanium.

The objective function considered in Ref. [147] represented a linear combination of the total strain energy and the peak effective stress normalized with respect to the spatial-varying yield stress. A ceramic-metal FGM considered in the solution included a ceramic and metal regions separated by a graded region. The yield stress of the graded region was a spatial function dependent on the volume fraction of constituent phases.

Cho and Park considered a mesh optimization for finite element studies of FGM aiming at an optimum material tailoring [148]. They proposed using a higher mesh density in the regions that require a larger flexibility in volume fraction variations.

Turteltaub [149] formulated the temperature optimization problem as finding a distribution of material properties (volume fraction of the constituent phases) that minimizes the difference between the target and actual temperature distributions. The minimization was conducted by the least-squares sense in the space-time space by assumption that material properties remain unaffected by temperature. The other paper by the same author illustrated that the loading history is important in optimization problems, even if the material remains in the elastic range [150]. The subject of an optimum material distribution in FGM coatings subject to thermal loading was considered in Ref. [151].

Qian and Batra considered the problem of optimization of a bidirectional FGM plate maximizing its natural frequencies [152]. In particular, it was shown that material grading in the axial direction is sufficient to achieve a maximum fundamental frequency of a plate clamped along one edge. A new approach to functional grading has been introduced by Batra and Jin who suggested to smoothly vary the fiber orientation through the thickness of an anisotropic plate [153]. In this paper, the first five natural frequencies were successfully maximized for plates of various boundary conditions by grading the fiber orientation.

While FGM may serve as an excellent optimization and material tailoring tool, the ability to incorporate optimization techniques and solutions in practical design depend on the capacity to manufacture these materials to required specifications. Conventional techniques are often incapable of adequately addressing this issue. As an example, consider Fig. 9 [154] that shows a designed and actual porosity gradient in a SiC fuel evaporated tube used in a gas turbine combustor where porous surface facilitates the evaporation rate. Both this and numerous other examples testify to the fact that even in a closely controlled manufacturing process, it may be difficult to achieve a designed grading in FGM.

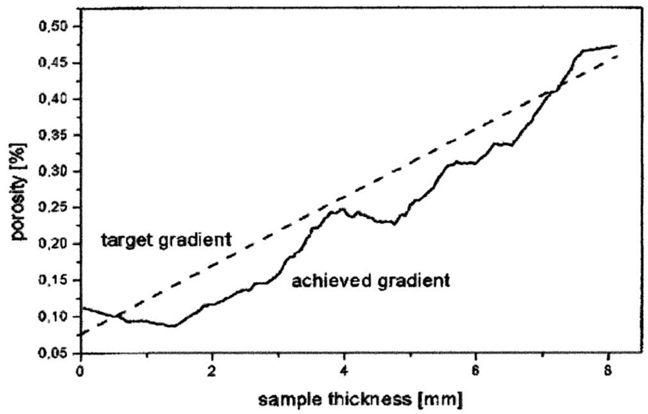

Fig. 9 Porosity gradient in a sintered ceramic sample produced by pressure filtration [154]

1.5 Manufacturing, Design, and Modeling Issues. A number of manufacturing techniques have been proposed for FGM, including electrophoretic deposition [155,156], chemical vapor deposition [157], spark plasma sintering [158,159], and centrifugal casting [160,161]. The electric field was combined with pressure to produce layered functionally graded $MoSi_2-SiC$ with robust interfaces that may be attributed to the specifics of the process characterized by a simultaneous synthesis and densification [162]. A relatively recent review of FGM manufacturing methods was published in 2003 [154]. As emphasized in this review, different grading approaches can be adopted dependent on the intended applications, including (a) porosity gradients, (b) chemical composition gradient of single-phase materials, and (c) volume fraction gradients of constitutive material phases. Table 1 outlines characteristics of conventional processing methods that are mostly based on the above-mentioned technologies.

While the methods listed above are usually used to manufacture FGM with the properties varying in the thickness direction, the in-plane variation of properties can also be achieved using ultraviolet irradiation [163].

A new method based on a differential heat treatment technique has recently been proposed [164]. Contrary to previously developed methods where the composition gradients are produced over the length corresponding to the required property gradients, this new method employs a differential temperature aging leading to a gradient in the microstructure and the corresponding property variations. Semisolid processing of intermetallic FGM has also been studied [165,166]. Laser treatment of the surface was used to produce functionally graded $TiC-Ni_3Al$ composites [167] and pulse laser deposition was employed to apply thin FGM films [168]. Electrochemical gradation of porous tungsten that was subsequently infiltrated with copper was applied to manufacturing W–Cu FGM [169]. Models of the pressureless sintering of ceramic-metal FGM have recently been developed by Pines and Bruck [170,171].

Examples of computer-aided design methods applicable to FGM can be found in the papers of Chen and Feng [172,173]. As emphasized in the latter paper, contrary to design of a homogeneous structure where the properties are known in advance, design of a FGM structure should include three steps:

a. configuration design
b. specifying required material properties in each region of structure
c. selection of material microstructure or constituent compositions for each region

A relationship between form and material features in heterogeneous objects including FGM was considered by Qian and Dutta who illustrated design techniques using physics-based B-spline object homogeneity modeling and direct face neighborhood alteration for constructive feature operations [174,175]. Siu and Tan introduced a source-based modeling technique for heterogeneous (FGM) objects where modifications of the material grading do not affect geometry of the object, so that such modifications can be

Table 1 FGM processing methods (Ref. 154). Overview of processing methods for FGMS

Process	Variability of transition function	Layer thickness[a]	Versatility in phase content	Type of FGM	Versatility in component geometry
Powder stacking	Very good	M, L	Very good	Bulk	Moderate
Sheet lamination	Very good	T, M[b]	Very good	Bulk	Moderate
Wet powder spraying	Very good	UT, T[b]	Very good	Bulk	Moderate
Slurry dipping	Very good	UT, T[b]	Very good	Coating	Good
Jet solidification	Very good	M, L	Very good	Bulk	Very good
Sedimentation/centrifuging	Good	C	Very good	Bulk	Poor
Filtration/slip casting	Very good	C	Very good	Bulk[c]	Good
Laser cladding	Very good	M	Very good	Bulk coating	Very good
Thermal spraying	Very good	T	Very good	Coating bulk	Good
Diffusion	Moderate	C	Very good	Joint, coating	Good
Directed solidification	Moderate	C	Moderate	Bulk	Poor
Electrochemical gradation	Moderate	C	Good	Bulk[c]	Good
Foaming of polymers	Moderate	C	Good	Bulk	Good
PVD, CVD	Very good	C	Very good	Coating	Moderate
GMFC process	Very good	M, L, C	Moderate	Bulk	Good

[a]L: large (>1 mm): M: medium (100–1000 μm): T: thin (10–100 μm). UT: very thin (<10 μm): C: continuous.
[b]Depending on available powder size.
[c]Maximum thickness is limited.

conveniently applied in design [176]. Several architectures of dynamically loaded FGM formed from Al 6061-TO and ceramic TiC were compared based on the effective stress considerations in Ref. [177].

Grinding was shown to be an effective method to achieve a desirable redistribution of residual stresses in alumina-based ceramic FGM disks [178]. As was shown in this paper, machining one surface of an Al_2O_3–ZrO_2 disk removing a 200 μm layer of the material could increase compressive stresses to 170 MPa while reducing maximum tensile stress in the "core" of the disk to 70 MPa. The approach to the turbine blade design enabling the engineer to control the process by specifying material variation constraints was considered by Qian and Dutta [179].

A design method combined with the optimization procedure is so-called topology optimization that has recently been applied to FGM objects. This method involves an application of a finite element or another numerical method combined with an optimization algorithm capable of specifying the material distribution within a domain subject to constraints, such as a constraint on the maximum stresses. In particular, a design of a rotating FGM disk maximizing its rotatory inertia was considered in Ref. [180]. A method of a material design in a rotating FGM disk subjected to a thermal load simulating the braking effect was suggested in Ref. [181] where the nonhomogeneous mechanical and thermal properties varied in the radial direction. The desired outcome of the design process included reduced weight, high-temperature radiation, and a relaxation of thermal stresses and centrifugal force.

The effect of residual thermal stresses on the properties of ceramic-metal FGM processed at high temperature may noticeably affect the subsequent behavior of the material. In particular, the effect of such stresses on the response of a particulate FGM subject to postprocessing cooling and subsequent heating above the room temperature was analyzed by Shabana and Noda [182]. This finite element analysis accounted both for the effect of temperature on the properties of constituent phases and for the plastic effects in the metallic matrix. An example of the results generated in this paper is shown for the plate composed of aluminum matrix with partially stabilized zirconium particles in Fig. 10 where the maximum tensile stress in the matrix is depicted as a function of the particle volume fraction and the regime, i.e., heating versus heating after cooling. The room and fabrication temperatures considered in this example were 300 K and 800 K, respectively. The particle distribution in a plate of thickness s was assumed to follow the law $f_p(y/s)=f_{p0}y^m/s^m$, where y is a coordinate counted from the surface. While cooling was assumed to result in a uniform temperature throughout the plate, the subsequent heating was applied at the surface $y=0$, while the opposite surface was kept at the room temperature. As is reflected in this figure, neglecting the history of the structure (i.e., neglecting the postprocessing cooling) may seriously affect the estimate of the stresses. Naturally, the particle volume fraction and its distribution (parameter m) are also important factors affecting the stress within FGM.

The issue of functionally graded piezoelectric actuators has also attracted significant attention. The advantages of functionally graded actuators are related to a possible reduction of discontinuity of stresses along the interface between dissimilar material layers that results in a premature delamination in conventional actuators. This discontinuity can be reduced and the lifetime of a piezoelectric actuator extended using functional grading techniques. The extensive work on the development of functionally graded piezoelectric actuators elucidating various aspects of their manufacturing, modeling, and design has been conducted during the last decade. Recent work in this area dealing with grading of one of the critical properties, such as conductivity, piezoelectric coefficient, permittivity, or porosity, is found in Refs. [183–188]. In particular, as was shown by Alexander and Brei [185], it is possible to reduce the interface stress by 68%, at the expense of a change in the tip displacement of only 23% in a cantilever piezoelectric beam by using material grading in the thickness direction. The subject of manufacturing, modeling, and design of piezoceramic functionally graded actuators using so-called dual electropiezo property gradient technique that emphasizes variations of both

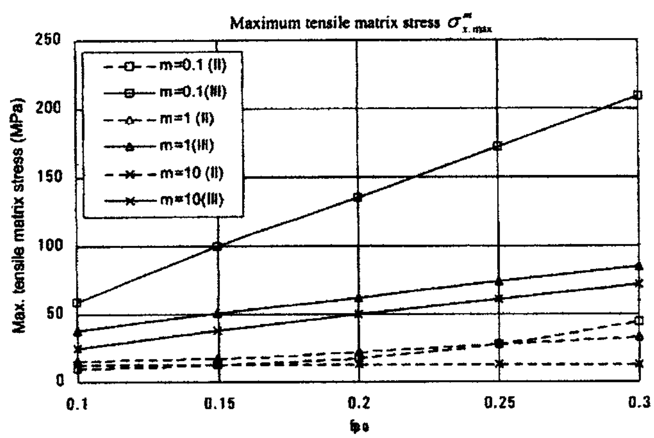

Fig. 10 The maximum tensile stress in the matrix as a function of the thermal regime and the particle volume fraction [182]. II =heating process; III=heating after cooling process.

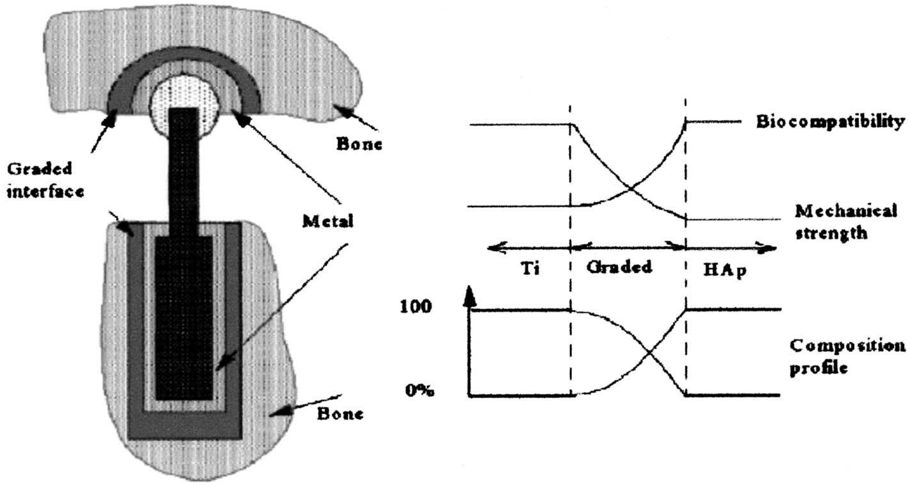

Fig. 11 Schematics of a FGM interface within a prosthesis [200]

piezoelectric coefficients and the electric permittivity was considered in Refs. [189,190]. Further work considering mechanics of functionally graded piezoelectric actuators is discussed in detail in the section on special problems below.

1.6 Testing. FGMs present a challenge to the engineer due to the necessity to accurately determine the distribution of constituent phases and/or properties throughout the material. A number of methods have been applied to solve this problem and to detect internal defects. Some of these methods that have recently been reported in FGM studies are referred to below.

The use of optical mapping to measure elastic modulus grading and to detect crack tip deformations and fracture parameters in glass-particle, epoxy-matrix composite was successfully used by Butcher et al. [191]. The holographic modification of the X-ray computer microtomography method was applied to obtain contrasted images of metal-matrix FGM used in automotive industry in Ref. [192] yielding accurate information about particle sizes and clusters. It was found that while a smaller size of particles results in a tendency to clustering, the mean size of the particle within the cluster affects the size of the cluster, i.e., larger particles correspond to larger cluster sizes. FGM glass/alumina coatings were characterized using the scanning electron microscopy and X-ray diffraction analysis [193]. X-ray diffraction analysis was also employed to determine residual stresses in FGM [194]. Thermal residual stresses within Al_2O_3/ZrO_2 FGM generated due to thermal mismatch of alumina and zirconia phases were accurately predicted by the Raman spectroscopy and found in excellent agreement with the theoretically predicted values [195].

Both four-point bending tests and acoustic emission analyses were applied to fracture studies of FGM TBCs [196]. Acoustic emissions were also used to detect damage due to thermal cycling of Cu/Al_2O_3 FGM considered as a potential heat sink material [197]. A cyclic surface heating by laser irradiation was used to test damage resistance of graded TBCs [198]. The properties of ceramic-metal FGM were also successfully predicted from ultrasonic measurements [199].

It is also necessary to recall here the previously mentioned neural network training and property validation based on measuring elastic surface waves [26,27] and the indentation testing [28,29].

1.7 Special Problems and Examples of Applications of Functionally Graded Material Structures. Potential applications of FGM are both diverse and numerous. Some examples of applications of FGM that have recently been reported include the following:

1. functionally graded prosthesis joint increasing adhesive strength and reducing pain, Fig. 11 [200]
2. functionally graded polyester-calcium phosphate materials for bone replacement with a controllable in vitro polyester degradation rate [201]
3. functionally graded TBCs for combustion chambers [202]
4. functionally graded layer between the Cr–MO shank and ceramic tip of a cutting tool improving the thermal strength, Fig. 12 [203]
5. functionally graded piezoelectric actuators [204]
6. FGM metal/ceramic armor [205]
7. functionally graded thermal protection systems for spacecraft, hypersonic, and supersonic planes [206]
8. functionally graded heated floor systems [207]

FGMs also find application as furnace liners and thermal shielding elements in microelectronics.

Instead of trying to outline and discuss all applications of FGM, we concentrate on three representative problems that have attracted close attention of investigators. The diversity of potential applications of FGM is reflected in these problems dealing with rotating FGM blades, smart FGM, and FGM TBCs.

1.7.1 Functionally Graded Material Blades. The problems of modeling and vibrations of FGM thin walled rotating blades that could be used in helicopters and turbomachinery applications and the associated subject of spinning circular cylindrical beams were considered by Librescu, and Oh and Song [208–212]. In these studies, the blade was modeled as a pretwisted thin-walled shear-

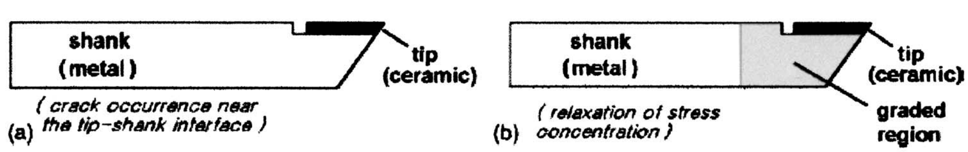

Fig. 12 Lathe metal cutting bites: (*a*) conventional bimaterial type; (*b*) FGM design [203]

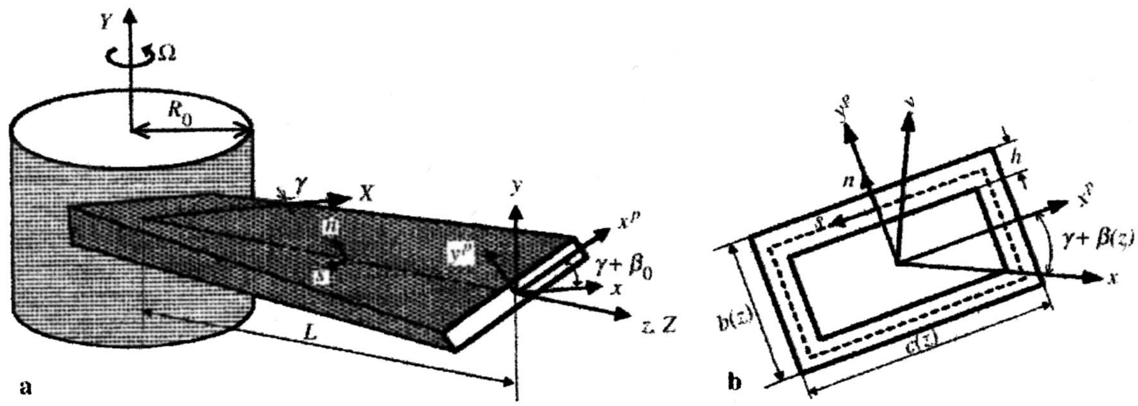

Fig. 13 Configuration of a tapered pretwisted turbine blade (*a*) and the cross section used in the analysis (*b*) [209]

deformable beam of a nonuniform cross section with material properties varying in the thickness direction according to a power law (Fig. 13). The thermal field was assumed steady state, and the rotation velocity was constant. The effects of material grading and blade taper ratios on the natural frequencies were elucidated in Refs. [208,209]. Besides free vibrations, the divergence and flutter instability of the blades, accounting for gyroscopic forces, were analyzed in Ref. [210] where both the natural frequencies and the spinning speed corresponding to the blade instability were significantly increased by appropriate variations in grading. The work on vibrations and spinning stability of blades was extended to cylindrical shafts (beams) in Ref. [211], while a comprehensive outline of the effort relevant to both rotating blades and spinning circular cylindrical beams was presented in Ref. [212].

1.7.2 Smart Functionally Graded Material. Using the FGM concept in smart structures represents another attractive potential application area. In particular, it is interesting to explore piezoelectric FGM structures where the piezoelectric effect could be used to suppress the dynamic or static response, while grading may result in reduced stresses and deformations. Numerous studies addressing various aspects of such problem have been published in 2000–2002. The transient response of a structure including FGM and piezoelectric layers was analyzed in Ref. [213]. The exact solution for stresses in a FGM piezoelectric plate subject to a combination of in-plane, bending, and twisting loads was presented by Lim and He [214]. Wang and Noda showed that local edge stresses and the stress discontinuity can be reduced in a FGM beam subjected to a uniform elevated temperature applied at the metallic surface and consisting of a metal layer and a piezoelectric actuator layer by introducing a functionally graded interlayer [215]. Active control of linear vibrations in a FGM plate with piezoelectric elements in the presence of temperature was considered using a first-order shear deformation theory in Ref. [216]. Other papers elucidating problems of free vibrations and stress analysis are Refs. [217–219]. Reddy and Cheng employed the transfer matrix and the asymptotic expansion method to find the three-dimensional response of FGM piezoelectric plates subjected to thermal or mechanical loading at the ceramic-rich surface [220]. In this paper, piezoelectric actuators bonded to the metal-rich surface were employed to suppress the response. A higher-order shear-deformable formulation was developed for FGM piezoelectric shells in Ref. [221].

The three-dimensional analysis of a functionally graded piezoelectric plate that is simply supported along all edges and subject to mechanical and electric loading was published by Zhong and Shang [222] who assumed that both mechanical and electric properties vary through the thickness according to the same exponential law. Stresses and displacements in a functionally graded bimorph were evaluated by the classical lamination theory and a finite element method and compared to a standard bimorph in Ref. [223]. An axisymmetric dynamic problem for a pyroelectric spherical functionally graded shell of an arbitrary thickness with properties varying proportionally to a power of the radial coordinate was solved in Ref. [224]. A response of FGM piezoelectric plates consisting of piezoelectric, graded, and substrate layers to electric or thermal actuation was analyzed accounting for the effect of temperature on the properties of the piezoelectric material in Ref. [225]. Stroh's formalism has recently been applied to a 3D analysis of a functionally graded simply supported piezoelectric laminated plate [226].

Geometrically nonlinear and linear vibrations of shear-deformable FGM plates consisting of a graded layer sandwiched between two piezoelectric facings used as actuators were studied in Ref. [227]. The formulation of the problem was based on the Reddy higher-order shear deformation theory, accounting for rotary inertia and transverse shear effects. A uniformly distributed temperature was included in the analysis, though its effect on the properties of constituent materials, including the piezoelectric properties that can be quite significant, was omitted. The solution was obtained for various boundary conditions and the effects of grading of the inner layer, geometry, and applied voltage on dynamics of a plate were illustrated. The same authors used their formulation in the previous paper to solve the problem of buckling and postbuckling behavior of three-layered plates with outer piezoelectric and inner FGM layers in Ref. [228]. The authors emphasized that the classical buckling phenomenon does not occur in such plates with bending-stretching coupling since they behave similarly to an imperfect column, i.e., deflections develop even if a small load is applied. An exception was the case of clamped plates where reactive moments at the edges neutralized prebuckling stress couples. Additionally, the problem of nonlinear bending of a FGM plate with piezoelectric actuator layers was solved using a similar formulation [229]. Both free and forced nonlinear vibrations of FGM plates designed with the inner FGM layer and outer piezoelectric layers were considered by Huang and Shen [230] using a higher-order shear deformation theory and von Karman's geometrically nonlinear formulation, while satisfying the in-plane boundary conditions in the integral sense. A higher volume fraction index was shown to result in smaller natural frequencies as could be expected due to an increased asymmetry of the structure. A higher temperature also resulted in lower natural frequencies.

In other recently published papers on smart FGM structures, postbuckling of FGM plates consisting of piezoelectric actuator layers and FGM ceramic-metal layers and subject to a combination of thermal, mechanical, and electric loads was also considered by Shen [231] using a higher-order shear deformation theory. The same author also analyzed postbuckling of a FGM cylindrical shell subject to an axial compression and a variable in the radial direction temperature [232]. The shear-deformable shell analyzed by a geometrically nonlinear von Karman–Donnell theory consisted of a FGM layer sandwiched between two piezoelectric layers. It is noted that while the previous two papers have provided a very detailed analysis, the effect of temperature on piezoelectric properties was disregarded. Various models modeling the response of hybrid layered piezoelectric beams have been considered in Ref. [233]. The analysis methodologies compared in this paper include the zigzag method, and third-order and first-order models. A solution for a FGM plate with an actuator layer of a piezoelectric composite bonded to one of the surfaces was recently published by Ray and Sachade [234] who predictably concluded that the actuator is more effective if it is bonded to a less rigid surface of the structure. While the papers referred in this section dealt with the issues of stress analysis and response of functionally graded actuators, the problems of design and manufacturing of these actuators have been considered in the papers listed in the corresponding section of this review.

The research on smart FGM using other than piezoelectric elements has been limited. Besides above-mentioned papers of Birman [10,11], Miyazaki and Watanabe considered SMA fiber FGM [235].

1.7.3 Functionally Graded Material Thermal Barrier Coatings. TBCs are typically used in applications where it becomes necessary to protect metallic or composite components in aircraft engines and power generators from excessive temperature (composite components of engines that have to be protected operate under less than 320°C and at air pressure of 0.4 MPa [236]). The problems that have to be addressed to design TBC include processing technology, heat transfer during and immediately after the processing, microstructure formation, residual thermal stresses, micromechanics of TBC, and thermomechanical response during the lifetime. FGM TBCs are attractive due to the potential for a reduction in thermal stresses, avoiding delamination and spallation tendencies and prevention of oxidation.

Among the issues relevant to FGM TBC, the heat transfer problem is simplified due to the fact that it is typically one-dimensional, occurring in the thickness direction. For example, if a FGM coating consists of a number of layers with constant volume fractions of individual components, the total thermal resistance can be evaluated as the sum of individual layer resistances [237]. It was also shown that viscoplastic effects often have to be accounted for in the stress and fracture analyses. When heat flux is applied to the surface of TBC, the stresses in the vicinity to the heated surface are usually compressive due to the constraint applied by the colder substrate. Accordingly, preheating of the substrate may be beneficial in certain situations.

An example of work on improved processing techniques for FGM TBS is the paper in Ref. [238] exploring the procedure where a FGM coating is produced by a combination of the detonation gun spray process and a new "shot control" method. The coating was sprayed as a multilayered system with a compositional gradient varying from pure metal adjacent to the substrate to a ceramic exposed layer. According to the authors, obvious interfaces between the layers of the coating could be eliminated using their technology.

The choice of an optimum manufacturing regime for plasma-sprayed zirconia/alumina FGM TBC was considered in Ref. [239]. The solution included the analysis of temperature and thermal stress distribution during cooling from the deposition to room temperature. It was illustrated that a lower cooling rate and preheating of the substrate can reduce residual stresses in a FGM coating. An additional reduction of stresses was achieved by introducing an Al_2O_3 interlayer. The stresses determined by finite element analysis (FEA) were found in a good agreement with experimental values. The same authors investigated beneficial effects of the Al_2O_3 interlayer on the oxidation resistance of TBC as a result of its low oxygen diffusivity [240,241]. Unfortunately, such interlayer has a relatively low fracture toughness that may facilitate interfacial fracture, so that the benefits and shortcomings of its application have to be more thoroughly investigated. Analytical studies of thermal residual stresses in FGM TBC have also recently been reported by Zhang et al. [242] who emphasized the benefits of a reduced mismatch between the coefficients of thermal expansion between the substrate and the adjacent coating on these stresses.

Other failure mechanisms in FGM TBC that have been investigated include spallation thoroughly studied by Pindera et al. [243,244] using the higher-order theory developed by the authors of these papers, including creep and relaxation effects in the constituent phases of TBC. As indicated by the authors, spallation is mainly due to a mismatch of the thermal expansion coefficients between the substrate and coating that could be reduced through an appropriate grading.

Thermal fatigue may occur in TBC subject to periodic temperature variations during their lifetime. As was shown by Khor and Gu [245], using a FGM coating may result in a five-time increase in the resistance to thermal fatigue compared to a conventional counterpart. Additionally, the oxidation resistance was improved as a result of grading in the results reported in this paper. It is interesting to note a scenario of damage to TBC subject to thermal cycling that involves the phases of vertical cracking, delamination, blistering, and spallation that follow each other in a sequence [246]. Detailed discussion of fracture in FGM TBC is outside the scope of this review, but relevant references and their brief description are included in the next section. Other damage modes in FGM TBC that have recently been studied include wear [247] and solid particle erosion.

1.8 Fracture Problems in Functionally Graded Material. The problems of fracture and fatigue of FGM are so important and they have been studied in depth to such extent that a separate review is needed to comprehensively address the state of art and to specify the most promising directions and critical needs in this area. Therefore, a discussion on these problems is mostly omitted from this review. Several recent papers are briefly discussed below to illustrate the variety and complexity of fracture problems. Other recent publications are listed in Table 2 where each paper is supplied with a one-sentence comment on its major emphasis.

The paper by Kim and Paulino [248] can serve as an example of a *finite element solution of fracture problems in FGM*. In this paper, the propagation of mixed-mode cracks in FGM was studied by a finite element approach combined with a remeshing algorithm reflecting changes in the path of the crack. The stress intensity factors were determined using the interaction integral method previously developed by the same authors. These factors were employed in a fracture criterion to specify the direction of crack propagation and the mesh used in the finite element method was automatically adjusted reflecting the new trajectory of the crack. The propagation of cracks in a quasibrittle FGM was considered by Comi and Mariani using FEA [249]. The stabilization of the crack propagating into an increasingly tougher material as well as a delay in the onset of initial cracking in tougher zones were reflected in numerical results shown in the paper.

The effect of *residual thermal stresses* on fracture has attracted a significant attention. This effect on the stress intensity factors for cracks developing from the surface of stepwise functionally graded alumina/zirconia materials was considered by Vena et al. [250].

Various aspects of *fatigue in FGM* are important in almost all applications involving these materials. For example, the subcritical crack growth as a result of mechanical or thermal cycling loading applied to a FGM coating bonded to a homogeneous substrate was considered using a three-dimensional finite element method in Ref. [251].

Transient and dynamic fractures have been intensively studied in recent years. An experimental and numerical investigation of functionally graded yttria stabilized zirconia bond coat alloy

Table 2 Representative papers on fracture and fatigue of FGM since 2000

Reference	Major emphasis (some of the papers referred to below could be identified with more than one area but such duplication is avoided for simplicity)
	T-stress effect in FGM
[255]	T stresses are evaluated in mixed-mode fracture mode by the interaction integral
[256]	The direction of the in-plane mixed-mode crack in FGM and the effect of T stresses are considered
	The interaction integral method in the analysis of fracture in FGM
[257]	The interaction integral is employed to evaluate T stresses in a mixed-mode fracture problem for an orthotropic FGM with straight or curved cracks
[258]	The interaction integral used to analyze a broad variety of crack problems in FGM
[259]	Mixed-mode stress intensity factors for arbitrary oriented cracks in FGM are found by the interaction integral
[260]	The nonequilibrium, incompatibility, and constant-constitutive tensor formulations applicable to the fracture analysis of FGM by the interaction integral method are compared
	Dynamic, transient, and viscous effects
[261]	Linear elastic-viscoelastic correspondence principle for FGM applied to the analysis of stress and displacement around a stationary crack tip
[262]	Continuation of the previously published work illustrating that the correspondence principle is invalid for FGM with "inseparable" in terms of space and time relaxation functions in shear and dilatation
[263]	Dynamic stress intensity factors found for a FGM coating crack under an antiplane impact
[264]	Analysis of the dynamic stress intensity factor for a crack in a FGM interface between two dissimilar cylinders subject to dynamic torsional impact
[265]	Transient stresses and dynamic stress intensity factor around a penny-shaped crack in a transversely isotropic FGM strip produced by torsional impact are evaluated
[266]	Dynamic Mode I and mixed-mode fracture in FGM are considered using the cohesive zone model implemented in a graded finite element method
[267]	Transient thermal stresses and stress intensity factors are evaluated for an edge crack in a FGM strip with varying thermal properties and constant elastic properties
[268]	Solution of Mode I fracture problem in a FGM viscoelastic strip by the correspondence principle
[269]	Crack growth in FGM studied by the weight function method, failure of a thermal barrier under cyclic thermal loading is considered
[270]	The dynamic fracture problem for a crack in FGM subjected to harmonic stress waves is considered and dynamic stress intensity factors are found
[271]	Dynamic stress intensity factors are determined for Mode I fracture for a crack that is perpendicular to the surfaces of an orthotropic FGM strip and subject to internal impact stresses
[272]	Effect of a functionally graded core on fracture resistance of sandwich structures subjected to impact loading is experimentally studied
[273]	Analysis of stresses and deformations associated with a crack propagating with a constant velocity along the direction of material property gradation
[274]	Dynamic stress intensity factors are found for a crack in a FGM coating that propagates in the direction perpendicular to a homogeneous substrate
[275]	The dynamic stress intensity factor and electric displacements are obtained for a functionally graded piezoelectric strip with a center crack
	Thermal effects on fracture in FGM
[276]	The analysis of fracture toughness of a FGM coating around a circular hole in an infinite elastic solid subject to thermal stresses
[277]	The study of a crack in a FGM plate with a bidirectional variation of the coefficient of thermal expansion subjected to steady thermal loading

Table 2 (*Continued.*)

Reference	Major emphasis (some of the papers referred to below could be identified with more than one area but such duplication is avoided for simplicity)
[278]	Thermomechanical stresses at the tip of a partially insulated crack are considered
[279]	Experimental investigation of fracture in FGM TBCs subjected to high heat flux and thermal cycling.
	Mixed-mode cracks in FGM
[280]	The path-independent J-integral method is extended to orthotropic FGM resulting in the stress intensity factors and energy release rates in Mode I and mixed-mode fracture problems
[281]	Self-similar crack growth within a FGM interface layer between dissimilar elastoplastic solids including the effect of mode mixity on the steady-state fracture
[282]	Mixed-mode stress intensity factors and strain energy release rates are obtained for a crack in a FGM coating bonded to a homogeneous substrate
[283]	Mixed-mode stress intensity factors, crack initiation angle, and T stress in FGM are determined by the interaction integral method combined with FEA
[284]	Finite element method solution with an automatic remeshing is employed to monitor mixed-mode crack propagation in FGM
	Experimental FGM fracture studies
[285]	Experimental study of fracture in functionally graded TBCs subject to thermal shock or thermal cycling reflecting on the effect of grading on the spallation life
[286]	Experimental study of effects of heat treatment promoting the formation of β-phase of titanium during aging on fatigue of a functionally graded Ti– 6Al–4V
	Miscellaneous problems
[287]	Stress intensity factors are found for an interfacial crack in antiplane loading mode
[288]	Modes I and II stress intensity factors are determined for a crack in a functionally graded interface between two homogeneous half-planes
[289]	The Schmidt method is applied to study the problem of an interfacial crack between inhomogeneous orthotropic media graded along the crack axis
[290]	Stress intensity factors are found for an orthotropic FGM with multiple arbitrary oriented and arbitrary shaped cracks by the modified crack-closure technique combined with the displacement correlation method
[291]	Stress intensity factors are determined for cracks in an orthotropic FGM strip subjected to crack surface pressure, fixed grip loading, and bending
[292]	Stress intensity factors are determined for a system of collinear interface cracks in a FGM composite consisting of layers with layerwise constant properties that is subject to thermal loading
[293]	Crack kinking in FGM as a result of stress relaxation is considered
[294]	Stress intensity factors associated with penny-shaped cracks perpendicular to a graded bimaterial interface are calculated by the boundary element method
[295]	The energy interaction method is applied to the mixed-mode fracture for an arbitrary oriented crack in FGM yielding stress intensity factors that were found in excellent agreement with FEA results
[296]	Constant-speed crack propagation in a 3D infinite FGM with the elastic modulus and density graded in the direction perpendicular to the crack and loading represented by normal and shear stresses at the crack surfaces
[297]	The Weibull statistics is applied to the prediction of fracture toughness and the average initiation angle of a crack in a brittle FGM
[298]	Stress intensity factors are determined for a semielliptical crack in an elastic FGM subject to tension, bending, and nonuniform temperature using a general domain method
[299]	Experimental study of Mode I fracture in polymer FGM specimens with various grading schemes

TBCs subject to transient thermal loading and accounting of viscoplastic effects was conducted by Kokini and Rangaraj [252] using laser thermal shock tests. The character of surface cracking, i.e., single versus multiple cracks, was affected by gradation of the coating, the tendency to multiple cracking increasing with increased gradation. Paulino and Zhang studied dynamic fracture in FGM using graded finite elements in the intact material and graded intrinsic cohesive elements in the fracture zone [253].

Experimental investigation of fracture in FGM is particularly valuable due to the complexity of the process and difficulties of its modeling and characterization. The onset and propagation of cracks in alumina/epoxy FGM were monitored using a four-point bending test under both monotonic or cyclic loading [254]. Both the crack growth rates and their trajectories were recorded. As follows from this paper, finite element predictions of the anticipated crack path generated by ANSYS for linear elastic FGM can be quite accurate. The deflection angles of the cracks generally increased in specimen with a larger grading gradient and when the crack onset occurred close to the compliant side.

2 Conclusions

The recent progress in the characterization, modeling, and analysis of FGM has been reviewed in this paper concentrating on research published since 2000. Due to the broad and rapidly developing field of FGM, these conclusions cannot encompass all significant directions, trends, and needs. Nevertheless, they reflect some of the observations of the authors based on the published research and their own analysis of the subject.

1. In the area of *homogenization* of particulate FGM, it is often possible to employ available techniques for composites without grading. However, in the case where the material has a significant gradient, the model (RVE) may become asymmetric, reflecting the corresponding property variations at the micromechanical level. Whatever approach to homogenization is adopted, the interaction between the particles should not be disregarded.
2. Material properties evaluated according to theoretical models often disagree with measured values of FGM constants. This indicates that a *probabilistic approach to homogenization* accounting for uncertainty in the actual material distribution throughout the volume may be justified.
3. In case the probabilistic homogenization approach suggested above is implemented, a need will arise in the application of *probabilistic mechanics* to the analysis of the response, fracture, and fatigue characteristics of FGM structures.
4. While there is a broad spectrum of successfully implemented *FGM manufacturing techniques*, there remains a need in the procedures and protocols that guarantee a reliable and predictable distribution of material constituent phases and properties throughout the structure.
5. The approach to the *heat transfer problems* of FGM should account for the effect of temperature on the material properties that in turn affect the solution of the heat conduction problem. Typical problems formulated accounting for temperature-dependent FGM properties can be solved using an iterative technique, although the exact solution may be found in relatively simple benchmark cases.
6. *Thermal residual stresses* should be accounted for in the analysis of FGM since they may affect local strength and fracture resistance of these materials. In the absence of a layered construction, the residual stresses one has to be concerned with represent microscopic stresses due to a thermal mismatch between the constituent materials (additionally, macromechanical residual stresses may arise if postprocessing deformations are restrained). If the properties change in a stepwise manner, macromechanical residual stresses due to different layerwise thermal expansion coefficients should also be incorporated in the analysis.
7. The *effect of temperature on the stress, stability and vibration problems of FGM* should be accounted for including explicit temperature-induced stress resultants and couples as well as the changes in material properties due to temperature. The latter changes are particularly important since FGM are heterogeneous materials. Therefore, if one of the constituent material phases is more affected by temperature than the other phase, a degree of property changes will be nonuniform throughout the material, even if it is subject to a uniform temperature.
8. *Optimization* problems are natural for FGM due to design opportunities open in the case of a combination of dissimilar materials that can be graded throughout the structure. However, the implementation of these solutions will depend on a reliable characterization and predictable manufacturing of these materials as discussed in the previous conclusions.
9. Usually, *FGMs form asymmetric structures*, with the exception of symmetrically graded composite and sandwich configurations. This implies the presence of the coupling effect, even in quasi-isotropic FGM. Accordingly, the response of FGM in static stress and stability problems as well as their dynamic behavior are affected by this inherited asymmetry. As a result, the analysis of compressed FGM structures should always be conducted accounting for prebuckling deformations. In particular, the classical Euler bifurcation buckling does not exist in thermally loaded FGM structures, with the exception of those with fully clamped boundaries.
10. The asymmetry introduced by grading may often result in higher deformations and stresses of a FGM structure as compared to its homogeneous symmetric counterpart with an overall identical material composition. This implies the necessity to *balance the advantages of grading against incurred disadvantages*. Therefore, it is essential to emphasize that while FGM are a useful tool in the arsenal of a designer they have their pros and cons and should be used accounting for all features produced in the structure by material grading.
11. *Nonsingular T stresses* in FGM with cracks are strongly affected by the material gradation. Furthermore, these stresses, material gradation, and stress intensity factors affect the crack initiation angle in fracture problems of FGM.

In conclusion, FGMs represent a rapidly developing area of science and engineering with numerous practical applications. The research needs in this area are uniquely numerous and diverse, but FGMs promise significant potential benefits that fully justify the necessary effort.

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