Potential energy

Mechanics:
\[ F_{\text{grav}} = mg \quad \text{U}_{\text{grav}} = mgy \]
\[ F_{\text{spring}} = -kx \quad U_{\text{spr}} = \frac{1}{2} kx^2 \]

Potential energy increases in the direction opposite to the force.
Electric potential energy

Two point charges:

\[ F = k \frac{|qq'|}{r^2} \]

\[ U = k \frac{qq'}{r} \]
Electric potential

\[ F = k \frac{|qq'|}{r^2} \quad \Rightarrow \quad E = \frac{k|q|}{r^2} \]

\[ U = k \frac{qq'}{r} \quad \Rightarrow \quad V = \frac{U}{q'} \]

Electric potential of point charge \( q \)
Electric Potential of a Point Charge

\[ V = K \frac{q}{r} = \frac{1}{4\pi \varepsilon_0} \frac{q}{r} \]

Electric potential at distance \( r \) from a point charge \( q \)
Unit: 1 Volt = \( \frac{1 \text{ Joule}}{1 \text{ Coulomb}} \)

Potential differences are created by separating positive and negative charges
Multiple charges

\[ V = \sum V_i = \sum k \frac{q_i}{r_i} = k \sum \frac{q_i}{r_i} \]

Remember HW wk 2 II 4?
Energy conservation

\[ K_i + U_i = K_f + U_f \]

\[ U = q V \]

\[ K_i + q V_i = K_f + q V_f \]

\[ K_f - K_i = -q (V_f - V_i) \]

\[ \Delta U > 0 \quad \Delta K < 0 \quad \text{slow down} \]

\[ \Delta U < 0 \quad \Delta K > 0 \quad \text{speed up} \]
Example

\[ V_i = 500 \, \text{V} \quad V_f = 400 \, \text{V} \]

\[ \Delta V = V_f - V_i = -100 \, \text{V} \]

\[ \Delta K = -q \Delta V > 0 \]

\[ \Delta K = -q \Delta V < 0 \]

Positive \( q \) speeds up

Negative \( q \) slows down
An electron moving through a potential difference of 1 Volt will gain kinetic energy of

\[ \Delta K = -q \Delta V = -(-e) \cdot 1V \]

\[ = 1.6 \times 10^{-19} C \cdot 1V \]

\[ 1eV = 1.6 \times 10^{-19} J \]
What is the speed of an 8.7MeV proton?

\[ k = \frac{1}{2} m \nu^2 \]

\[ \nu = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2 \times 8.7 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}}{1.7 \times 10^{-27} \text{ kg}}} \]

\[ = 4.1 \times 10^7 \text{ m/s} \]

very fast

\[ \text{Nothing can be faster than } 3 \times 10^8 \frac{\text{m}}{\text{s}}, \text{ speed of light in vacuum} \]