

Name: Solution February 17, 2023

Total Score: 120 /120

Constants: electron mass $m_e = 9.11 \times 10^{-31}$ kg

proton mass $m_p = 1.67 \times 10^{-27}$ kg

$e = 1.602 \times 10^{-19}$ C

$\epsilon_0 = 8.85 \times 10^{-12}$ C²/N•m²

$k = 9.0 \times 10^9$ N•m²/C²

$F = k \frac{|q_1 q_2|}{r^2}$

$E = k \frac{|q|}{r^2}$

$\vec{F} = q\vec{E}$

$U = k \frac{q_1 q_2}{r}$

$V = k \frac{q}{r}$

$U = qV$

$C = \frac{Q}{\Delta V}$

$C = \kappa \epsilon_0 \frac{A}{d}$

$\Delta V = Ed$

$U = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2 = \frac{1}{2} \frac{Q^2}{C}$

$K = \frac{1}{2} m v^2$

$\Delta K = -q \Delta V$ parallel: $C_{eq} = \sum_i C_i$

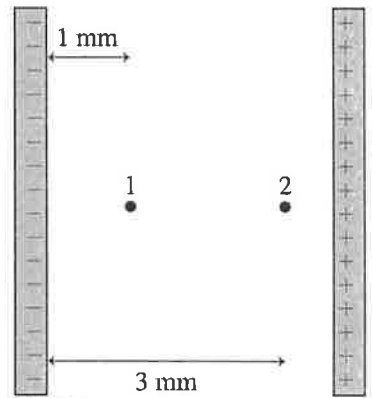
series: $\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$

D 1. (5) An electron is moving in a uniform electric field and slowing down. Its potential energy _____ and it moves towards _____ electric potential.

- A) increases, higher
- B) decreases, higher
- C) decreases, lower
- D) increases, lower

C 3.(5) The figure shows a parallel plate capacitor. Let $V=0$ be the electric potential at the negative plate. Which of the following is true about the of the electric potential V_1 and V_2 and the electric field strengths E_1 and E_2 at the two points, respectively?

- A) $V_1 = V_2$ and $E_1 = E_2$
- B) $V_1 = \frac{1}{3} V_2$ and $E_1 = \frac{1}{3} E_2$
- C) $V_1 = \frac{1}{3} V_2$ and $E_1 = E_2$
- D) $V_1 = V_2$ and $E_1 = \frac{1}{3} E_2$



A 3. (5) Which of the following is **FALSE**?

- A) The electric field vector points towards higher potential.
- B) The surface of a conductor is an equipotential surface.
- C) Equipotentials are perpendicular to the electric field vector.
- D) The electric field inside a conductor in electrostatic equilibrium is zero.

B 4. (5) If the potential difference across a capacitor is doubled, the capacitance

- A) doubles
- B) remains unchanged
- C) halves
- D) decreases by a factor of four

20 /20 points for this page

5. A parallel plate capacitor consists of two circular plates of radius 10cm that are spaced 2mm apart with air between the plates. The electric field between the plates is 1.6×10^5 V/m.

a) (5) Calculate the voltage drop across the capacitor.

$$\Delta V = Ed = 1.6 \times 10^5 \frac{V}{m} \cdot 2 \times 10^{-3} m = 3.2 \times 10^2 V = 320V$$

b) (5) Calculate the capacitance.

$$C = \frac{k\epsilon_0 A}{d} = \frac{1.885 \times 10^{-12} \frac{C^2}{Nm^2} \cdot \pi (0.1m)^2}{2 \times 10^{-3} m} = 1.39 \times 10^{-10} F$$

c) (5) Calculate the amount of charge store on each plate.

$$Q = C \Delta V = 4.4 \times 10^{-8} C$$

d)(5) The capacitor is remains connected to the battery, and the plates are moved further apart. Which of these quantities remains unchanged? Circle the correct answer.

charge

capacitance

potential difference

electric field

Q ↓

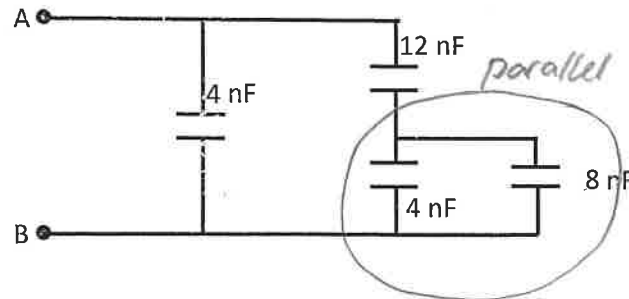
C ↓

$$\Delta V = Ed$$

$$E = \frac{\Delta V}{d} \quad \downarrow$$

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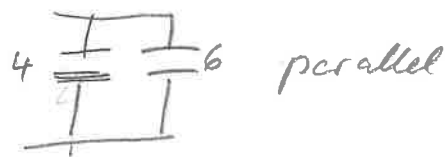
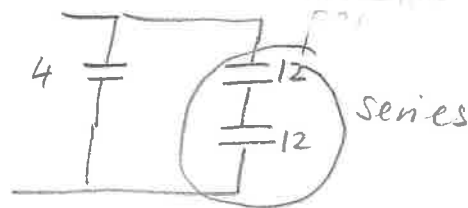
6. (20) For the capacitor circuit in the figure,
 $C_1 = 4 \text{ nF}$, $C_2 = 12 \text{ nF}$, $C_3 = 4 \text{ nF}$, and $C_4 = 8 \text{ nF}$.
 a)(15) Find the equivalent capacitance.



Parallel $C_{34} = 4 \text{ nF} + 8 \text{ nF} = 12 \text{ nF}$

Series: $\frac{1}{C_{234}} = \frac{1}{C_2} + \frac{1}{C_{34}} = \frac{1}{12 \text{ nF}} + \frac{1}{12 \text{ nF}}$
 $= \frac{2}{12 \text{ nF}} \Rightarrow C_{234} = 6 \text{ nF}$

Parallel $C_{eq} = 4 \text{ nF} + 6 \text{ nF} = 10 \text{ nF}$



- b) (5) If the applied voltage $V_{ab} = 36 \text{ V}$, find the total charge on the system.

$$Q = C_{eq} V = 10 \text{ nF} \cdot 36 \text{ V} = 360 \text{ nC}$$

- 7) a)(15) The potential difference between two plates of a parallel plate capacitor equals $1,500 \text{ V}$. A proton is launched from the positive plate with a speed of $1 \times 10^5 \text{ m/s}$. Derive a symbolic answer in terms of system parameters and calculate a numerical value for the speed with which the proton strikes the negative plate.

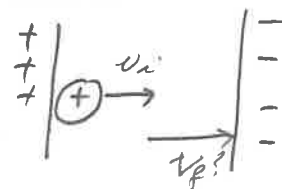
$$K_i + U_i = K_f + U_f$$

$$\Delta K = -q \Delta V$$

$$\frac{1}{2} m (v_f^2 - v_i^2) = -e (V_f - V_i)$$

$$v_f = \sqrt{v_i^2 - \frac{2e}{m} (V_f - V_i)} = \sqrt{(1 \times 10^5 \frac{\text{m}}{\text{s}})^2 - \frac{2 \cdot 1.6 \times 10^{-19} \text{ C}}{1.67 \times 10^{-27} \text{ kg}} \cdot (-1,500 \text{ V})}$$

$$v_f = 5.5 \times 10^5 \frac{\text{m}}{\text{s}}$$

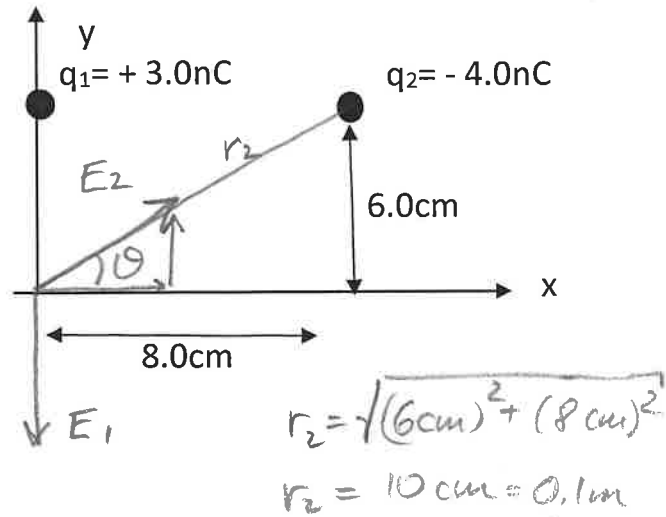


- b)(5) During this process, what is the proton's change in kinetic energy in electron volt?

$$1500 \text{ eV}$$

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8. (40) Two charges, $q_1 = +3.0 \text{ nC}$ and $q_2 = -4.0 \text{ nC}$, are located as shown in the figure. Charge q_1 is located on the y -axis at $y = 6.0 \text{ cm}$. Charge q_2 is at $x = 8.0 \text{ cm}$, $y = 6.0 \text{ cm}$.



a) (5) At the origin, draw the electric field vectors created by each of the charges.

b) (10) Calculate the magnitudes of the electric fields created by each of the charges at the origin.

$$E_1 = \frac{kq_1}{r_1^2} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{3 \times 10^{-9} \text{C}}{(6 \times 10^{-2} \text{m})^2}$$

$$E_1 = 7,500 \frac{\text{N}}{\text{C}}$$

$$E_2 = \frac{k|q_2|}{r_2^2} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{4 \times 10^{-9} \text{C}}{(0.1 \text{m})^2} = 3,600 \frac{\text{N}}{\text{C}}$$

c) (10) Calculate the x - and y - components of the net electric field at the origin.

$$E_{1x} = 0 \quad E_{1y} = -E_1 = -7500 \frac{\text{N}}{\text{C}}$$

$$E_{2x} = E_2 \cos \theta = 2880 \frac{\text{N}}{\text{C}}$$

$$E_{2y} = E_2 \sin \theta = 2160 \frac{\text{N}}{\text{C}}$$

$$\cos \theta = \frac{8}{10} = 0.8$$

$$\sin \theta = \frac{6}{10} = 0.6$$

$$E_{\text{net}x} = E_{1x} + E_{2x} = 2880 \frac{\text{N}}{\text{C}} \quad E_{\text{net}y} = E_{1y} + E_{2y} = -5340 \frac{\text{N}}{\text{C}}$$

d) (5) Calculate the magnitude of the net electric field at the origin.

$$E_{\text{net}} = \sqrt{E_{\text{net}x}^2 + E_{\text{net}y}^2} = 6070 \frac{\text{N}}{\text{C}}$$

e) (10) Calculate the electric potential at the origin.

$$V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \left(\frac{3 \times 10^{-9} \text{C}}{0.06 \text{m}} - \frac{4 \times 10^{-9} \text{C}}{0.1 \text{m}} \right) \quad /40 \text{ points for this page}$$

$$V = 450 \text{V} - 360 \text{V} = 90 \text{V}$$