Today's agenda:

Electric potential energy.

You must be able to use electric potential energy in work-energy calculations.

Electric potential.

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The electron volt.

You must be able to use the electron volt as an alternative unit of energy.

Potential energy vs. electric potential

Two new quantities:

- (electric) potential energy U
- electric potential V

Do not mix them up!

(electric) potential energy ≠ electric potential

(Electric) potential energy

Work done by electric (Coulomb) force:

$$\left[W_E\right]_{i\to f} = \int_{r_i}^{r_f} \vec{F}_E \cdot d\vec{\ell}$$

- **independent** of path -> force is **conservative**

Define **potential energy U:** (unit J or Nm)

$$U_f - U_i = -\left[W_E\right]_{i\to f} = -\int_{r_i}^{r_f} \vec{F}_E \cdot d\vec{\ell}$$

equivalently: $\vec{F}_E = -\dfrac{\partial U}{\partial \vec{r}}$

- potential energy defined w.r.t. (initial) **reference state**

Potential energy of two point charges

- two point charges q_1 and q_2, initially at infinite distance
- moved to distance r_{12}

$$U_f - U_i = -\int_{r_i}^{r_f} \vec{F}_E \cdot d\vec{\ell} = -\int_{\infty}^{r_{12}} \frac{k\, q_1 q_2}{r^2}\, dr = \frac{k\, q_1 q_2}{r_{12}}$$

potential energy of two point charges

$$\boxed{U(r_{12}) = k\,\frac{q_1 q_2}{r_{12}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}}$$

works for positive and negative charges

potential energy is zero in **reference state** when particles are infinitely far apart

You **must** use this convention if you want to use the equation for potential energy of point charges! If you use the above equation, you are “automatically” using this convention.

Potential energy of point charge in electric field

- force on point (test) charge: $\vec{F} = q\vec{E}$

$$U_f - U_i = -\int_{r_i}^{r_f} \vec{F} \cdot d\vec{\ell} = -q\int_{r_i}^{r_f} \vec{E} \cdot d\vec{\ell}$$

potential energy of point charge in electric field:

$$\boxed{U_f - U_i = -q\int_{i}^{f} \vec{E} \cdot d\vec{\ell}}$$

Example: calculate the electric potential energy of two protons separated by a typical proton-proton intranuclear distance of $2x10^{-15}$ m.

$+1.15x10^{-13}$ J

To be worked at the blackboard in lecture.

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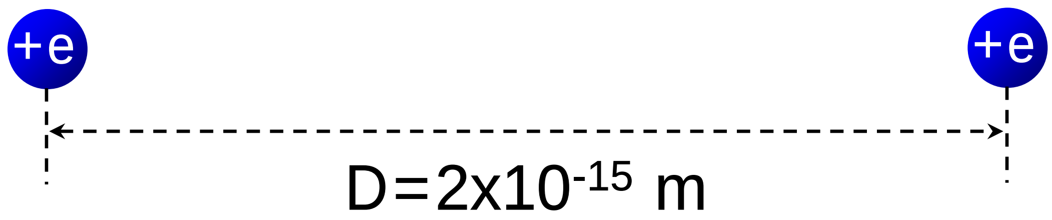


$$U = k\,\frac{q_1 q_2}{r_{12}} = k\,\frac{(+e)(+e)}{D} = 9\times10^{9}\,\frac{\left(+1.6\times10^{-19}\right)\left(+1.6\times10^{-19}\right)}{\left(2\times10^{-15}\right)}$$

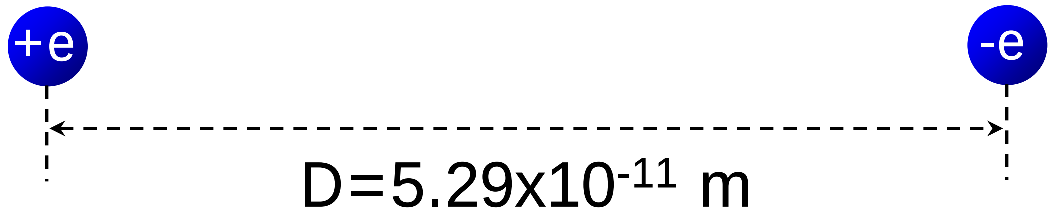
$$U = +1.15\times10^{-13}\ \text{J}$$

What is the meaning of the + sign in the result?

Example: calculate the electric potential energy of a hydrogen atom (electron-proton distance is 5.29×10^{-11} m). -4.36×10^{-18} J

To be worked at the blackboard in lecture.

Example: calculate the electric potential energy of a hydrogen atom (electron-proton distance is $5.29\text{x}10^{-11}$ m). $-4.36\text{x}10^{-18}$ J



$$U = k\,\frac{q_1 q_2}{r_{12}} = k\,\frac{(+e)(-e)}{D} = 9\times10^{9}\,\frac{\left(+1.6\times10^{-19}\right)\left(-1.6\times10^{-19}\right)}{\left(5.29\times10^{-11}\right)}$$

$$U = -4.36\times10^{-18}\ \text{J}$$

What is the meaning of the - sign in the result? Is that a small energy? I’ll have more to say about the energy at the end of the lecture.

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Work-energy problems

- system of charged particle has electric potential energy
- if charges move, kinetic and potential energies change

Energy conservation law:

$$E_f - E_i = (K_f + U_f) - (K_i + U_i) = \left[W_{other}\right]_{i \to f}$$

This is from Physics 1135.

Important: Distinguish conservative work and external work

$$\Delta U = U_f - U_i = -\left[W_{\text{conservative}}\right]_{i\to f}$$

change in potential energy is defined as the negative of the work done by the conservative force which is associated with the potential energy (today, **the electric force**).

If an external force moves an object "against" the conservative force,* and the object's kinetic energy remains constant, then

$$\left[W_{\text{external}}\right]_{i\to f} = -\left[W_{\text{conservative}}\right]_{i\to f}$$

Always ask yourself which work you are calculating!

*for example, if you "slowly" lift a book, or "slowly" push two negatively charged balloons together

Example: two isolated protons are constrained to be a distance $D = 2x10^{-10}$ meters apart (a typical atom-atom distance in a solid). If the protons are released from rest, what maximum speed do they achieve, and how far apart are they when they reach this maximum speed?

$2.63x10^4$ m/s

To be worked at the blackboard in lecture...

Example: two isolated protons are constrained to be a distance $D = 2 \times 10^{-10}$ meters apart (a typical atom-atom distance in a solid). If the protons are released from rest, what maximum speed do they achieve, and how far apart are they when they reach this maximum speed?

2.63×10^{4} m/s

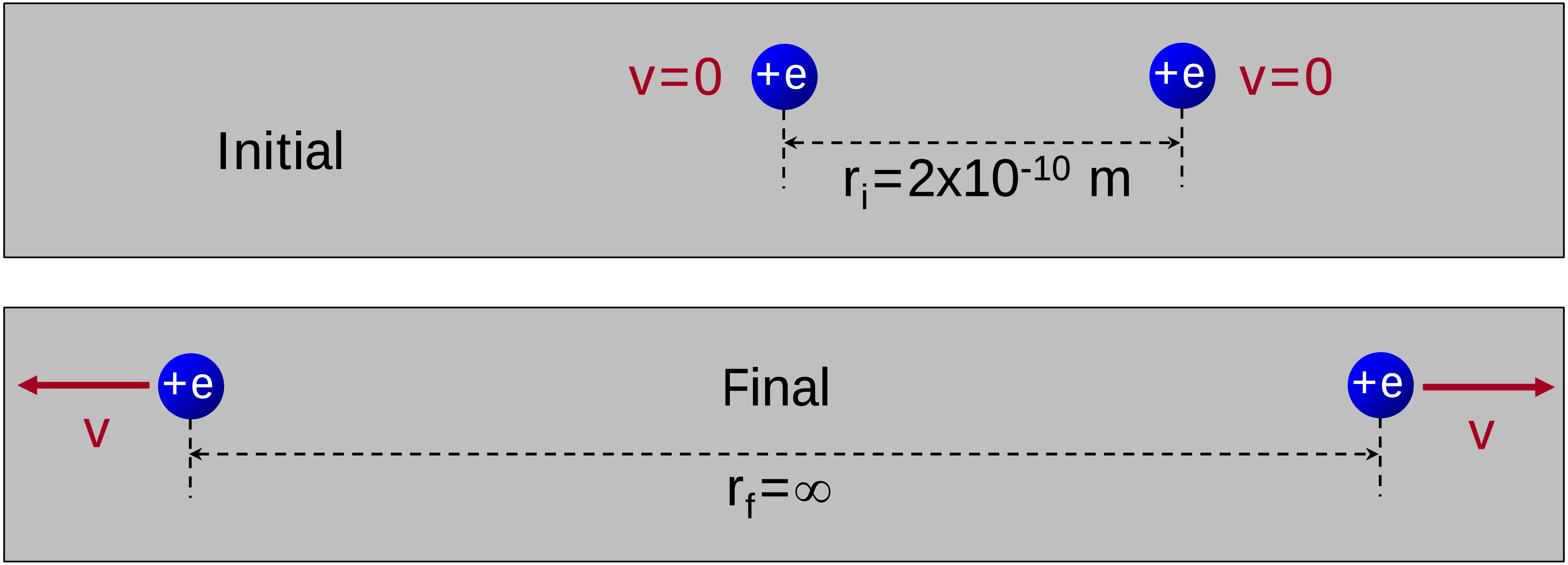
We need to do some thinking first.

What is the proton’s potential energy when they reach their maximum speed?

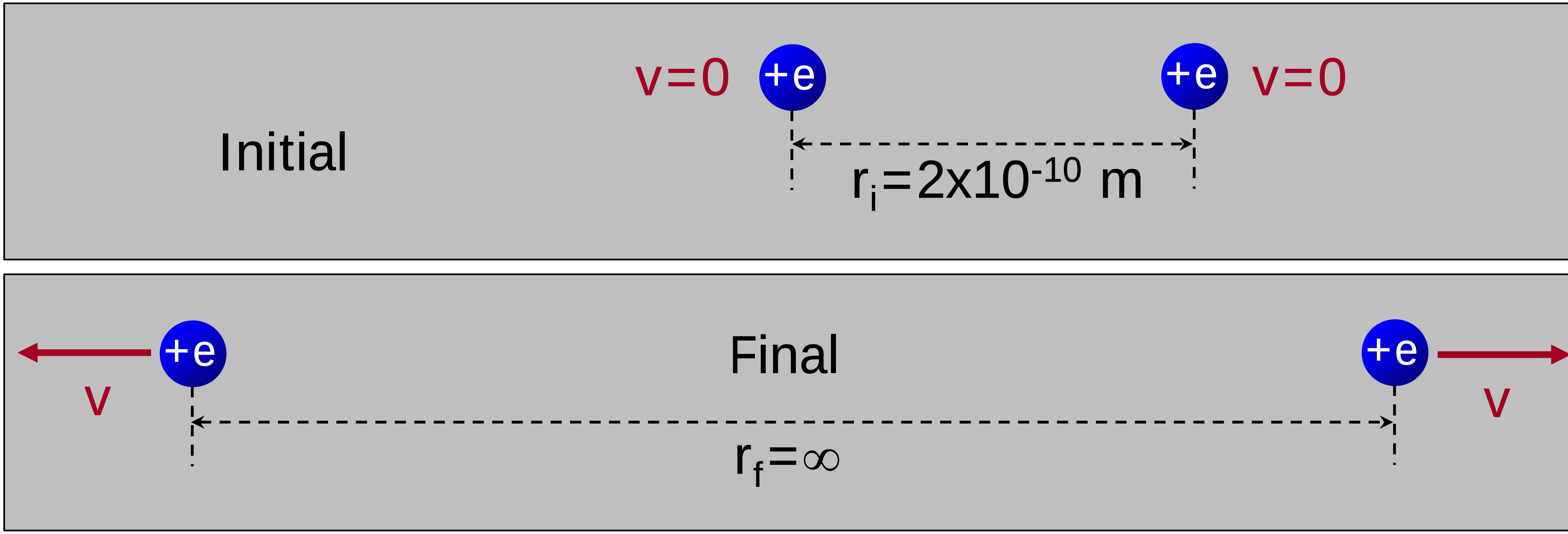
How far apart are the protons when they reach their maximum speed?

Example: two isolated protons are constrained to be a distance $D = 2x10^{-10}$ meters apart (a typical atom-atom distance in a solid). If the protons are released from rest, what maximum speed do they achieve, and how far apart are they when they reach this maximum speed?

$2.63x10^{4}$ m/s



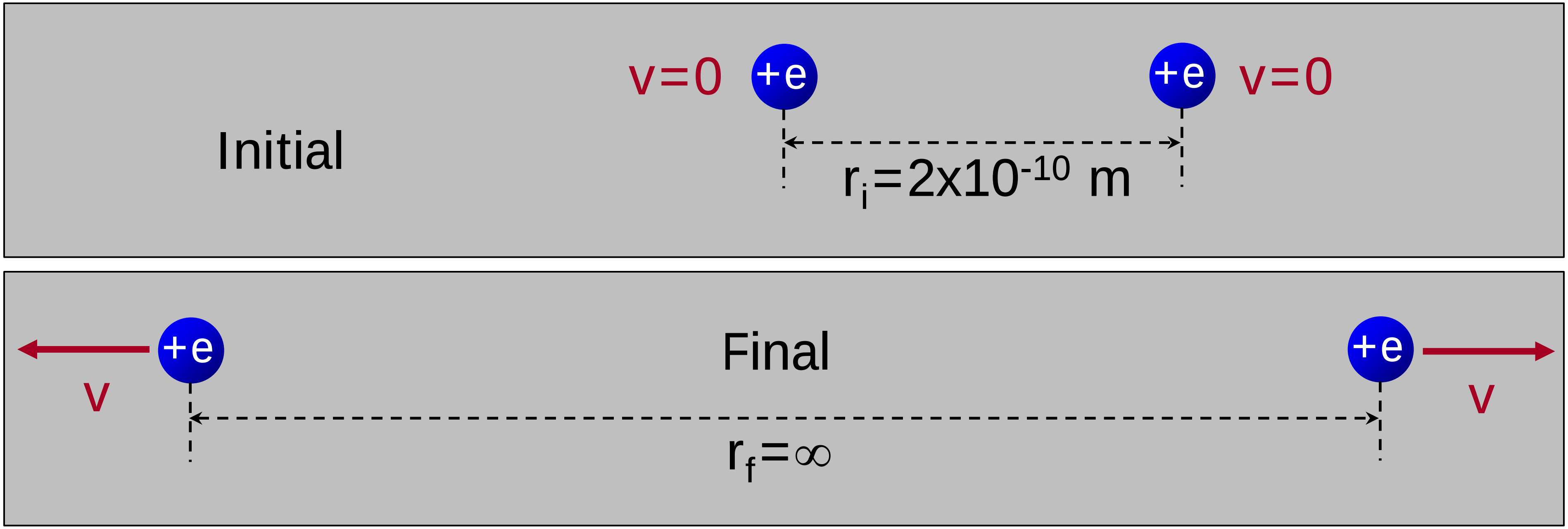
Why are the two speeds the same? There is a conservation of momentum problem buried in here!



$$E_f - E_i = \left[W_{other}\right]_{i\to f}$$

$$K_f + \cancel{U_f}^{\,0} - \left(\cancel{K_i}^{\,0} + U_i\right) = \left[\cancel{W_{other}}^{\,0}\right]_{i\to f}$$

$$K_f = U_i$$



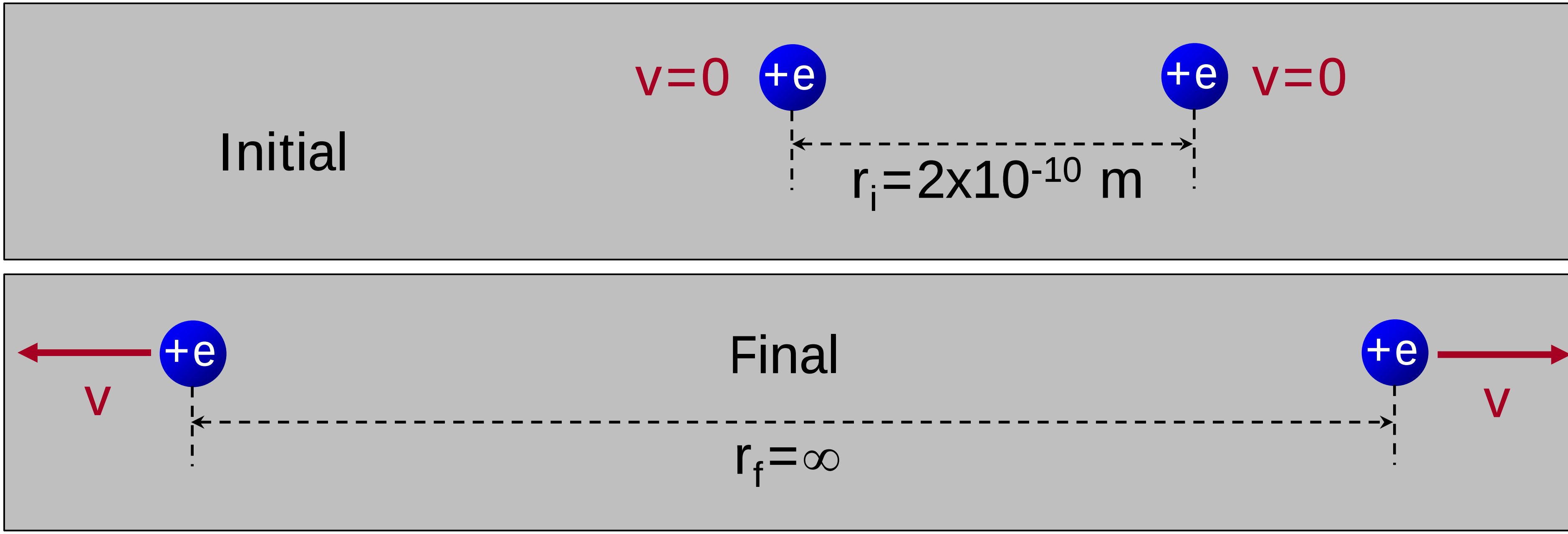
$$K_f = U_i$$

How many objects are moving in the final state? Two.

How many K_f terms are there? Two.

How many **pairs** of charged particles in the initial state? One.

How many U_i terms are there? One.

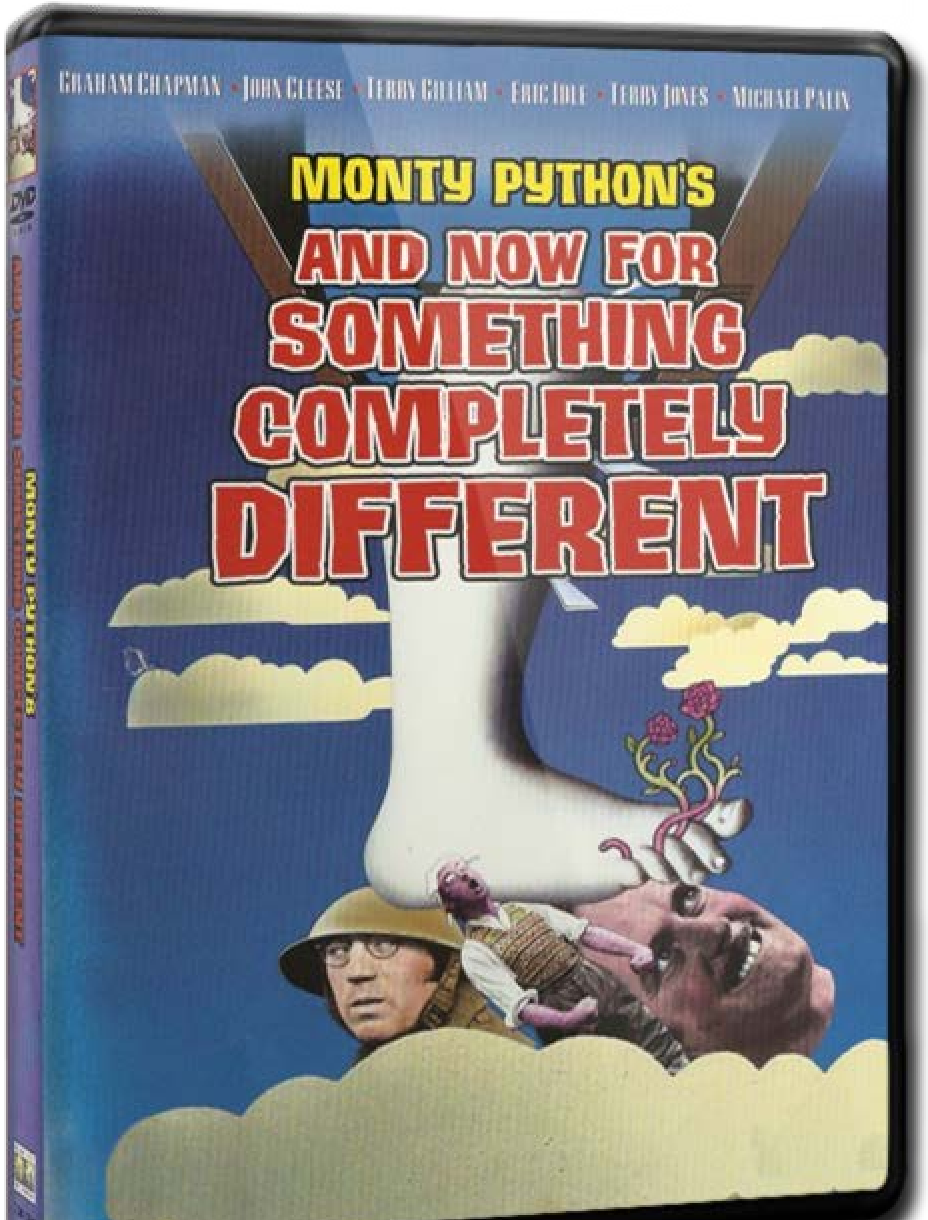


$$K_f = U_i$$

$$2\left(\frac{1}{2}m_p v^2\right) = k\frac{(+e)(+e)}{r_i}$$

$$v = \sqrt{\frac{ke^2}{m_p r_i}} = \sqrt{\frac{(9\times10^{9})(1.6\times10^{-19})^2}{(1.67\times10^{-27})(2\times10^{-10})}} = 2.63\times10^{4}\ \frac{m}{s}$$

Is that fast, or slow?


GRAHAM CHAPMAN • JOHN CLEESE • TERRY GILLIAM • ERIC IDLE • TERRY JONES • MICHAEL PALIN
MONTY PYTHON'S
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Electric Potential

Lecture 1: defined electric field by force it exerts on test charge

$$\vec{F} = q\vec{E}$$

Now: define **electric potential V** via potential energy of test charge

$$U = qV$$

$$\boxed{U_f - U_i = q\,(V_f - V_i)}$$

source charges create electric potential V, test charge q feels the potential, this produces potential energy U

unit of electric potential: Nm/C = V   (Volt)

Electric Potential of a point charge:

$$V(r) = \frac{U(r)}{q_1} = \frac{1}{q_1}\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{12}}$$

q_1 is the test charge, q_2 is the source charge

so that the electric potential of a point charge q is

$$\boxed{V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\ .}$$

Only valid for a point charge!

Relation between electric potential and electric field

$$V_f - V_i = \frac{U_f - U_i}{q} = \frac{-\int_{r_a}^{r_b} \vec{F}_E \cdot d\vec{\ell}}{q} = -\int_{r_a}^{r_b} \frac{\vec{F}_E}{q} \cdot d\vec{\ell} = -\int_{r_a}^{r_b} \vec{E} \cdot d\vec{\ell} \ .$$

$$\boxed{V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{\ell}}$$

Two conceptual examples.

Example: a proton is released in a region in space where there is an electric potential. Describe the subsequent motion of the proton.

The proton will move towards the region of lower potential. As it moves, its potential energy will decrease, and its kinetic energy and speed will increase.

Example: a electron is released in a region in space where there is an electric potential. Describe the subsequent motion of the electron.

The electron will move towards the region of higher potential. As it moves, its potential energy will decrease, and its kinetic energy and speed will increase.

Protons fall down, electrons fall up.

What is the potential due to the proton in the hydrogen atom at the electron's position (5.29×10^{-11} m away from the proton)? 27.2V

To be worked at the blackboard in lecture.

Important note:

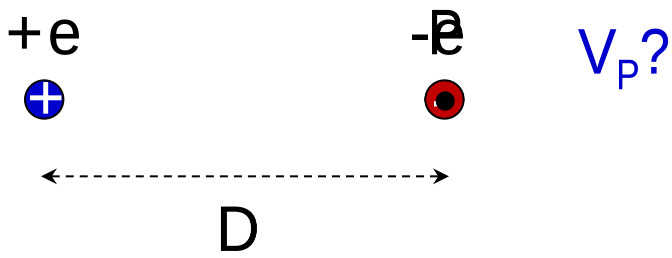
V this is the symbol for electrical potential

V this is the symbol for the unit (volts) of electrical potential

v this is the symbol for magnitude of velocity, or speed

Don't get your v's and V's mixed up! Hint: write your speed v's as script v's, like this (or however you want to clearly indicate a lowercase v): 

What is the potential due to the proton in the hydrogen atom at the electron's position (5.29×10^{-11} m away from the proton)? 27.2V



$$V_p = \frac{kq}{r} = \frac{k(+e)}{D} = \frac{(9 \times 10^{9})(+1.6 \times 10^{-9})}{(5.29 \times 10^{-11})} = 27.2\ \mathrm{V}$$

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Electric Potential Energy of a System of Charges

Electric potential energy comes from the interaction between pairs of charged particles, so you have to add the potential energies of **each pair** of charged particles in the system.

(Could be a pain to calculate!)

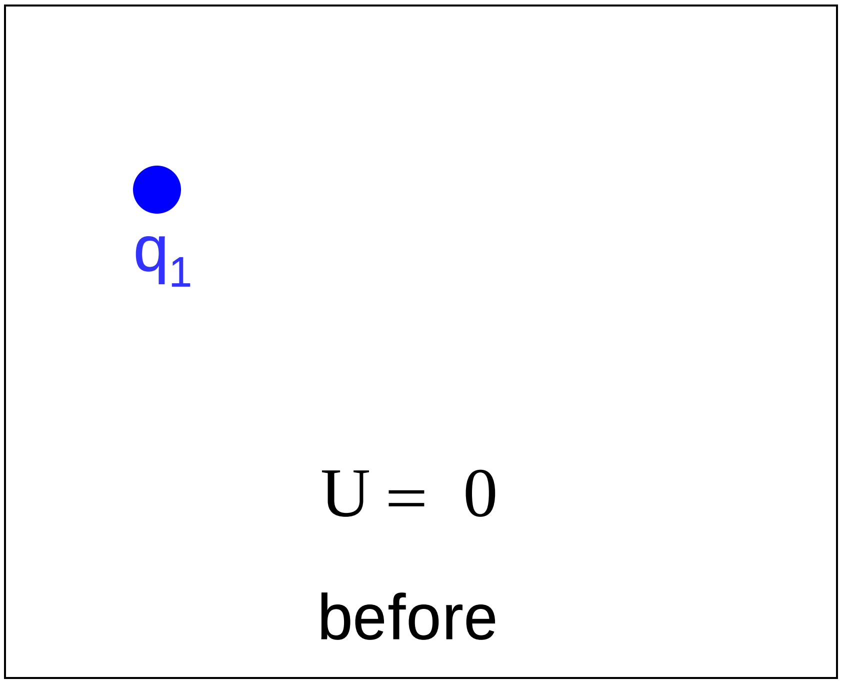
Electric Potential of a System of Charges

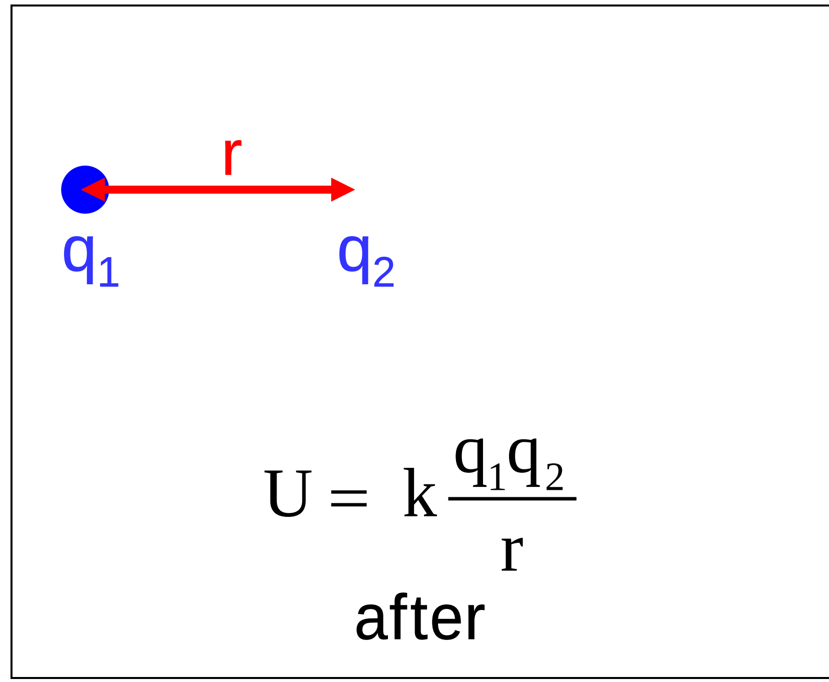
The potential due to a particle depends only on the charge of that particle and where it is relative to some reference point.

The electric potential of a system of charges is simply the sum of the potential of each charge. (Much easier to calculate!)

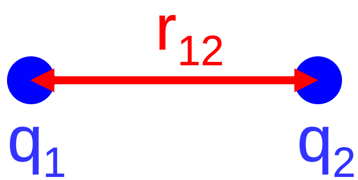
Example: electric potential energy of three charged particles

A single charged particle has no electrical potential energy. To find the electric potential energy for a system of two charges, we bring a second charge in from an infinite distance away:



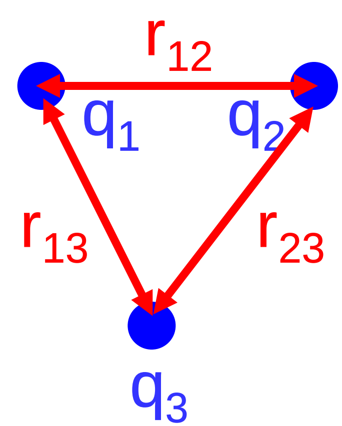


To find the electric potential energy for a system of three charges, we bring a third charge in from an infinite distance away:



before

$$U = k\frac{q_1 q_2}{r_{12}}$$



after

$$U = k\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

We have to add the potential energies of each pair of charged particles.

Electric Potential of a Charge Distribution (details next lecture)

Collection of charges: $V_P = \dfrac{1}{4\pi\varepsilon_0} \sum_i \dfrac{q_i}{r_i}.$

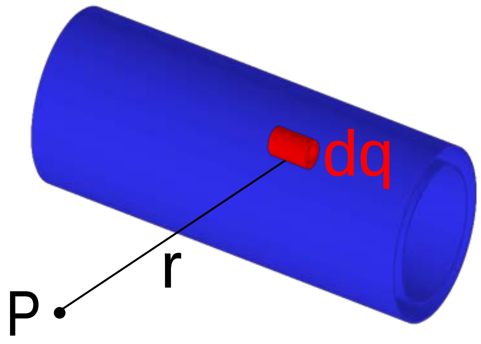
P is the point at which V is to be calculated, and r_i is the distance of the i^{th} charge from P.

Charge distribution:

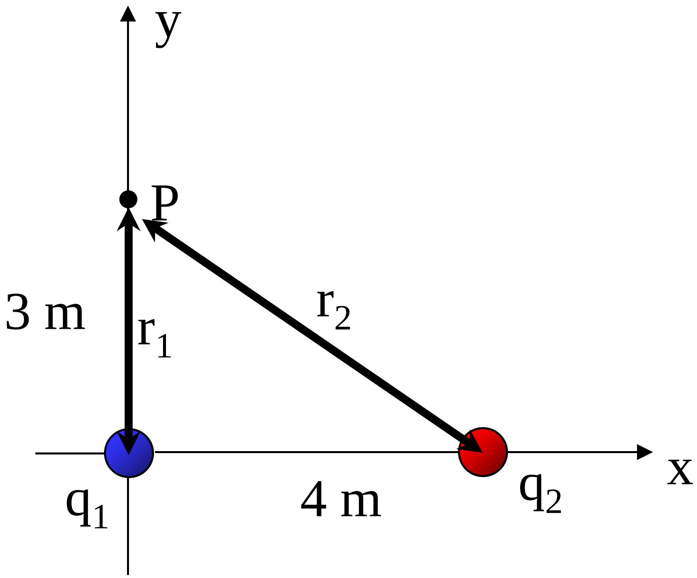
$$\boxed{V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r}\ .}$$

Potential at point P.

We'll work with this next lecture.



Example: a 1 μC point charge is located at the origin and a -4 μC point charge 4 meters along the +x axis. Calculate the electric potential at a point P, 3 meters along the +y axis.

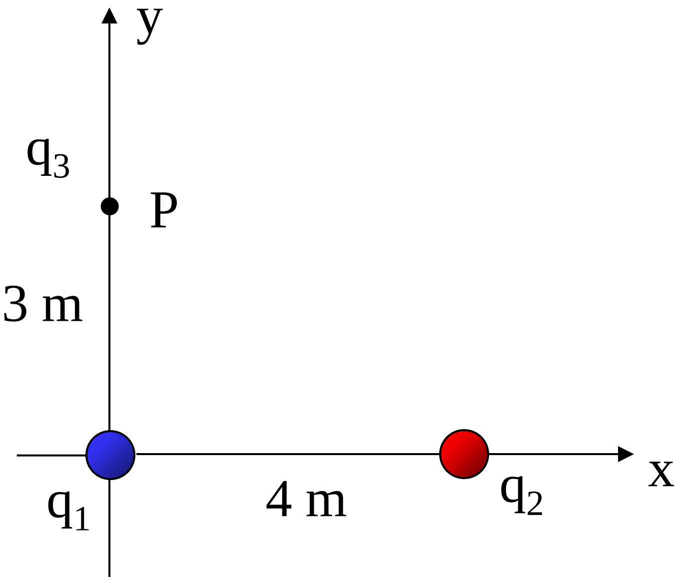


$$V_P = k\sum_i \frac{q_i}{r_i} = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$= 9\times10^9\left(\frac{1\times10^{-6}}{3} + \frac{-4\times10^{-6}}{5}\right)$$

$$= -4.2\times10^3\ \text{V}$$

Example: how much work is required to bring a +3 μC point charge from infinity to point P? (And what assumption must we make?)



$$W_{\text{external}} = \Delta E = \cancelto{0}{\Delta K} + \Delta U$$

$$W_{\text{external}} = \Delta U = q_3 \Delta V$$

$$W_{\text{external}} = q_3 \left(V_P - \cancelto{0}{V_\infty} \right)$$

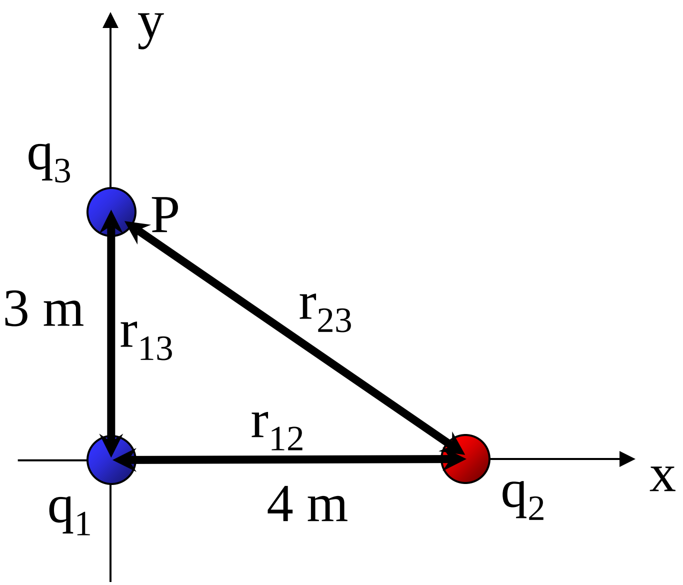
$$W_{\text{external}} = 3 \times 10^{-6} \left(-4.2 \times 10^{3} \right)$$

$$W_{\text{external}} = -1.26 \times 10^{-3}\,\text{J}$$

The work done by the external force was negative, so the work done by the electric field was positive. The electric field "pulled" q_3 in (keep in mind $|q_2|$ is 4 times $|q_1|$).

Positive work would have to be done by an external force to remove q_3 from P.

Example: find the total potential energy of the system of three charges.



$$U = k\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$U = 9\times10^{9}\left(\frac{\left(1\times10^{-6}\right)\left(-4\times10^{-6}\right)}{4} + \frac{\left(1\times10^{-6}\right)\left(3\times10^{-6}\right)}{3} + \frac{\left(-4\times10^{-6}\right)\left(3\times10^{-6}\right)}{5}\right)$$

$$U = -2.16\times10^{-2}\ \mathrm{J}$$

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The Electron Volt

An electron volt (eV) is the energy acquired by a particle of charge e when it moves through a potential difference of 1 volt.

$$\Delta U = q\Delta V$$

$$1 \text{ eV} = \left(1.6\times10^{-19}\text{C}\right)\left(1 \text{ V}\right)$$

$$\boxed{1 \text{ eV} = 1.6\times10^{-19}\text{J}}$$

This is a very small amount of energy on a macroscopic scale, but electrons in atoms typically have a few eV (10’s to 1000’s) of energy.

Example: on slide 9 we found that the potential energy of the hydrogen atom is about -4.36x10^{-18} joules. How many electron volts is that?

$$U = -4.36\times10^{-18}\text{J} = \left(-4.36\times10^{-18}\text{J}\right)\left(\frac{1\text{ eV}}{1.6\times10^{-19}\text{J}}\right) \approx -27.2\text{ eV}$$

"Hold it! I learned in Chemistry (or high school physics) that the ground-state energy of the hydrogen atom is -13.6 eV. Did you make a physics mistake?"

The ground-state energy of the hydrogen atom includes the positive kinetic energy of the electron, which happens to have a magnitude of half the potential energy. Add KE+PE to get ground state energy.

Homework Hints!

You'll need to use starting equations from Physics 1135!

Remember your Physics 1135 hammer equation?

$$E_f - E_i = [W_{other}]_{i \to f}$$

What "goes into" E_f and E_i? What "goes into" W_{other}?

This is also handy: $$U_f - U_i = -[W_c]_{i \to f}$$

Homework Hints!

Work-Energy Theorem:

$$\left[W_{net}\right]_{i \to f} = \Delta K$$

"Potential of a with respect to b" means V_a - V_b