Lecture 21: Torque

- Cross product
- Torque
- Relationship between torque and angular acceleration
- Problem solving
What causes rotation?

Demo: bolt and wrench

Need:
- Force
- Distance
- Perpendicular component
Vector cross product: magnitude

\[ \vec{A} \times \vec{B} = \vec{C} \]

\[ C = AB \sin \theta = A_{\perp B}B = AB_{\perp A} \]
**Vector cross product: Direction**

\[ \vec{A} \times \vec{B} = \vec{C} \]

\( \vec{C} \) is perpendicular to both \( \vec{A} \) and \( \vec{B} \)

**Direction:** right hand rule

thumb \( \times \) index finger = middle finger
Torque

\[ \vec{\tau} = \vec{r} \times \vec{F} \]

\[ |\vec{\tau}| = rF \sin \theta = rF_\perp = r_\perp F \]

moment arm

\( \vec{r} = r \sin \theta \)

line of action

\( F_\perp = F \sin \theta \)

\( F \)

\( \theta \)
Direction of torque

Right hand rule:

or easier:

If force tends to produce rotation in the positive $z$-direction, $\tau_z$ is positive:

$$\tau_z = + r F \sin(\theta)$$

If force tends to produce rotation in the negative $z$-direction, $\tau_z$ is negative:

$$\tau_z = - r F \sin(\theta)$$

Indicate positive $z$-direction through curved arrow.
\[ \tau_{1z} = +F_1 r_{1\perp} = +F_{1\perp} r_1 \]

\[ \tau_{2z} = -F_2 r_{2\perp} = -F_{2\perp} r_2 \]
Angular acceleration of rigid object

Rigid object that can rotate about $z$-axis.
$I_z$ moment of inertia about $z$-axis

$$\sum \tau_z = I_z \alpha_z$$

Compare to $\Sigma F_x = ma_x$

Begin with extended free-body diagram that shows forces and where they act on the object
Example 1:

A uniform bar of length $L$ and mass $M$ can freely rotate about frictionless horizontal axis $O$ at its end. The bar is initially in a horizontal position, is released from rest, and swings down under the influence of gravity. What is the initial angular acceleration of the bar just after it is released from rest?

$I_{\text{bar}} = \frac{1}{3} M L^2$ about $O$
Example 2: Rolling w/o slipping

An object of mass $M$, radius $R$ and moment of inertia $I$ is rolling without slipping down incline that makes an angle $\theta$ with the horizontal. Derive an expression for the object’s linear acceleration.
Example 3: Coupled objects

A small disk of radius $r$ is glued onto a large disk of radius $R$ that is mounted on a fixed axle through its center. The combined moment of inertia of the disks is $I$. A string is wrapped around the edge of the small disk, and a box of mass $m$ is tied to the end of the string. The string does not slip on the disk.

Find the acceleration of the box after it is released from rest.