

Name: Solution March 15, 2023

Total Score: 120 /120

$e = 1.602 \times 10^{-19} \text{ C}$      $I = \frac{\Delta q}{\Delta t}$      $I = \frac{V}{R}$      $R = \rho \frac{L}{A}$      $P = IV = \frac{V^2}{R} = I^2 R$

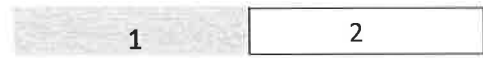
series:  $R_{eq} = \sum_i R_i$     parallel:  $\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$      $C = \frac{Q}{V}$

Discharging     $Q = Q_0 e^{-\frac{t}{RC}}$      $I = I_0 e^{-\frac{t}{RC}}$      $V_C = V_{C0} e^{-\frac{t}{RC}}$

Charging     $Q = Q_f (1 - e^{-\frac{t}{RC}})$      $I = I_0 e^{-\frac{t}{RC}}$      $V_C = V_{Cf} (1 - e^{-\frac{t}{RC}})$

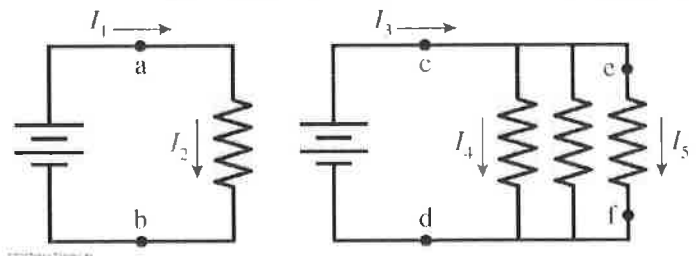
B 1. (5) Current flows through a resistor which consists of two segments of equal lengths and equal radii in series. The resistivity of segment 1 is three times that of segment 2,  $\rho_2 = 3\rho_1$ . If the current in segment 1 is  $I_1$ , how big is the current in segment 2 ?

- A)  $\frac{1}{3} I_1$     B)  $I_1$     C)  $3 I_1$     D)  $6 I_1$



B 2. (5) In the two circuits on the right, the batteries are identical and all resistors are identical. Which of the statements is **true**?

- A)  $I_1 > I_2$     B)  $I_3 = 3 I_1$   
 C)  $I_3 = I_1$     D)  $I_1 > I_4$



C 3. (5) The potential difference across a length of wire is decreased. Which of the following does **not** decrease as well?

- A) Electric field in the wire    B) Power dissipated in the wire  
 C) Resistance of the wire    D) Current through the wire

D 4. (5) The time constant of an RC circuit is the time it takes

- A) for the capacitor to be completely charged.  
 B) for the current to reach its maximum value.  
 C) for the current to drop to zero.  
 D) for the current to decrease to 37% of its initial value.

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5) (20) A nichrome wire is 50 cm long. The resistivity of nichrome is  $1.5 \times 10^{-6} \Omega \cdot \text{m}$ . The potential difference between the ends is 8.0V, and a current of 4A is flowing through the wire.

a) (5) Calculate the resistance of the wire.

$$I = \frac{V}{R} \quad R = \frac{V}{I} = \frac{8V}{4A} = 2\Omega$$

b) (5) Calculate the diameter of the wire's circular cross section.

$$R = \rho \frac{L}{A} = \rho \frac{L}{\frac{1}{4}\pi d^2}$$
$$d = \sqrt{\frac{4\rho L}{\pi R}} = \sqrt{\frac{4 \cdot 1.5 \times 10^{-6} \Omega \cdot \text{m} \cdot 0.5 \text{m}}{\pi \cdot 2\Omega}} = 6.9 \times 10^{-4} \text{m}$$

c) (5) Calculate the power dissipated in the wire.

$$P = IV = 4A \cdot 8V = 32W$$

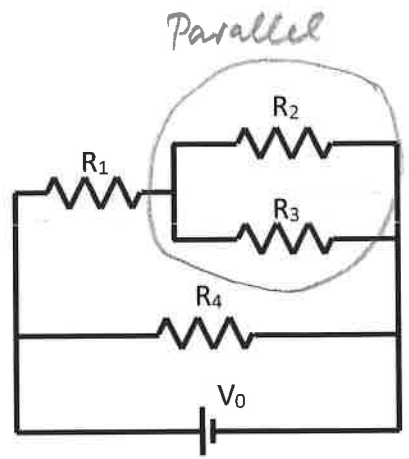
d) (5) How many electrons per second pass through the cross section of the wire?

$$I = \frac{\Delta q}{\Delta t} = \frac{Ne}{\Delta t}$$

$$N = \frac{I \Delta t}{e} = \frac{4A \cdot 1s}{1.6 \times 10^{-19} C} = 2.5 \times 10^{19}$$

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6. a) (20) For the resistor circuit shown,  $R_1 = 2.0 \Omega$ ,  $R_2 = 3.0 \Omega$ ,  $R_3 = 6.0 \Omega$ , and  $R_4 = 4.0 \Omega$ . The battery provides a potential difference of  $V_0 = 12 \text{ V}$ .



(a) (20) Find the equivalent resistance.

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{3\Omega} + \frac{1}{6\Omega} = \frac{2+1}{6\Omega} = \frac{1}{2\Omega}$$

$$R_{23} = 2\Omega$$

Series:  $R_1 + R_{23} = 2\Omega + 2\Omega = 4\Omega = R_{123}$

parallel:  $\frac{1}{R_{eq}} = \frac{1}{R_{123}} + \frac{1}{R_4} = \frac{1}{4\Omega} + \frac{1}{4\Omega} = \frac{2}{4\Omega} \Rightarrow R_{eq} = 2\Omega$

b) (20) Calculate the potential drop across each of the resistors and the current through each resistor. Put the answers in the table below.

$R_4$ :  $V_4 = V_0$        $I_4 = \frac{12V}{4\Omega} = 3A$

total current:  $I_{tot} = \frac{V_0}{R_{eq}} = \frac{12V}{2\Omega} = 6A$

$\Rightarrow$  leaves 3A for other branches

$V_1 = I_1 R_1 = 3A \cdot 2\Omega = 6V \Rightarrow$  leaves 6V for  $R_{23}$

$I_2 = \frac{6V}{R_2} = \frac{6V}{3\Omega} = 2A$

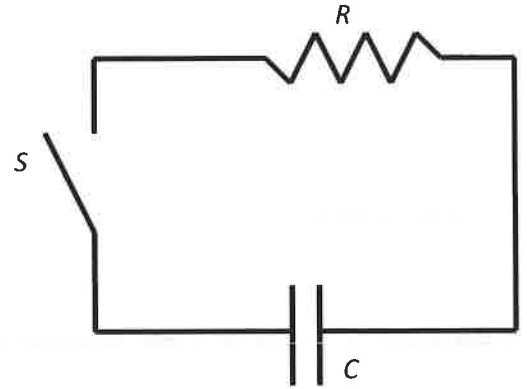
$I_3 = \frac{6V}{R_3} = \frac{6V}{6\Omega} = 1A$

adds to 3A

$V_1 =$ 6V	$I_1 =$ 3A
$V_2 =$ 6V	$I_2 =$ 2A
$V_3 =$ 6V	$I_3 =$ 1A
$V_4 =$ 12V	$I_4 =$ 3A

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7. (40) A capacitor of capacitance  $C = 4 \mu\text{F}$  has been charged so that the potential difference between its plates is 100 V. The capacitor is then connected to a  $5\text{k}\Omega$  resistor as shown. The switch  $S$  is closed and the capacitor begins to discharge.



a) (5) Calculate the time constant of the circuit.

$$\tau = RC = 5 \times 10^3 \Omega \cdot 4 \times 10^{-6} \text{ F} = 0.02 \text{ s}$$

b) (5) Determine the charge on the capacitor when the switch is closed at  $t=0$ .

$$C = \frac{Q}{\Delta V} \quad Q = C \cdot V_0 = 4 \mu\text{F} \cdot 100 \text{ V} = 400 \mu\text{C}$$

c) (5) Calculate the current through the resistor immediately after the switch is closed.

$$I = \frac{\Delta V}{R} = \frac{100 \text{ V}}{5 \times 10^3 \Omega} = 20 \text{ mA}$$

d) (10) Calculate the current 10 ms after the switch is closed.

$$I = I_0 e^{-\frac{10 \text{ ms}}{20 \text{ ms}}} = 20 \text{ mA} e^{-\frac{1}{2}} = 12 \text{ mA}$$

e) (15) Calculate the time after which the charge has decreased to one third its maximum value.

$$Q = Q_0 e^{-\frac{t}{RC}} = \frac{1}{3} Q_0$$

$$e^{-\frac{t}{RC}} = \frac{1}{3}$$

$$-\frac{t}{RC} = \ln \frac{1}{3}$$

$$t = -RC \ln \frac{1}{3} = RC \ln 3$$

$$t = 20 \text{ ms} \cdot \ln 3$$

$$t = 22 \text{ ms}$$

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