Plates and Shells

Victor Birman

Engineering Education Center, Missouri University of Science and Technology, St. Louis, MO, USA

1 INTRODUCTION

Plates and shells represent principal elements of aerospace structures, including fuselages of planes and missiles, control surfaces, bulkheads, helicopter blades, and others. The multiple applications, shapes, and materials found in plate and shell structures dictate the necessity of a comprehensive approach to their analysis reflected in relevant theories and methodologies.

Various aspects of the theory and analysis of these structures are found in the books by Timoshenko and Woinowsky-Krieger (1959), Novozhilov (1964), Dym (1974), Libai and Simmonds (1998), Ugural (1999), Ventsel and Krauthammer (2001), and Reddy (2007).

All theories of plates and shells rely on the following relationships:

1. Kinematic equations determine a displacement field throughout the structure experiencing deformations. A particular theory is based on assumptions adopted for the kinematic equations. For example, plates and shells are often analyzed by assumption that the thickness of the structure remains constant under loading, implying that deflections along a normal to the middle surface of the structure (the surface that is equidistant from the outer surfaces) are constant.
2. Strain-displacement relationships reflect the magnitude of deformations relative to characteristic dimensions of the structure. These relationships are affected by kinematic equations, enabling us to introduce such effects as transverse shear deformability into the analysis. If displacements of the plate or shell are relatively small, the strains can be assumed linear functions of displacements. However, in case of large displacements, strains are nonlinear functions of deformations. Linear strain-displacement relationships always represent a particular case of a more general nonlinear formulation.
3. The constitutive relations account for the physical properties of the material defining the stress tensor in terms of the strain tensor. Thus, elastic, elastic–plastic, viscoelastic, viscoplasctic, shape memory, piezoelectric and other materials can be characterized by the appropriate theory. With the exception of sandwich structures with a soft core, the thickness of the plate or shell can usually be assumed unaffected by deformations, that is, the normal strain $\varepsilon_z = 0$ (the Cartesian coordinate system adopted in the analysis of rectangular plates and the cylindrical coordinate system used for cylindrical shells are shown in Figures 1 and 2, respectively). In nearly all plates and shells found in applications, the applied pressure and normal stresses through the thickness are

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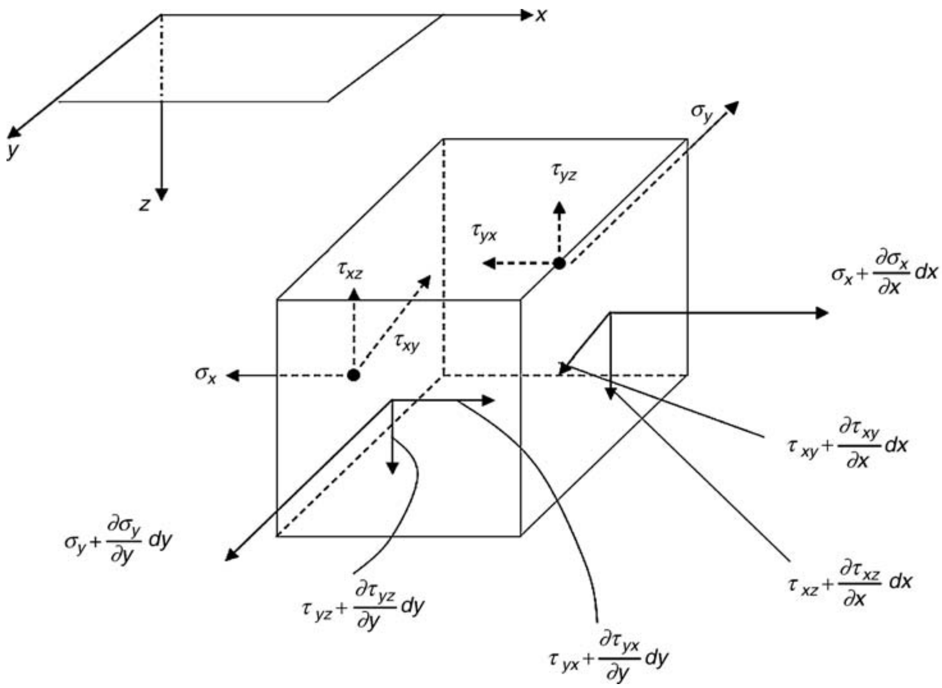

Figure 1. Cartesian coordinate system and stresses acting on an infinitesimal element (for simplicity, stresses on the top and bottom surfaces $z = \text{const}$ are not shown).

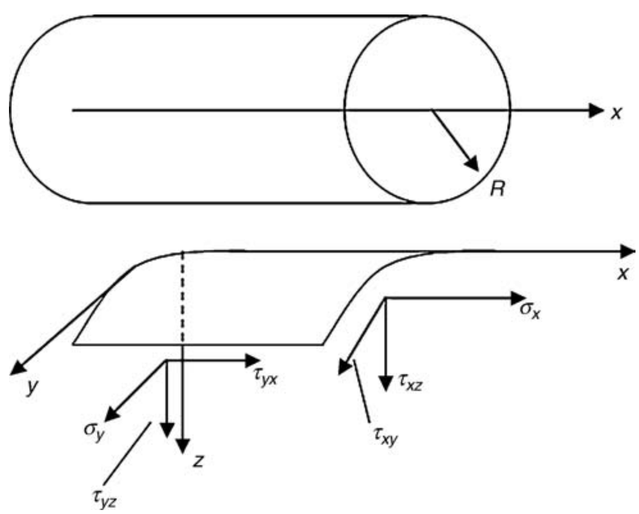

Figure 2. Cylindrical coordinate system and stress notation.

at least an order of magnitude higher than the maximum in plane stresses, so that $\sigma_z \approx 0$. The reciprocity law of shear stresses dictating the symmetry of the stress tensor implies that$\tau_{mn} = \tau_{nm}$ $(m, n = x, y, z)$.

4. Equilibrium equations (equations of motion in dynamic problems) and the boundary conditions can be derived by two methods. One of these methods employs the analysis of equilibrium of an infinitesimal element detached from the structure where the effect of adjacent parts is represented by the stresses applied to the element (Figure 1). The increments in the stresses on the opposite faces of the element shown in Figure 1 indicate that they are continuous functions of the coordinates. The other method is the energy approach using the Hamilton principle. An alternative to using equilibrium equations for the analysis is the application of one of the energy methods, such as the Rayleigh–Ritz method. The use of energy methods is conditioned on specifying the expressions for the potential energy of the structure (kinetic energy is also necessary in dynamic problems).

There are five equations of equilibrium including those enforcing the equilibrium of forces acting on an infinitesimal element in the *x*, *y*, and *z*-directions and the equilibrium of moments about the x and y coordinate axes. The sixth equation of equilibrium of moments about the z-axis is identically satisfied since $\tau_{xy} = \tau_{yx}$.

Equilibrium equations for plates and shells involve functions of stresses, that is, the forces and moments per unit width of the cross section (they are called stress resultants and stress couples, respectively). These stress resultants and stress couples are defined by integrals of the stresses and their moments with respect to the middle surface through the thickness. For example, in the cylindrical coordinate system (Figure 2) the in plane stress resultants $N_i,\ N_{ij}$, transverse stress resultants Q_i and stress couples $M_i,\ M_{ij}$ are

$$\begin{bmatrix} N_x \\ M_x \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x \left(1 - \frac{z}{R}\right) \begin{bmatrix} 1 \\ z \end{bmatrix} \mathrm{d}z, \quad \begin{bmatrix} N_y \\ M_y \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y \begin{bmatrix} 1 \\ z \end{bmatrix} \mathrm{d}z,$$
$$\begin{bmatrix} N_{xy} \\ M_{xy} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy} \left(1 - \frac{z}{R}\right) \begin{bmatrix} 1 \\ z \end{bmatrix} \mathrm{d}z, \quad \begin{bmatrix} N_{yx} \\ M_{yx} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{yx} \begin{bmatrix} 1 \\ z \end{bmatrix} \mathrm{d}z,$$
$$Q_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xz} \left(1 - \frac{z}{R}\right) dz, \qquad Q_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{yz} \mathrm{d}z \tag{1}$$

where $\sigma_i,\ \tau_{in}\ (i = x, y,\ n = x, y, z)$ are normal and shear stresses oriented similarly to those in Figure 1, R is the radius of the middle surface, and h is the thickness of the shell. In the case of a flat plate, $R = 0$ and$N_{xy} = N_{yx},\ M_{xy} = M_{yx}$. While these stress resultants and couples are not equal to each other in cylindrical shells, in thin shells the ratio $\frac{z}{R} << 1$ since$-\frac{h}{2} \le z \le \frac{h}{2}$, so that the differences between N_{xy} and N_{yx} and between M_{xy} and M_{yx} are negligible. The simplified version of equations (1) becomes

$$\begin{bmatrix} N_x \\ M_x \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x \begin{bmatrix} 1 \\ z \end{bmatrix} \mathrm{d}z, \qquad \begin{bmatrix} N_y \\ M_y \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y \begin{bmatrix} 1 \\ z \end{bmatrix} \mathrm{d}z,$$
$$\begin{bmatrix} N_{xy} \\ M_{xy} \end{bmatrix} = \begin{bmatrix} N_{yx} \\ M_{yx} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy} \begin{bmatrix} 1 \\ z \end{bmatrix} \mathrm{d}z, \quad Q_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xz} \mathrm{d}z,$$
$$Q_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{yz} \mathrm{d}z \tag{2}$$

The stress resultants and stress couples defined by equations (2) are depicted in Figure 3 in a Cartesian coordinate system.

The difference between results obtained by various shell theories is usually small, with the exception of the Donnell theory that is less reliable for long cylindrical shells (e.g., Bert and Kim, 1995). Here we consider cylindrical shells using the Love shell theory. Eliminating transverse shear stress resultants Q_x and Q_y from the moment equilibrium equations, the remaining three equations for the cylindrical shell subject to transverse pressure become

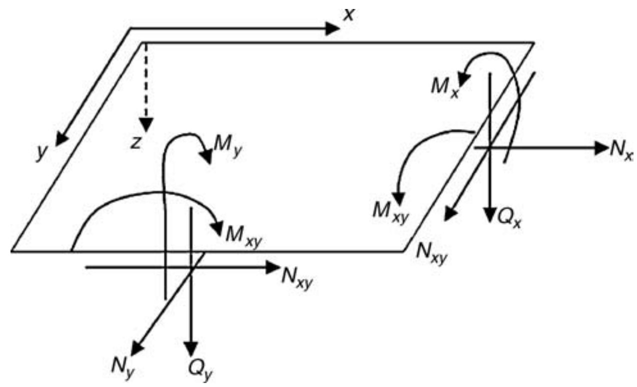

Figure 3. Stress resultants and stress couples in the Cartesian coordinate system (for simplicity, stress resultants and couples are shown only on two edges of an infinitesimal element).

$$N_{x,x} + N_{xy,y} = 0, \qquad N_{xy,x} + N_{y,y} + \frac{M_{xy,x}}{R} + \frac{M_{y,y}}{R} = 0,$$
$$M_{x,xx} + 2M_{xy,xy} + M_{y,yy} - \frac{N_y}{R} + q = 0 \tag{3}$$

where q is pressure applied in the perpendicular direction to the middle surface and $(\ldots),_i \equiv \partial(\ldots)/\partial i, \quad i = x,\ y$. Three equations of equilibrium (3) are not sufficient to determine six stress resultants and stress couples they contain, that is, the problem is statically indeterminate. This obstacle is eliminated if the equations of equilibrium are expressed in terms of displacements or by the use of the stress function illustrated below for a flat plate. Equations of equilibrium of flat plates in the Cartesian coordinate system are obtained from equation (3) if $R = \infty$.

2 CLASSICAL THEORY OF PLATES AND SHELLS

The classical theory of thin plates and shells is based on the Kirchhoff–Love hypothesis. Two assumptions involved in this hypothesis are:

1. A cross-section perpendicular to the middle surface prior to deformation remains plane and perpendicular to the deformed middle surface (Figure 4). This assumption can also be formulated in the sense that a normal to the middle surface remains straight and normal to this surface. Accordingly, transverse shear strains in the planes xz and yz are equal to zero. Moreover, the length of the normal remains constant($\varepsilon_z = 0$).
2. The transverse normal stress $\sigma_z = 0$.

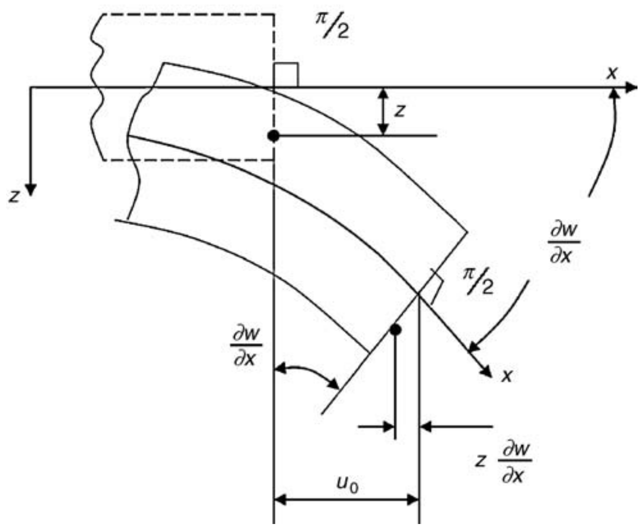

Figure 4. Deformation of the cross section in the xz plane according to the classical plate theory.

The kinematic equations reflecting this hypothesis define the following vector of displacements

$$\boldsymbol{u} = \begin{bmatrix} u(x, y, z) \\ v(x, y, z) \\ w(x, y) \end{bmatrix} = \begin{bmatrix} u_0(x, y) - zw(x, y)_{,x} \\ v_0(x, y) - zw(x, y)_{,y} \\ w(x, y) \end{bmatrix} \quad (4)$$

where u_0, v_0, and w are in plane displacements of the middle surface in the x- and y-directions and the deflection in the z-direction, respectively, while u and v are in plane displacements of a point located at a distance z from the middle surface. For example, the first equation (4) can easily be interpreted by identifying the appropriate terms illustrated in Figure 4.

The first Kirchhoff–Love assumption that transverse shear strains are equal to zero implies that transverse shear stresses are absent in isotropic and composite materials. In conjunction with the second assumption, this means that the shell or plate is in the state of plane stress.

The in plane strains obtained using the geometrically nonlinear Green strain tensor and neglecting higher-order terms are

$$\varepsilon_x = \varepsilon_x^0 + z\kappa_x, \quad \varepsilon_y = \varepsilon_y^0 + z\kappa_y, \quad \gamma_{xy} = \gamma_{xy}^0 + z\kappa_{xy} \quad (5)$$

where in plane strains in the middle surface and the changes of curvature and twist are

$$\varepsilon_x^0 = u_{0,x} + \frac{1}{2}w_{,x}^2, \qquad \varepsilon_y^0 = v_{0,y} + \frac{w}{R} + \frac{1}{2}w_{,y}^2,$$
$$\gamma_{xy}^0 = u_{0,y} + v_{0,x} + w_{,x}w_{,y} \quad (6)$$

and

$$\kappa_x = -w_{,xx}, \quad \kappa_y = -w_{,yy} + \frac{v_{,y}}{R}, \quad \kappa_{xy} = -2w_{,xy} + \frac{v_{,x}}{R} \quad (7)$$

respectively.

Upon the substitution of equations (5) into the plane-stress Hookean constitutive relations and the subsequent integration of the stresses according to (2), the stress resultants and stress couples become

$$N_x = \frac{Eh}{1-\nu^2}\left(\varepsilon_x^0 + \nu\varepsilon_y^0\right), \quad N_y = \frac{Eh}{1-\nu^2}\left(\varepsilon_y^0 + \nu\varepsilon_x^0\right),$$
$$N_{xy} = Gh\gamma_{xy}^0,$$
$$M_x = D\left(\kappa_x + \nu\kappa_y\right), \quad M_y = D\left(\kappa_y + \nu\kappa_x\right),$$
$$M_{xy} = D\frac{1-\nu}{2}\kappa_{xy} \quad (8)$$

where E and ν are the modulus of elasticity and the Poisson ratio of the material, respectively, and the so-called cylindrical stiffness is $D = \frac{Eh^3}{12(1-\nu^2)}$. Transverse shear stress resultants Q_i are equal to zero according to the assumption of plane stress.

3 BENDING AND BUCKLING OF THIN ISOTROPIC PLATES

This section illustrates geometrically linear and nonlinear analyses using rectangular plates as an example. Other geometries may employ different coordinate systems and accordingly modified equations, but the principal aspects of the analysis remain unaltered.

3.1 Geometrically linear problem

Limiting the analysis to relatively small deflections we can use the linear version of the strain-displacement relationships (5), (6), and (7). Then substituting the stress resultants and stress couples given by equations (8) into equations of equilibrium (3) where $R = \infty$, we obtain three equations in terms of displacements u_0, v_0, and w. It can be shown that the first two equations contain only in plane displacements u_0, v_0, while the third equation includes only transverse deflection w.

Consider the case where the plate is subject to combined loading, including pressure q and in plane axial and shear stress resultants $\bar{N}_x$, $\bar{N}_y$ and $\bar{N}_{xy}$ (Figure 5). Then the third

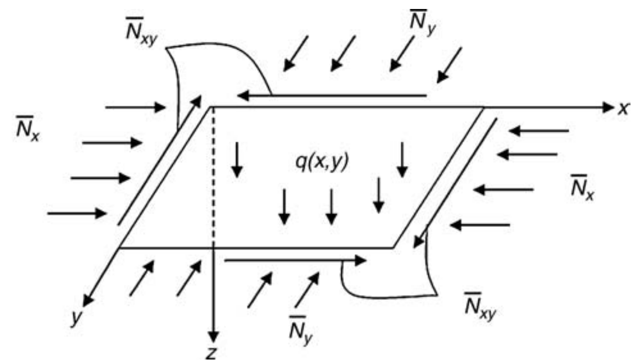

Figure 5. Rectangular plate subject to transverse pressure and in-plane loads.

equilibrium equation obtained from (3) becomes

$$D\left(w_{,xxxx} + 2w_{,xxyy} + w_{,yyyy}\right) = q + \underline{\overline{N}_x w_{,xx} + \overline{N}_y w_{,yy} + 2\overline{N}_{xy} w_{,xy}} \quad (9)$$

where the underlined terms in the right side represent the projections of the applied stress resultants in the direction perpendicular to the plate surface.

The solution of equation (9) must satisfy the boundary conditions. Similar to equations of equilibrium, these conditions are uncoupled in geometrically linear problems. The conditions involving transverse deflections, bending stress couples, and transverse shear stress resultants are unaffected by in plane displacements and stresses. Typical conditions include:

1. Simply supported edge:

$$w = M_n = 0 \quad (10a)$$

2. Clamped edge:

$$w = w_{,n} = 0 \quad (10b)$$

3. Free edge:

$$M_n = 0, \quad Q_n = M_{n,n} + 2M_{nt,t} + \overline{N}_n w_{,n} + \overline{N}_{nt} w_{,t} = 0 \quad (10c)$$

where n and t denote the normal and tangential directions to the edge, respectively. For example, for edges $x = \text{const}$, $n \equiv x$, $t \equiv y$. Other possible boundary conditions could be specified if the edge was supported by a flexible beam providing an elastic translational and/or rotational support. In geometrically linear bending problems without in plane loading, in plane equilibrium equations are satisfied if $u_0 = v_0 = 0$. If the plate is subject to in plane loads, in plane equilibrium implies that these loads remain constant throughout the plate.

Typical problems considered below include bending of a rectangular plate simply supported along the edges $x = 0,\ x = a,\ y = 0,\ y = b$ (so-called Navier's solution) and buckling of such plate subject to compressive loads ($q = \overline{N}_{xy} = 0$).

Consider bending of a plate that is simply supported along all edges. Both the applied pressure and the transverse deflection can be represented by double Fourier series, that is

$$q = \sum_{m,n} q_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$
$$w = \sum_{m,n} W_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \quad (11)$$

where m and n are integer numbers.

Boundary conditions (10a) are identically satisfied (the stress couples are expressed in terms of deflections by (7) and (8)). The substitution of (11) into (9) yields uncoupled expressions for the amplitudes of harmonics in the series for deflections ($\overline{N}_{xy} = 0$):

$$W_{mn} = \frac{q_{mn}}{D\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^2 + \overline{N}_x\left(\frac{m\pi}{a}\right)^2 + \overline{N}_y\left(\frac{n\pi}{b}\right)^2} \quad (12)$$

As follows from (12), deflections of the plate decrease if it is subject to tensile in plane loads $\overline{N}_x > 0,\ \overline{N}_y > 0$. On the contrary, compression results in an increase of deflections.

Particular cases of bending due to pressure (without in plane loads) and buckling can be obtained from (12). In the buckling problem $q_{mn} = 0$ and the combination of critical loads corresponding to the mn-th mode shape of instability is available from the requirement that the denominator of (12) must be equal to zero (this results in infinite deflections as is characteristic for linear buckling problems). The actual mode shape of buckling is affected by the aspect ratio of the plate (e.g., Jones, 2006).

If the plate is subject to a combination of transverse pressure and in plane compressive loads approaching the critical (buckling) combination, deflections become very large, necessitating a geometrically nonlinear analysis (Figure 6). As is shown in Figure 6, deflections produced by pressure only (point A), increase under compression and asymptotically approach infinity as the compressive load approaches the buckling value (point C). However, as deflections and stresses increase, the material becomes plastic and the plate collapses (in Figure 6 the collapse point is identified by *B). Accordingly, in relatively rigid plates, plastic failure may occur prior to the emergence of noticeable geometrically nonlinear effects.

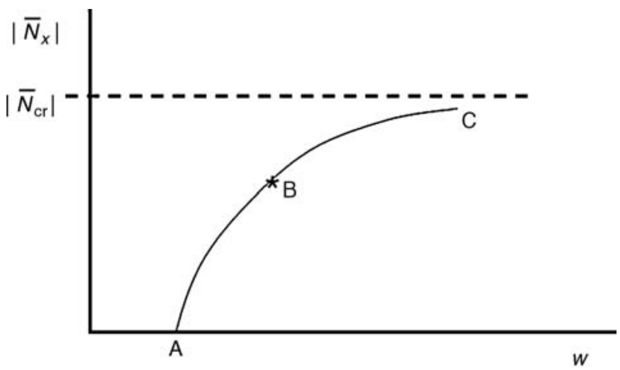

Figure 6. Deflections of a plate subject to transverse pressure and compression in the x-direction.

The common feature of linear problems for isotropic plates of all geometries is uncoupling of equations of equilibrium and boundary conditions. As a result, the solution is reduced to the analysis of transverse deflections and associated stress couples and transverse shear stress resultants, similar to the case of rectangular plates considered above.

3.2 Geometrically nonlinear problem

If the plate experiences large deflections, nonlinear terms cannot be neglected in the middle plane strain-displacement relations (6). The nonlinearity affects both the equations of equilibrium as well as natural (static) boundary conditions for stress resultants and stress couples. In the nonlinear formulation uncoupling of equations of equilibrium and boundary conditions is not possible due to the interaction between bending and stretching of the plate.

The solution is often found by introducing the stress function φ that identically satisfies in-plane equilibrium equations, that is, the first two equations (3) where $R = \infty$:

$$N_x = \varphi_{,yy}, \qquad N_y = \varphi_{,xx}, \qquad N_{xy} = -\varphi_{,xy} \tag{13}$$

This reduces the problem to just one equation of equilibrium and the additional compatibility equation that guarantees a single-valued solution for displacements. For a rectangular plate, these equations are

$$\begin{aligned} D\nabla^4 w &= w_{,xx}\varphi_{,yy} + w_{,yy}\varphi_{,xx} - 2w_{,xy}\varphi_{,xy} + q \\ \frac{1}{Eh}\nabla^4\varphi &= w_{,xy}^2 - w_{,xx}w_{,yy} \end{aligned} \tag{14}$$

Equations (14) include two unknowns, the deflection and the stress function. Boundary conditions can also be expressed in terms of these unknowns (i.e., the problem is statically determinate). Analytical solutions are usually complicated due to difficulties in satisfying boundary conditions related to in plane displacements and stress resultants. In particular, in plane natural boundary conditions can usually be satisfied only in the integral sense.

The other approach to the nonlinear analysis of plates and shells utilizes energy methods. The presence of geometrically nonlinear terms does not alter the approach to the solution compared to the linear problem (e.g., the Rayleigh–Ritz method is applicable to both linear and nonlinear problems). However, the resulting nonlinear mathematical formulation usually necessitates a numerical solution.

A geometrically nonlinear analysis results in deflections and stresses that are smaller than their counterparts obtained by the linear solution. Accordingly, the linear solution of bending problems is conservative. In the problems of buckling, the nonlinear solution illustrates a radically different behavior of plates and shells. An increase of the load beyond the buckling value results in gradually increasing deflections and stresses in flat plates. Accordingly, a plate can operate in the postbuckling regime under increasing loads until failure due to the loss of strength. On the contrary, a shell subject to buckling loads experiences snap-through, resulting in large and often irrecoverable deformations that can be identified with failure.

Initial imperfections, that is, deviations from a perfect geometric shape that often occur as a result of the manufacturing process, affects the response of plates and shells, particularly in buckling problems. These imperfections do not qualitatively change the postbuckling response of flat plates but result in larger stresses and deflections than in the case of a perfect structure. However, imperfections can be responsible for a drastic reduction in the buckling load of the shell (Hutchinson, Tennyson and Muggeridge, 1971).

4 PLATES AND SHELLS WITH STIFFENERS AND CUT-OUTS

4.1 Stiffened plates and shells

Plates and shells are often reinforced by frames and stringers (Figure 7). Such stiffeners enable a designer to reduce the

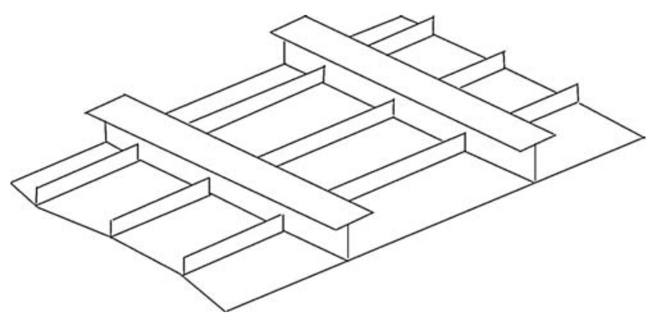

Figure 7. Stiffened plate supported by frames (T-profiles) and blade stringers.

thickness of the skin and develop the structure capable of withstanding prescribed loads that is lighter than its unstiffened counterpart. Alternatively, stiffeners can be used to increase the load-carrying capacity compared to an equal-weight unstiffened structure.

The analysis of stiffened structures is conducted considering deformations and stresses (or stress resultants and couples) in the skin and stiffeners. The continuity requirement for deformations and stresses along the junctions between the skin and stiffeners ensures that the structure is treated as one unit. An alternative approach is based on "smearing" the stiffness of the stiffeners, replacing the actual stiffened plate or shell with the fictitious "equivalent" structure with uniform stiffness in the direction of each system of parallel reinforcements (Bert, Kim and Birman, 1995). Besides the application of equations of equilibrium, energy methods can be applied to the analysis of stiffened plates and shells. These methods can incorporate the energy contributed by the stiffeners modeling them as discrete beam-type structures (e.g., Li and Kettle, 2002) or using the smeared stiffeners technique.

Design of a stiffened plate or shell represents a balancing act. Stiff and heavy stiffeners may actually present conditions approaching rigid support along the junction with the skin. If the stiffeners provide rigid support to the structure (usually, simple support), a further increase of their stiffness is counterproductive since failure takes place in the panels supported by these stiffeners. On the other hand, exceedingly light stiffeners may fail prior to the panels of the structure they support (such cases are rare).

In general, enhanced stiffness and strength of the stiffened structure outweigh the detrimental effect due to the additional weight contributed by the stiffeners. Accordingly, stiffened structures are used in the majority of applications. An exception is found in plates and shells subject to membrane tensile loading where uniform state of tensile stresses makes unstiffened design preferable. Stiffeners are also avoided in sandwich plates and shells as discussed in paragraph 5.2.

4.2 Openings and cut-outs in plates and shells

Openings and cut-outs are often found in plate and shell structures due to the necessity to accommodate equipment, provide space for hatches and windows or reduce weight. The presence of an opening invariably weakens the structure due to a local stress concentration as well as reduced global stiffness (e.g., a plate with a large opening subject to bending experiences larger deflections than the otherwise identical plate without an opening). The stress concentration may result in cracks originating from the openings and cut-outs, for example, cracks initiated at the corners of a rectangular opening where the theoretical elastic stress concentration factor is equal to infinity. Linear elastic solutions have limited applicability for the evaluation of local stresses around the openings due to a combination of physically and geometrically nonlinear effects. For example, the stress concentration factor in a circular cylindrical shell with a circular hole subjected to axial tension is unaffected by tensile load in the linear elastic formulation. However, it is abruptly reduced if geometric nonlinearity is accounted for, while the effect of physical nonlinearity, (i.e., plasticity) can be even more pronounced (Chernyshenko, Storozhuk and Kharenko, 2008).

The effect of openings is affected by their size. A large opening can alter the global response of the plate or shell, while a small opening has negligible effect on the stiffness, but is still undesirable due to the local stress concentration.

Both global and local detrimental effects of openings and cut-outs can be reduced by stiffening the structure. Stiffeners usually encompass openings, compensating for the loss of stiffness and reducing local stresses. The problem of local stress concentration can also be reduced by avoiding sharp corners. For example, rectangular openings typically have rounded rather than rectangular corners.

5 COMPOSITE AND SANDWICH PLATES AND SHELLS

5.1 Composite structures

Isotropic materials have identical strength and stiffness in all directions. In contrast, composites, such as polymer-matrix carbon/epoxy and glass/epoxy, metal-matrix boron/aluminum or ceramic-matrix carbon/carbon composites are anisotropic materials that exhibit different properties along and across the fibers. Accordingly, it is possible to tailor their response to achieve maximum structural efficiency.

The kinematic, strain-displacement, and equilibrium equations in terms of stress resultants and stress couples are not explicitly affected by the material. However, the properties of the material may be reflected in the choice of some of these equations. For example, a composite plate with low transverse shear stiffness may be studied by the first-order or higher-order theories discussed in the next paragraph, while a metallic plate of the same geometry could be confidently analyzed using the classical theory. The anisotropy of composites directly affects the constitutive relations. Furthermore, laminated composite materials have different strengths in different directions relative to the fiber orientation, affecting the choice of the strength criterion.

Composites usually have relatively low transverse shear stiffness. Thus the Kirchhoff–Love assumption may become invalid, necessitating the use of so-called first-order and higher-order theories (Reddy, 2004) or a three-dimensional analysis. The theories referred to above introduce various assumptions regarding the shape of a normal to the middle surface upon deformations. According to the first-order theory, the normal shown in Figure 4 remains straight but rotates relative to the middle surface, so that the right angle between this normal and the middle surface is not retained during deformation (transverse shear strain). Higher-order theories discard the assumption that the normal remains straight and represent its shape by various polynomials, the thickness coordinate being the argument.

The methods of bending and buckling analyses applicable to isotropic plates and shells remain applicable to composite structures. However, the potential for structural optimization is facilitated by changing the sequence and orientation of layers and using a broad choice of available fiber and matrix materials that provide multiple opportunities to designers. Composite plates and shells exhibit unique modes of failure not typical in isotropic counterparts. For example, delamination cracks can originate in the structure subject to a low-velocity impact or in the vicinity of discontinuities associated with the concentration of transverse shear or peeling stresses.

Composite plates and shells are often reinforced by stiffeners. The shape of such stiffeners may resemble that found in isotropic structures but unique configurations, such as hat-stiffeners, are also found. Delamination cracks may originate from the edges of stiffeners due to the presence of a local concentration of transverse shear and peeling stresses.

5.2 Sandwich structures

A special class of composite plates and shells that has found numerous applications in the aerospace industry is sandwich structures (Figure 8). These structures are characterized by two thin and stiff facings separated by a light thick core. A typical core is polymeric or honeycomb and its purpose is mainly to combine the facings into one structure and support them, preventing local deformation and failure modes. The facings usually carry in plane loads, acting similarly to flanges of an I-beam. The core works primarily in transverse shear and it can participate in the transfer and redistribution of transverse normal loads applied to the facings. The analogy between the core of a sandwich structure and the web of an I-beam is evident. The fact that the facings are separated by a relatively thick core guarantees high stiffness of sandwich structures that often results in improved performance when compared to stiffened structures.

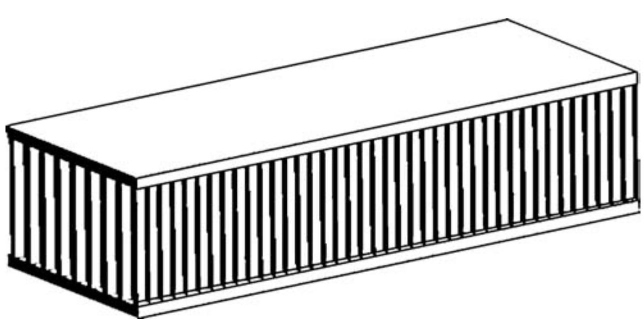

Figure 8. Sandwich panel with a honeycomb core.

Transverse shear stiffness of the core is relatively low, so that the Kirchhoff–Love assumptions become inaccurate. Accordingly, the analysis is usually conducted using the first or higher-order theories (Reddy, 2004) or by a three-dimensional finite element method.

Sandwich plates and shells have a number of unique failure modes, including wrinkling of the facings due to compressive or in plane shear loads, crushing of the core, and dimpling of the facings over honeycomb cells (Vinson, 1999). In addition to these failure modes, the structure is also examined for potential “conventional” failure, such as the loss of strength or global buckling. Contrary to conventional composite and isotropic plates and shells, sandwich structures do not need stiffeners since they possess very high stiffness.

6 SUMMARY

Isotropic and composite plates and shells are major elements of aerospace structures. Plates and shells are usually reinforced by stiffeners, providing them with superior strength and stiffness compared to unstiffened counterparts. Sandwich plates and shells are increasingly used in aerospace applications, competing with stiffened structures in terms of strength and stiffness. Cut-outs that are often found in plates and shells result in stress concentrations and reduced stiffness that are alleviated by stiffening the cut-out edge and avoiding sharp corners. Plates and shells are complex structures that require a numerical analysis, except for several benchmark geometries and boundary conditions. Thin plates and shells found in aerospace applications often operate under loads that result in large deformations and geometrically nonlinear effects. Some of promising new technologies for plates and shells utilize smart materials (piezoelectrics, shape memory alloys, etc.) and nanomaterials.

RELATED CHAPTERS

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Structural Stability
Elements of Structural Dynamics
Finite Element Analysis of Composite Plates and Shells

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