Exam 2: Tuesday, March 21, 5:00-6:00 PM

Test rooms:

Instructor	Sections	Room
Dr. Hale	F, H	104 Physics
Dr. Kurter	B, N	125 BCH
Dr. Madison	K, M	199 Toomey
Dr. Parris	J, L	B-10 Bertelsmeyer*
Mr. Upshaw	A, C, E, G	G-3 Schrenk
Dr. Waddill	D	120 BCH
Special Accommodations		Testing Center

(Contact me a.s.a.p. if you need accommodations different than for exam 1)

*new room

Exam 2 will cover chapters 24.3 to 27
(energy stored in capacitors to forces and torques on currents)

Today’s agenda:

Magnetic Field Due To A Current Loop.

You must be able to apply the Biot-Savart Law to calculate the magnetic field of a current loop.

Ampere’s Law.

You must be able to use Ampere’s Law to calculate the magnetic field for high-symmetry current configurations.

Solenoids.

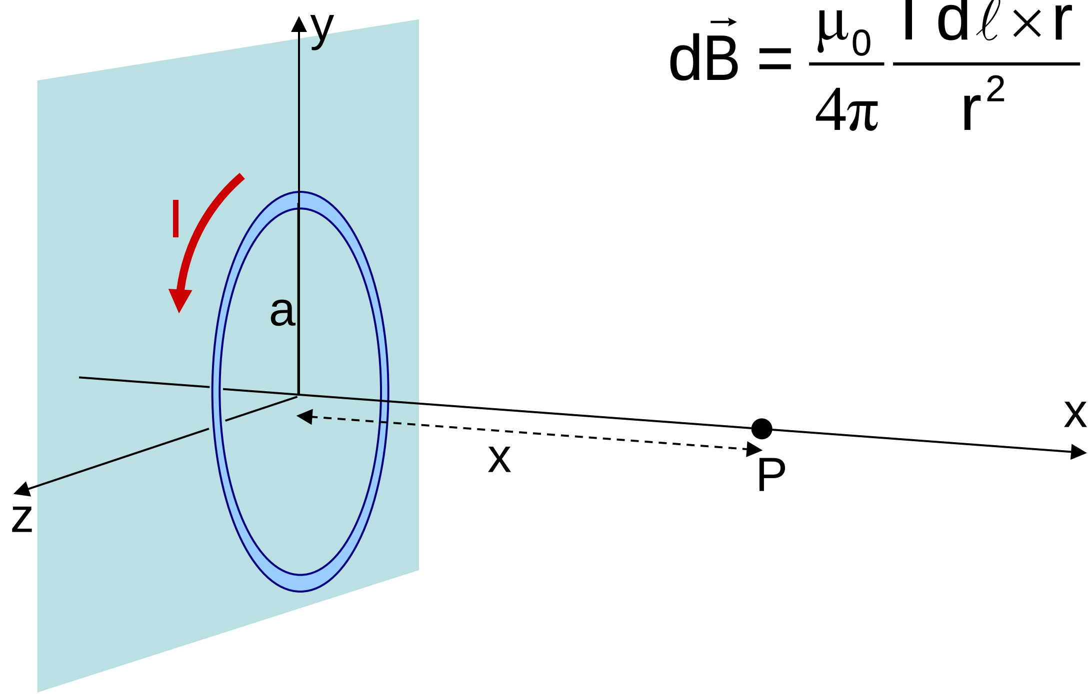
You must be able to use Ampere’s Law to calculate the magnetic field of solenoids and toroids.

Magnetic Field of a Current Loop

A circular ring of radius a carries a current I as shown. Calculate the magnetic field at a point P along the axis of the ring at a distance x from its center.

Draw a figure. Write down the starting equation. It tells you what to do next.

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$$

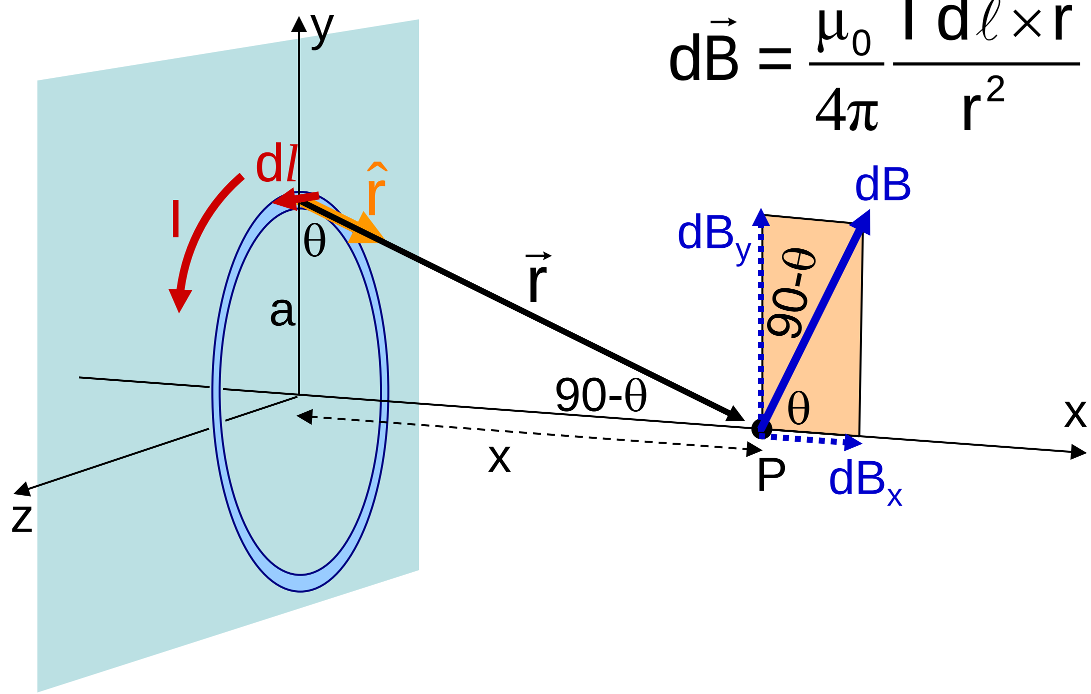


Magnetic Field of a Current Loop

A circular ring of radius a carries a current I as shown. Calculate the magnetic field at a point P along the axis of the ring at a distance x from its center.

Complicated diagram! You are supposed to visualize the ring lying in the yz plane.

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$$



$d\vec{l}$ is in the yz plane. $\hat{r}$ is in the xy plane and is perpendicular to $d\vec{l}$.* Thus $\left| d\vec{\ell} \times \hat{r} \right| = d\ell$.

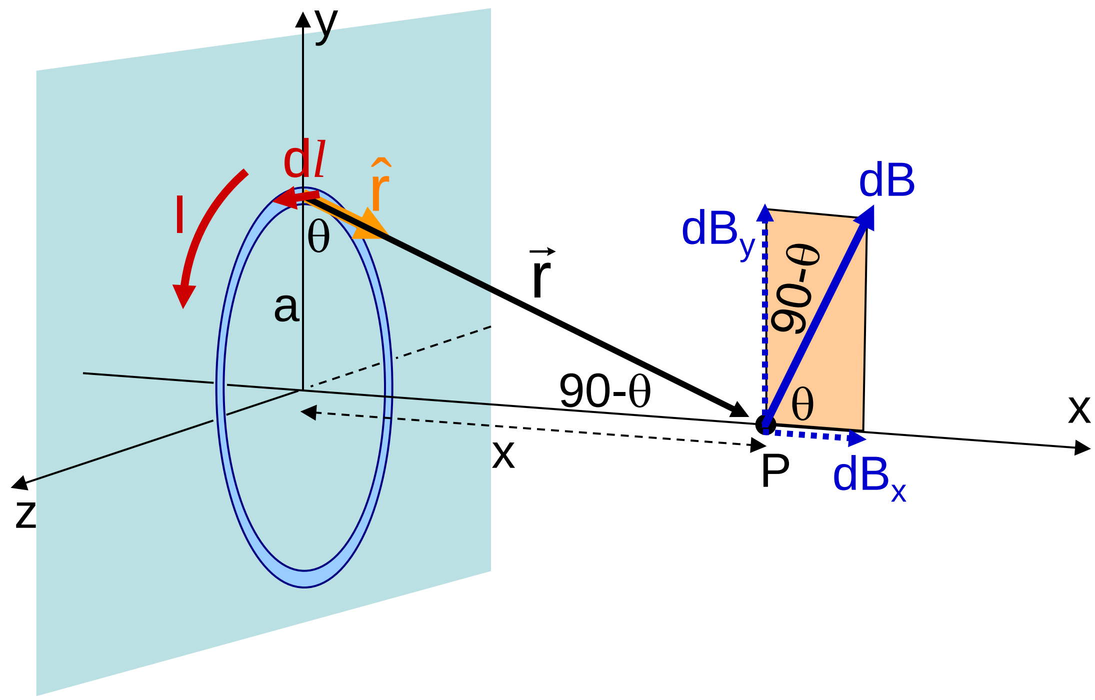
Also, $d\vec{B}$ must lie in the xy plane* (perpendicular to $d\vec{l}$) and is perpendicular to $\vec{r}$.

*Only when $d\vec{l}$ is centered on the y-axis!

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{I\, d\ell}{r^2}$$

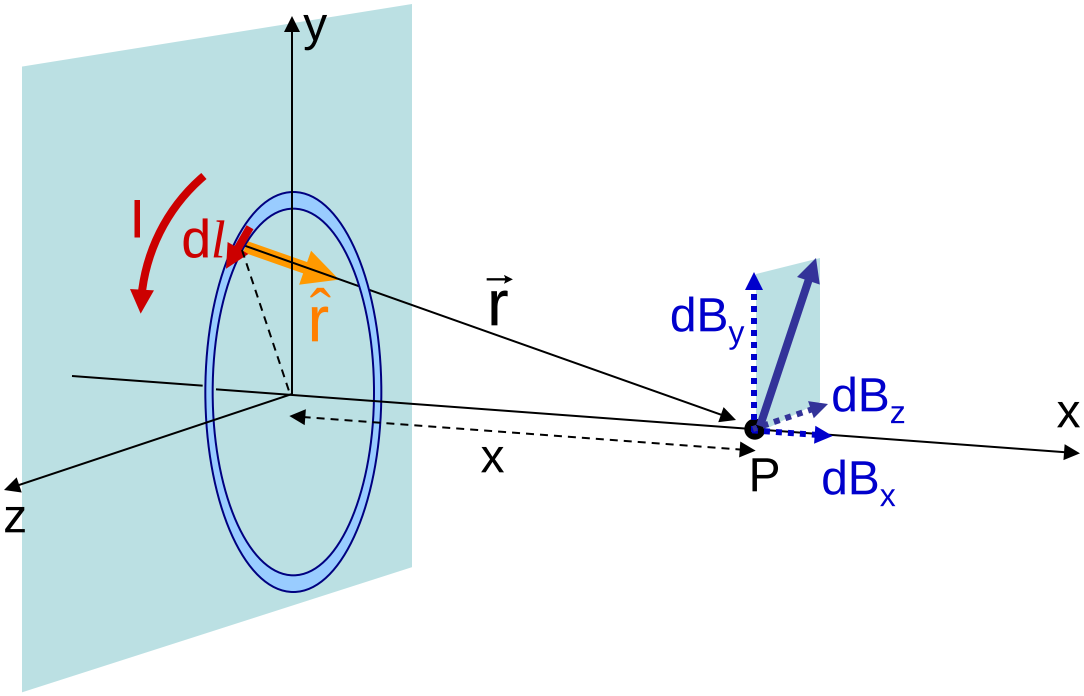
$$dB = \frac{\mu_0}{4\pi}\frac{I\, d\ell}{\left(x^2 + a^2\right)}$$



$$dB_x = \frac{\mu_0}{4\pi}\frac{I\, d\ell}{\left(x^2 + a^2\right)}\cos\theta = \frac{\mu_0}{4\pi}\frac{I\, d\ell}{\left(x^2 + a^2\right)}\frac{a}{\left(x^2 + a^2\right)^{1/2}}$$

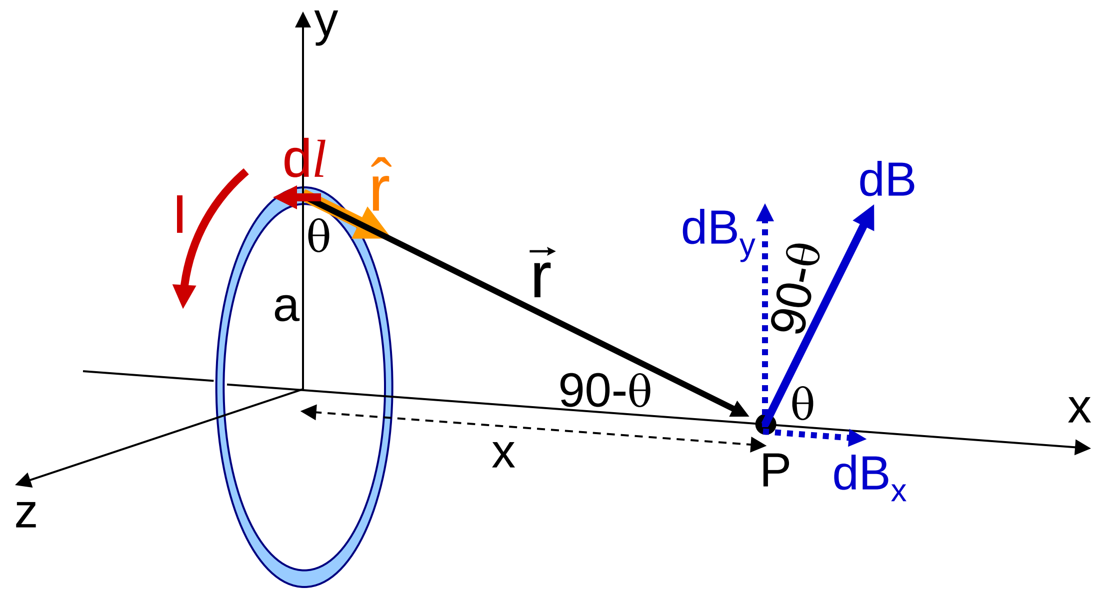
By symmetry, B_y will be 0. Do you see why?

Use symmetry to find B_y. Don't try to integrate dB_y to get B_y. See here for the reason.



When $d\vec{l}$ is not centered at z=0, there will be a z-component to the magnetic field, but by symmetry B_z will be zero.

$$dB_x = \frac{\mu_0}{4\pi}\frac{I\,a\,d\ell}{\left(x^2+a^2\right)^{3/2}}$$



$$B_x = \int_{\text{ring}} dB_x$$

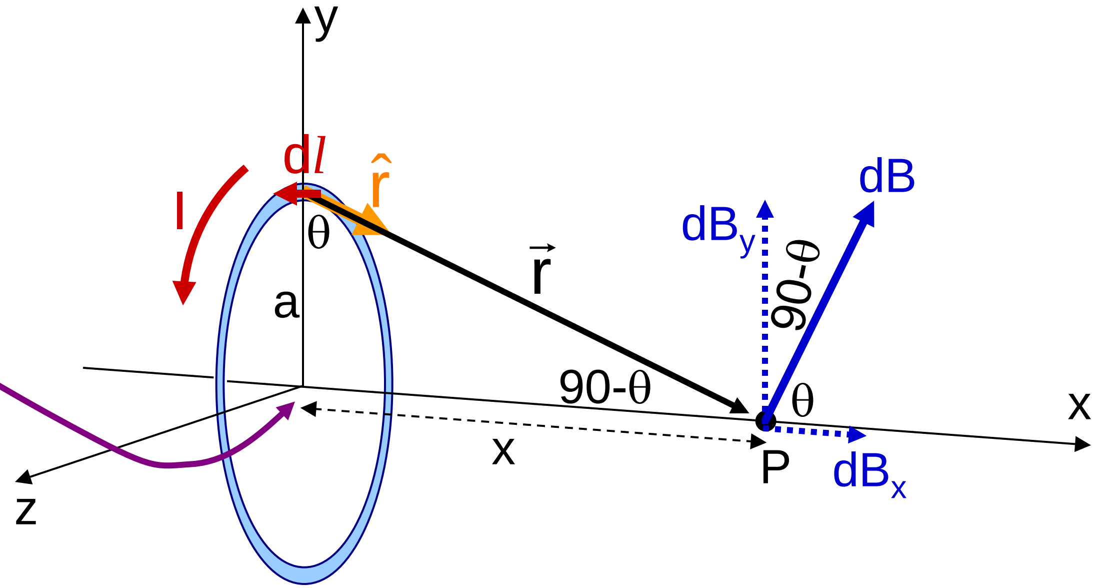
I, x, and a are constant as you integrate around the ring!

$$B_x = \frac{\mu_0}{4\pi}\frac{I\,a}{\left(x^2+a^2\right)^{3/2}}\int_{\text{ring}} d\ell = \frac{\mu_0}{4\pi}\frac{I\,a}{\left(x^2+a^2\right)^{3/2}}2\pi a$$

$$B_x = \frac{\mu_0\,I\,a^2}{2\left(x^2+a^2\right)^{3/2}}$$

This is not on your starting equation sheet

At the center of the ring, x=0.



$$B_{x,center} = \frac{\mu_0 \, I \, a^2}{2\left(a^2\right)^{3/2}}$$

$$B_{x,center} = \frac{\mu_0 \, I \, a^2}{2a^3} = \frac{\mu_0 \, I}{2a}$$

For N **tightly** packed concentric rings (a tight coil)...

$$B_{x,center} = \frac{\mu_0 \, N \, I}{2a}$$

This is not on your starting equation sheet.
For homework, if a problem requires this equation, you need to derive it!

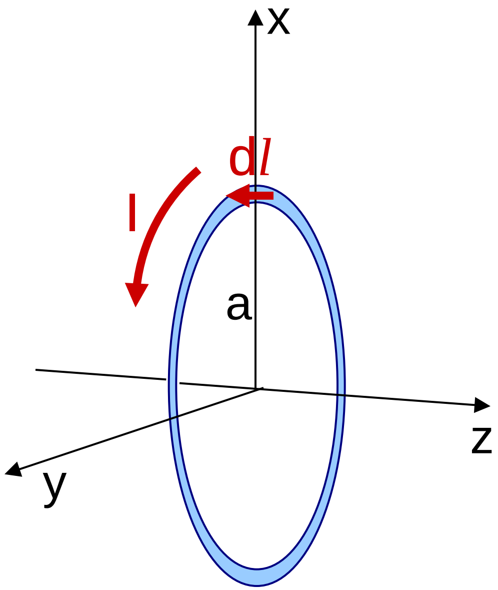
Magnetic Field at the center of a Current Loop

A circular ring of radius a lies in the xy plane and carries a current I as shown. Calculate the magnetic field at the center of the loop.

If you **only** look at the center of the loop, derivation is simpler.

Work on blackboard!

The direction of the magnetic field will be different if the plane of the loop is not in the xy plane.



Today’s agenda:

Magnetic Field Due To A Current Loop.

You must be able to apply the Biot-Savart Law to calculate the magnetic field of a current loop.

Ampere’s Law.

You must be able to use Ampere’s Law to calculate the magnetic field for high-symmetry current configurations.

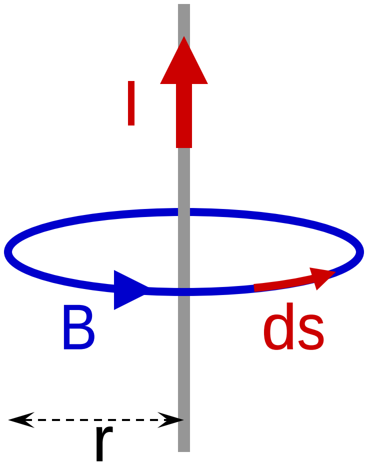
Solenoids.

You must be able to use Ampere’s Law to calculate the magnetic field of solenoids and toroids.

Recall:

- magnetic field of long straight wire:

$$B = \frac{\mu_0 I}{2\pi r}$$ winds around the wire



Line integral of B over a closed circular path around wire:

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = B(2\pi r)$$

$$\vec{B} \parallel d\vec{s}$$

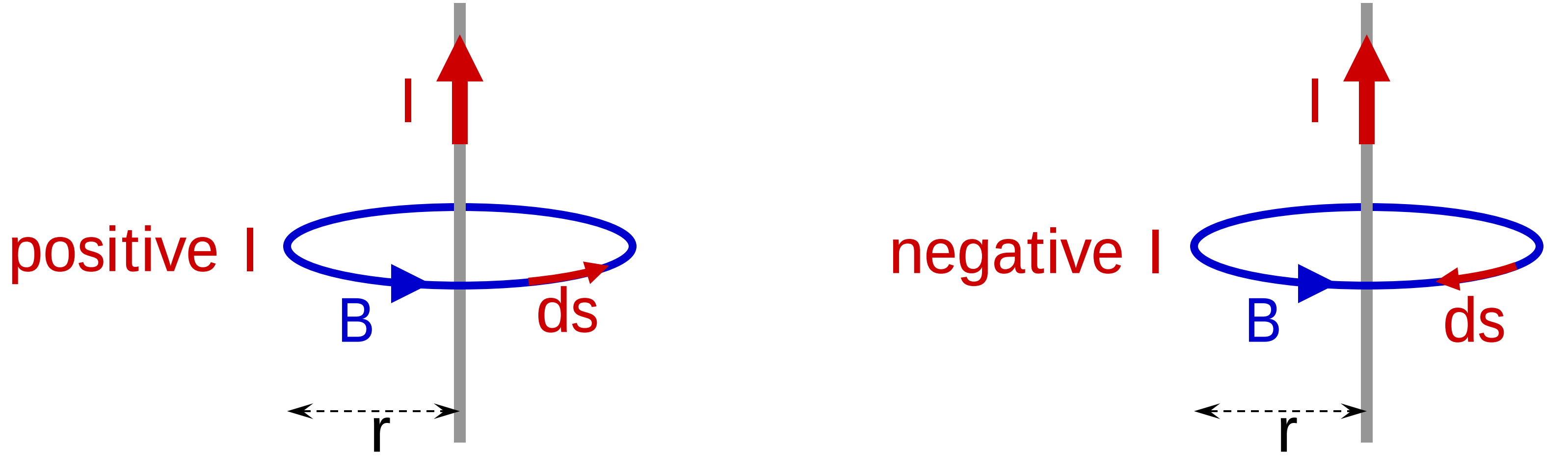
$$\oint \vec{B} \cdot d\vec{s} = \left(\frac{\mu_0 I}{2\pi r} \right) (2\pi r) = \mu_0 I$$

Is this an accident, valid only for this particular situation?

Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \, I_{encl}$$ Ampere's Law

- I_{encl} is total current that passes through surface bounded by closed path of integration.
- **law of nature:** holds for any closed path and any current distribution
- current I counts positive if integration direction is the same as the direction of $\vec{B}$ from the right hand rule



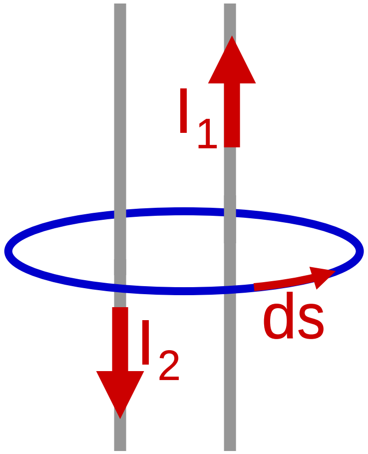
- starting equation on your OSE sheet contains second term:

$$\boxed{\oint \vec{B}\cdot d\vec{s} = \mu_0 \left[I_{encl} + \cancelto{0}{\kappa\varepsilon_0 \frac{d\Phi_E}{dt}} \right]}$$

The reason for the 2nd term on the right will become apparent later. Set it equal to zero for now.

- if path includes more than one source of current, add all currents (with correct sign).

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 \left(I_1 - I_2 \right)$$



- Ampere’s law can be used **to calculate magnetic fields** in **high-symmetry situations**

Recipe for using Ampere’s law to find magnetic fields

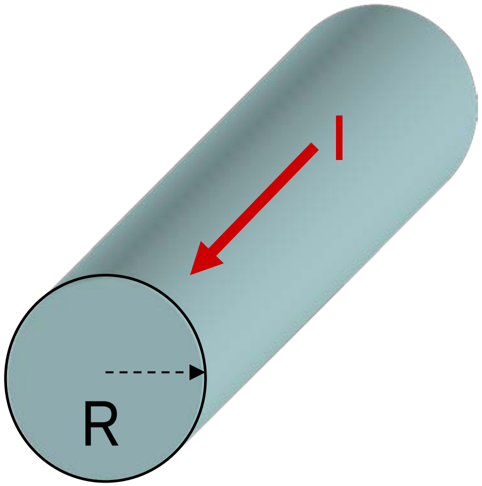
- requires **high-symmetry situations** so that line integral can be disentangled
- analogous to Gauss’ law calculations for electric field

1. Use symmetry to find direction of magnetic field

2. Choose Amperian path such that
   (a) it respects the symmetry, usually $\vec{B} \,||\, d\vec{s}$
   (b) and goes through point of interest

3. Start from Amperes law, perform integration, solve for B

Example: a cylindrical wire of radius R carries a current I that is uniformly distributed over the wire's cross section. Calculate the magnetic field inside and outside the wire.

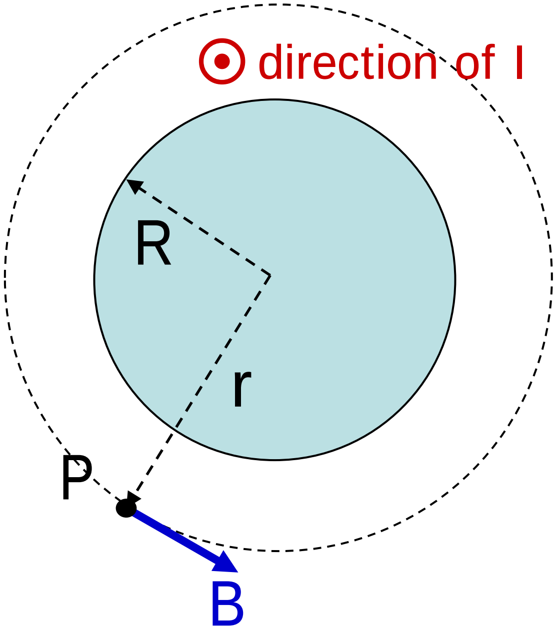


Example: a cylindrical wire of radius R carries a current I that is uniformly distributed over the wire's cross section. Calculate the magnetic field inside and outside the wire.

Outside the wire:

1. B field tangential to circles around wire

2. Chose circular Amperian path around wire through P



3. Integrate:

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = 2\pi r B = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

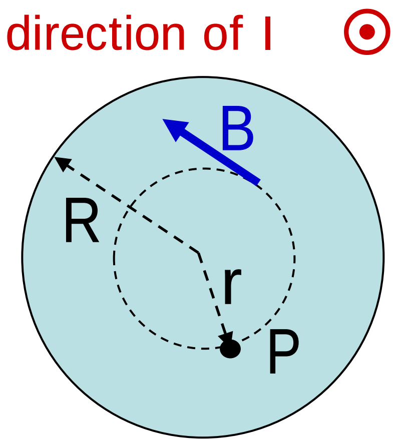
a lot easier than using Biot-Savart Law!

Example: a cylindrical wire of radius R carries a current I that is uniformly distributed over the wire's cross section. Calculate the magnetic field inside and outside the wire.

Inside the wire:

- Only part of current enclosed by Amperian path

$$I_{encl} = I\frac{(A\text{ enclosed by } r)}{(A\text{ enclosed by } R)} = I\frac{(\pi r^2)}{(\pi R^2)} = I\frac{r^2}{R^2}$$



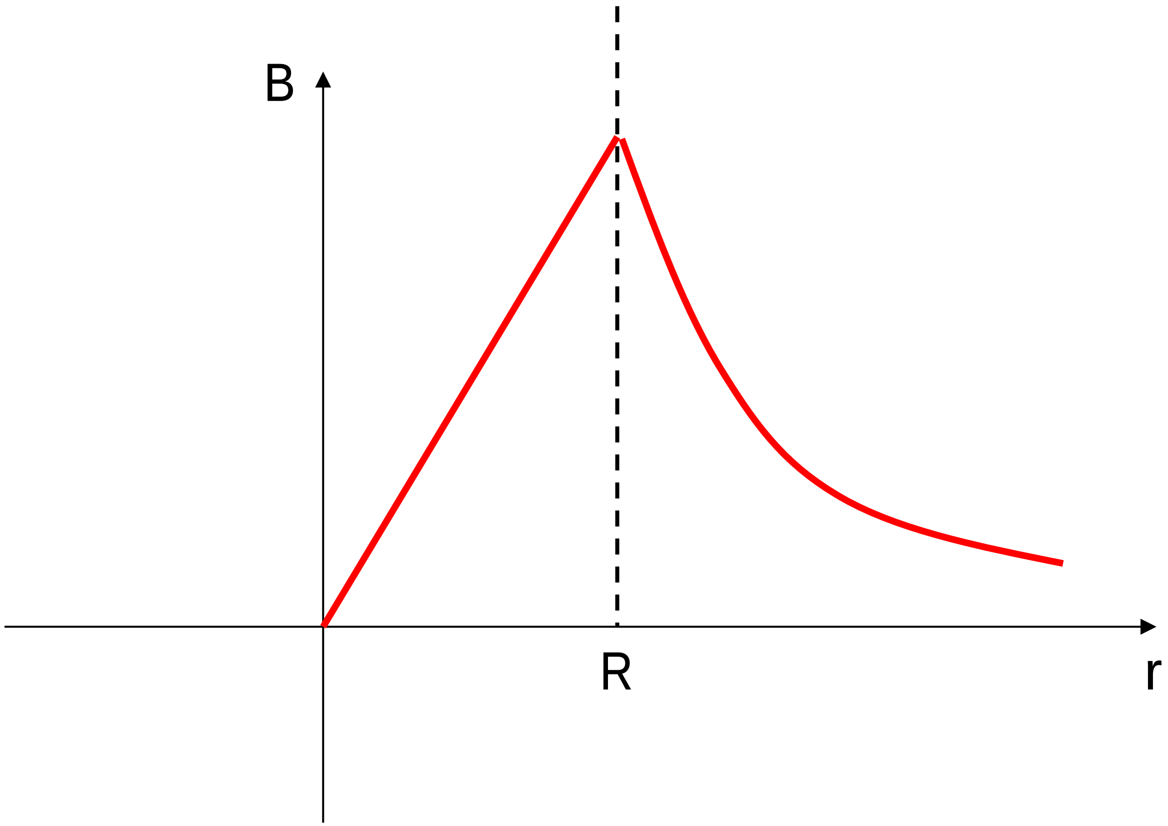
Ampere's law:

$$\oint \vec{B}\cdot d\vec{s} = B\oint ds = 2\pi r B = \mu_0\, I_{encl} = \mu_0\, I\frac{r^2}{R^2}$$

Solve for B:

$$B = \mu_0\, I\frac{r^2}{2\pi r R^2} = \mu_0\, I\frac{r}{2\pi R^2} = \frac{\mu_0\, I}{2\pi R^2}r$$

Plot:



Calculating Electric and Magnetic Fields

Electric Field

in general: Coulomb's Law

for high symmetry configurations: Gauss' Law
(surface integral)

Magnetic Field

in general: Biot-Savart Law

for high symmetry configurations: Ampere's Law
(line integral)

This analogy is rather flawed because Ampere's Law is not really the "Gauss' Law of magnetism."

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Ampere’s Law.

You must be able to use Ampere’s Law to calculate the magnetic field for high-symmetry current configurations.

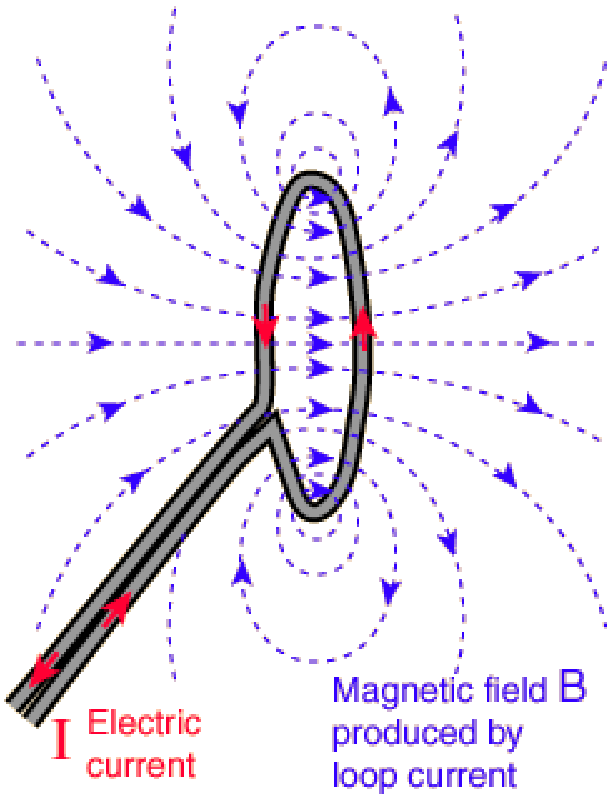
Solenoids.

You must be able to use Ampere’s Law to calculate the magnetic field of solenoids and toroids. You must be able to use the magnetic field equations derived with Ampere’s Law to make numerical magnetic field calculations for solenoids and toroids.

Magnetic Field of a Solenoid

A solenoid is made of many loops of wire, packed closely to form long cylinder.

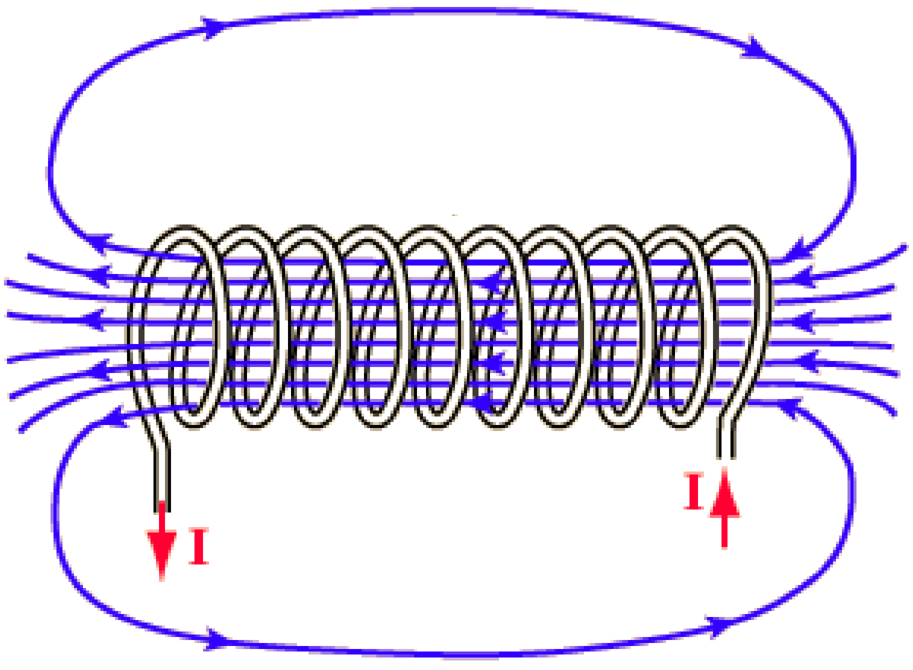
Single loop:



images from hyperphysics.

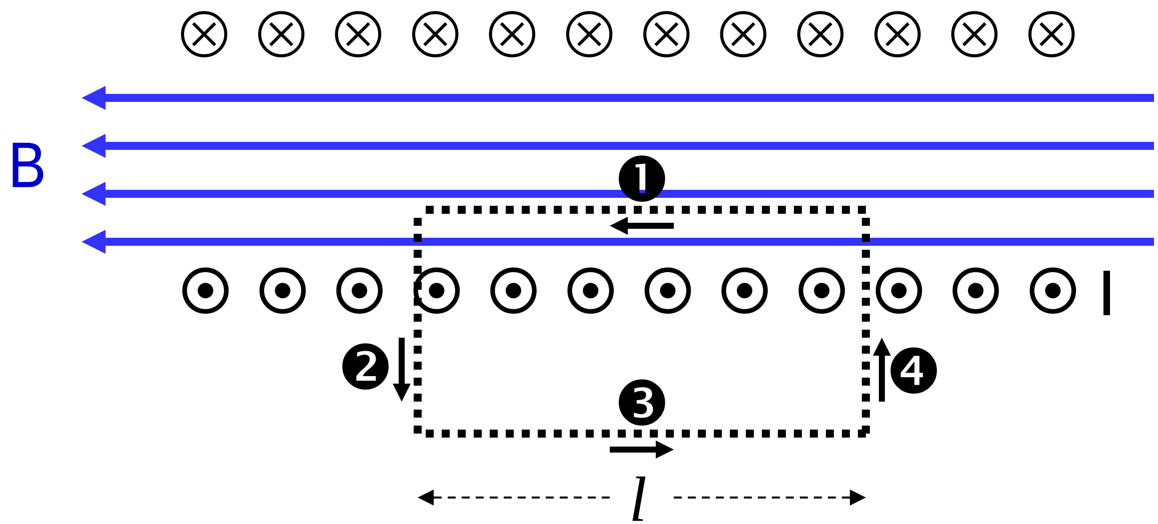
*But not so closely that you can use $B = \frac{\mu_0 N I}{2a}$

Stack many loops to make a solenoid:



The magnetic field is concentrated into a nearly uniform field in the center of a long solenoid. The field outside is weak and divergent.

Ought to remind you of the magnetic field of a bar magnet.



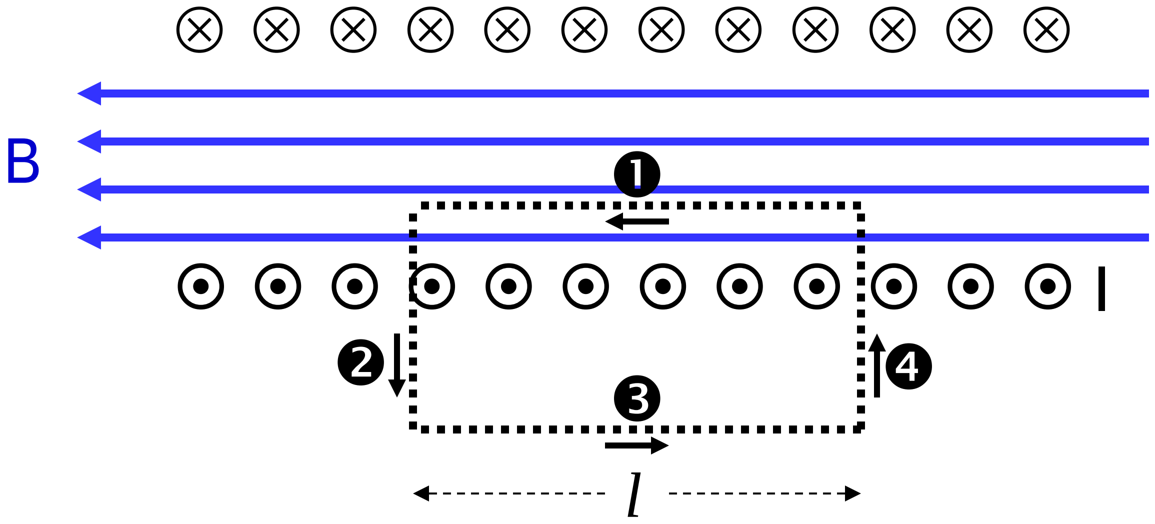
Use Ampere's law to calculate the magnetic field of a solenoid:

$$\oint \vec{B}\cdot d\vec{s} = \int_1 \vec{B}\cdot d\vec{s} + \int_2 \vec{B}\cdot d\vec{s} + \int_3 \vec{B}\cdot d\vec{s} + \int_4 \vec{B}\cdot d\vec{s}$$

$$\oint \vec{B}\cdot d\vec{s} = B\ell + 0 + 0 + 0 = \mu_0 I_{\text{enclosed}}$$

$$B\ell = \mu_0 N I$$

N is the number of loops enclosed by our surface.



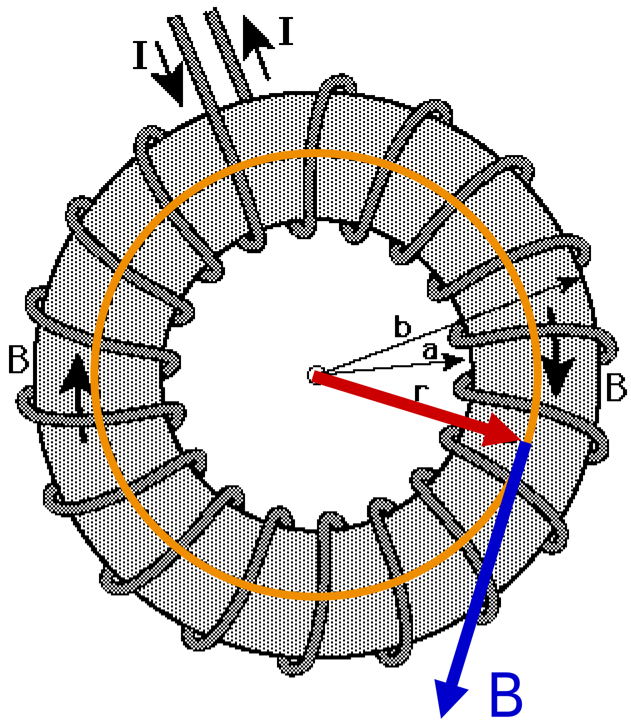
$$B = \mu_0 \frac{N}{\ell} I$$

$$B = \mu_0 n I$$

Magnetic field of a solenoid of length l , N loops, current I. $n=N/l$ (number of turns per unit length).

The magnetic field inside a long solenoid does not depend on the position inside the solenoid (if end effects are neglected).

A toroid* is just a solenoid "hooked up" to itself.



$$\oint \vec{B}\cdot d\vec{s} = \mu_0\, I_{enclosed} = \mu_0 N I$$

$$\oint \vec{B}\cdot d\vec{s} = B\int ds = B(2\pi r)$$

$$B(2\pi r) = \mu_0 N I$$

$$\boxed{B = \frac{\mu_0 N I}{2\pi r}}$$

Magnetic field inside a toroid of N loops, current I.

The magnetic field inside a toroid is not subject to end effects, but is not constant inside (because it depends on r).

*Your text calls this a "toroidal solenoid."

Example: a thin 10-cm long solenoid has a total of 400 turns of wire and carries a current of 2 A. Calculate the magnetic field inside near the center.

$$B = \mu_0 \frac{N}{\ell} I$$

$$B = \left(4\pi \times 10^{-7}\ \frac{T \cdot m}{A}\right) \frac{(400)}{(0.1\,m)} (2\ A)$$

$$\boxed{B = 0.01\ T}$$

"Help! Too many similar starting equations!"

$$B = \frac{\mu_0 I}{2\pi r}$$

long straight wire

use Ampere's law (or note the lack of N)

$$B = \mu_0 \frac{N}{\ell} I$$

solenoid, length l, N turns

field inside a solenoid is constant

$$B = \mu_0 n I$$

solenoid, n turns per unit length

field inside a solenoid is constant

$$B = \frac{\mu_0 N I}{2\pi r}$$

toroid, N loops

field inside a toroid depends on position (r)