Announcements

- grade spreadsheets with exam 2 scores will be posted today on the Physics 2135 web site
- you need your PIN to find your grade

- preliminary exam average is about 66%
- scores ranged from 31 to 200

- exam 2 will be returned in recitation Thursday
- check that points were added correctly
- regrade requests are **due by Thursday** after spring break

On a separate sheet of paper, explain the reason for your request. This should be based on the work shown on paper, not what was in your head. Attach to the exam and hand it to your recitation instructor by next Thursday.

Today's agenda:

Induced current and emf.
You must understand how changing magnetic flux can induce an emf, and be able to determine the direction of the induced emf.

Faraday's Law.
You must be able to use Faraday's Law to calculate the emf induced in a circuit.

Lenz's Law.
You must be able to use Lenz's Law to determine the direction induced current, and therefore induced emf.

Generators.
You must understand how generators work, and use Faraday's Law to calculate numerical values of parameters associated with generators.
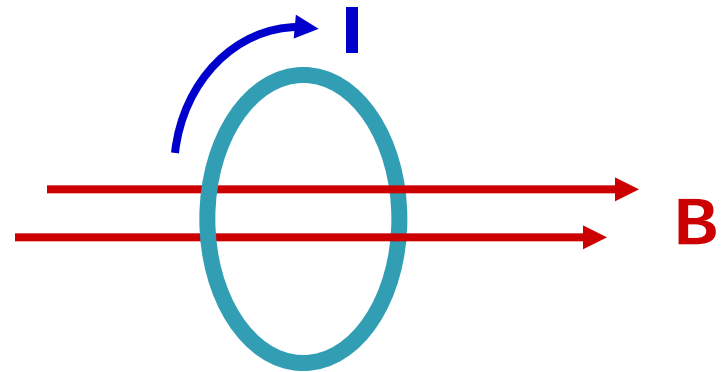
Back emf.
You must be able to use Lenz's law to explain back emf.

Magnetic Induction

- in lectures 16 and 17, we found that electric current can give rise to a magnetic field

**Question:** Can magnetic field give rise to an electric current?

Observation: electric current is induced if magnetic flux through closed circuit (e.g., loop of wire) **changes**
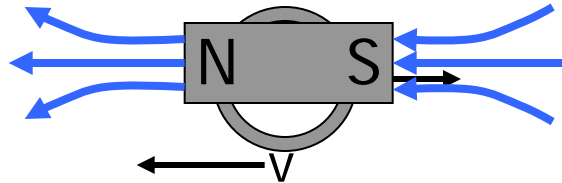
- **constant** magnetic flux does **not** induce a current

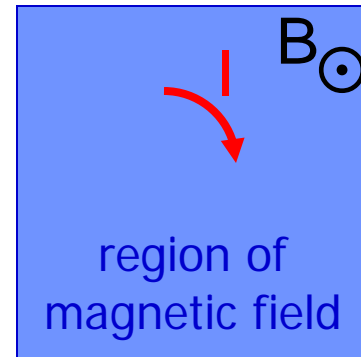# "Change" may or may not require observable (macroscopic) motion

- a magnet may move through a loop of wire
- a loop of wire may be moved through a magnetic field
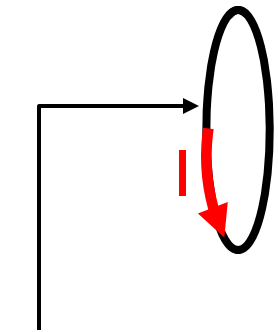
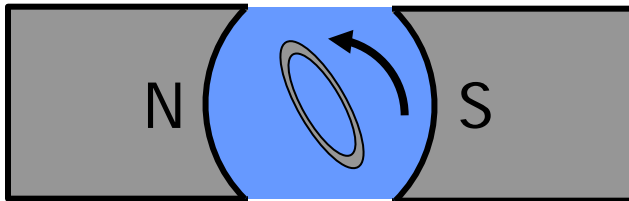this part of the loop is
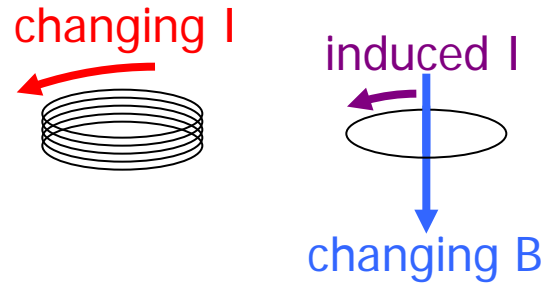closest to your eyes

move magnet toward coil

$B_\odot$

region of
magnetic field

change area of loop
inside magnetic field

rotate coil in
magnetic field

changing I

induced I

changing B

- changing current in wire loop gives rise to changing magnetic field which can induce a current in another nearby loop of wire

In the this case, nothing observable (to your eye) is moving, although, of course microscopically, electrons are in motion.

Induced current is produced by a changing magnetic flux.

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Lenz's Law.
You must be able to use Lenz's Law to determine the direction induced current, and therefore induced emf.

Generators.
You must understand how generators work, and use Faraday's Law to calculate numerical values of parameters associated with generators.

Back emf.
You must be able to use Lenz's law to explain back emf.

Quantify the induced emf (voltage)

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

Faraday's Law of Magnetic Induction

$\Phi_B = \int \vec{B} \cdot d\vec{A}$ is magnetic flux through N closed loops of wire

Two ways to find **sign of the induced emf:**

- directly from Faraday's law by using right hand rule and keeping track of sign of Φ_B (explained in your text, page 959 of the 14th edition)

- more easily from Lentz's law (see below)

Example: move a magnet towards a wire coil of area A=0.002 m² with 5 turns. The field changes at a rate of dB/dt =0.4 T/s. Find the magnitude of the induced emf.

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -N\frac{d\int \vec{B}\cdot d\vec{A}}{dt}$$
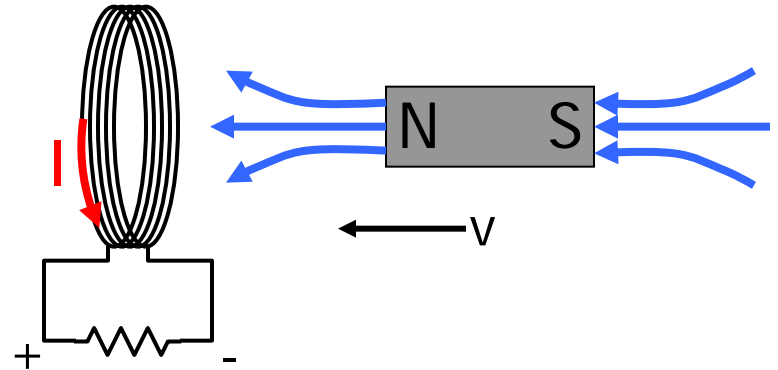
$$\varepsilon = -N\frac{d(BA)}{dt} \qquad \text{(what assumptions did I make here?)}$$

$$\varepsilon = -N A\frac{dB}{dt}$$

$$\varepsilon = -5\left(0.002 \text{ m}^2\right)\left(0.4\frac{T}{s}\right) = \text{-0.004 V}$$

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Lenz's Law.

You must be able to use Lenz's Law to determine the direction induced current, and therefore induced emf.

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You must understand how generators work, and use Faraday's Law to calculate numerical values of parameters associated with generators.

Back emf.

You must be able to use Lenz's law to explain back emf.

Lenz's law—An induced emf gives rise to a current whose magnetic field opposes the **change** in flux.

induced magnetic field

I

N   S

+   -
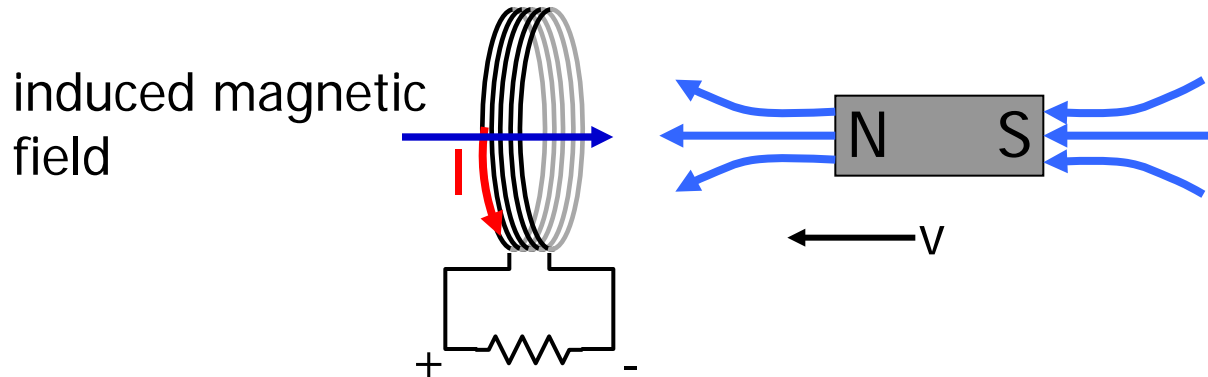
More general formulation of Lenz's law:
Any induction effect opposes its cause!

If Lenz's law were not true, then an increase in magnetic field would produce a current, which would further increase the magnetic field, further increasing the current, making the magnetic field still bigger… **violates energy conservation**
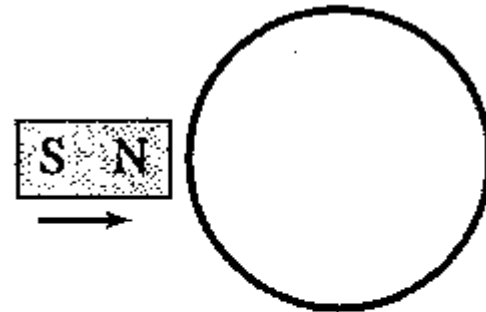
In which direction is the current induced in the coil?
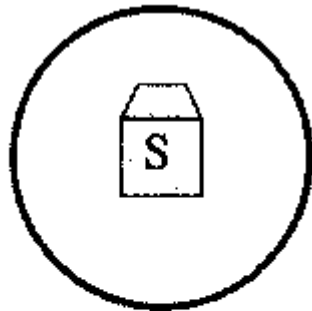
(a)

N magnetic pole
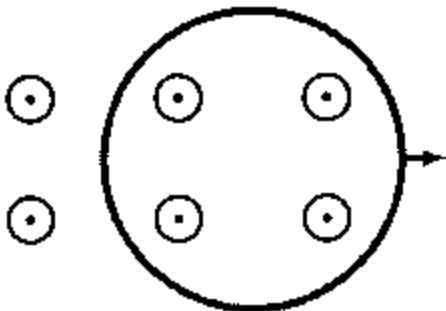moving toward coil
from above the page

(counterclockwise)

(b)

N magnetic pole moving
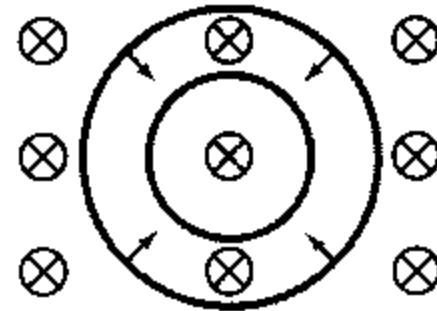toward the coil in the plane
of the page

(no current)

Important: cause of induction effect is **change** in flux!

(c)
Pulling the coil to the right
out of a magnetic field
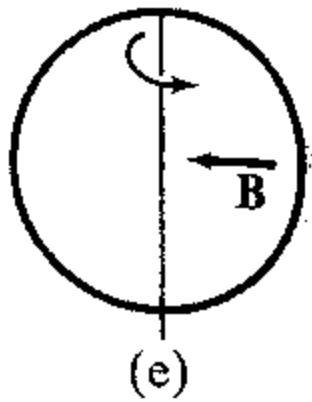that points out of the page

(counterclockwise)

(d)
Shrinking a coil in a
magnetic field pointing
into the page

(clockwise)

Important: cause of induction effect is **change** in flux!

(e)

Rotating the coil about the vertical diameter by pulling the left side toward the reader and pushing the right side away from the reader in a magnetic field that points from right to left in the plane of the page.

(counterclockwise)

Today's agenda:

Induced emf.
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Lenz's Law.
You must be able to use Lenz's Law to determine the direction induced current, and therefore induced emf.
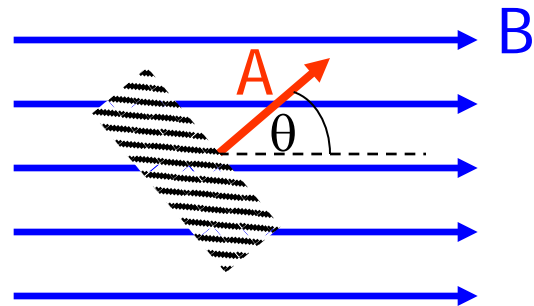
Generators.
You must understand how generators work, and use Faraday's Law to calculate numerical values of parameters associated with generators.

Back emf.
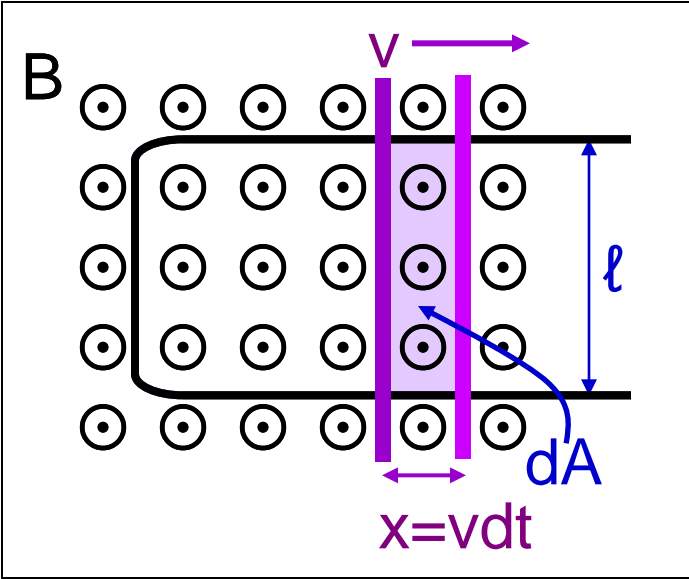You must be able to use Lenz's law to explain back emf.

- a generator is a device that produces electricity (specifically, an emf) from motion

- your text introduces four ways of producing induced emf:

1. Flux change if a wire loop rotates in a magnetic field
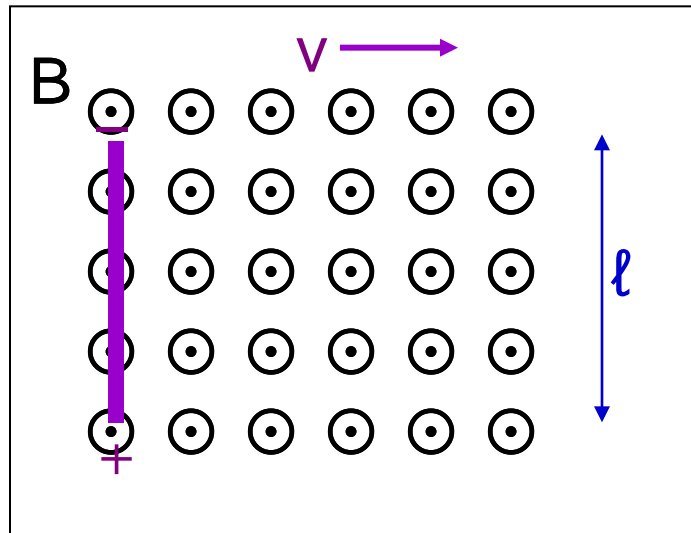
B

A

θ

Rotating loop generator

side view

2. Flux change if a wire loop expands or contracts in a magnetic field


slide-wire generator

3. Conductor moving in a magnetic field: motional emf

B

V

ℓ

motion produces voltage
difference between wire ends

This is called "motional" emf, because the emf results from the motion of the conductor in the magnetic field. The other three examples in this lecture are "induced" emf because they involve the change of magnetic flux through a current loop, as described by Faraday's Law.
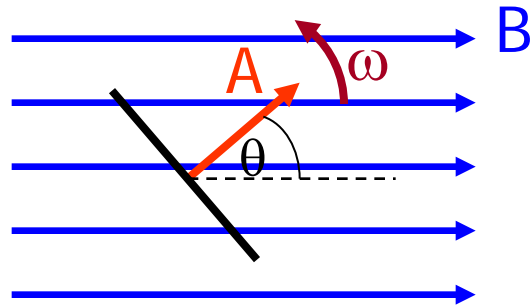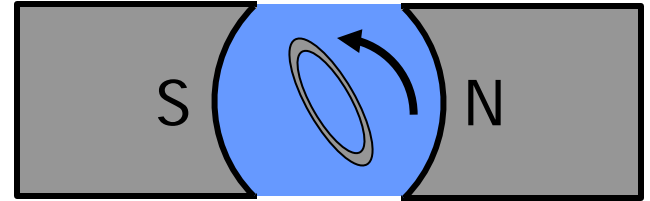
4. Flux change if a wire loop moves into or out of a magnetic field
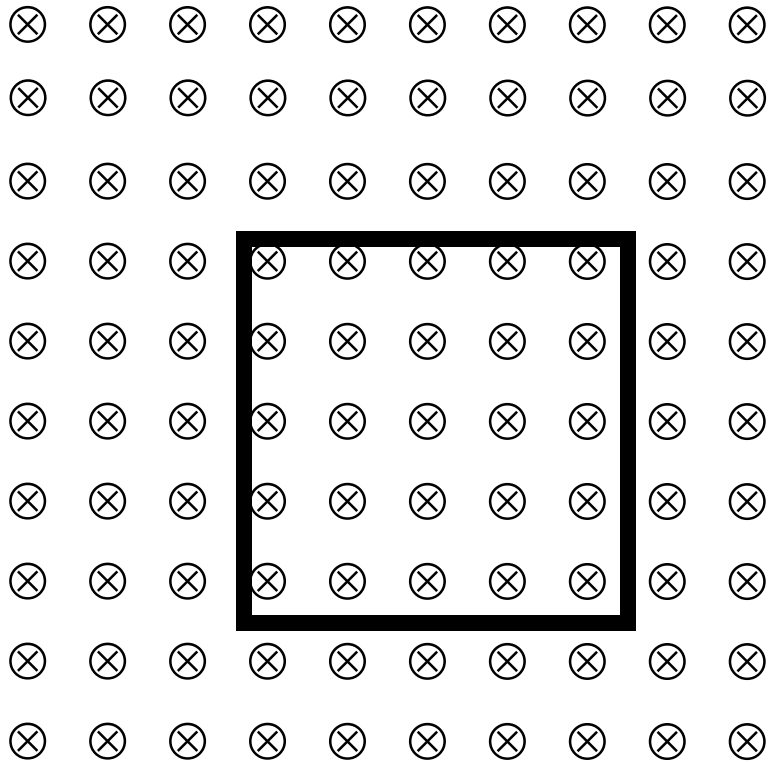
Let's look in detail at each of these four ways of using flux change or motion to produce an emf.

Rotating loop generators

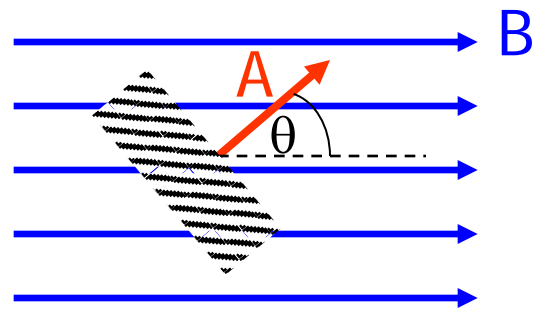Take a loop of wire in a magnetic field and rotate it with an angular speed ω.

side view

$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos(\theta)$$

Choose $\theta_0 = 0$. Then

$$\theta = \theta_0 + \omega t = \omega t \,.$$

$$\Phi_B = BA\cos(\omega t)$$

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

side view

If there are N loops in the coil

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

$$\varepsilon = -N\frac{d(BA\cos(\omega t))}{dt}$$

$$\varepsilon = NBA\,\omega\sin(\omega t)$$

$|\varepsilon|$ is maximum when $\theta = \omega t = 90°$ or $270°$; i.e., when Φ_B is zero. The *rate* at which the magnetic flux is changing is then maximum. ε is zero when the magnetic flux is maximum.

emf, current and power from a generator

$$\varepsilon = NBA\,\omega\,\sin(\omega t)$$

$$I = \frac{\varepsilon}{R} = \frac{NBA\omega}{R}\sin(\omega t)$$

$$P = \varepsilon I = \frac{1}{R}\left[NBA\omega\,\sin(\omega t)\right]^2$$

Example: the armature of a 60 Hz ac generator rotates in a 0.15 T magnetic field. If the area of the coil is 2×10^{-2} m^2, how many loops must the coil contain if the peak output is to be ε_{max} = 170 V?

$$\varepsilon = N B A \omega \sin(\omega t)$$

Not a starting equation. Needs to be derived.

$$\varepsilon_{max} = N B A \omega$$

$$N = \frac{\varepsilon_{max}}{B A \omega}$$

$$N = \frac{(170 \text{ V})}{(0.15 \text{ T})(2 \times 10^{-2} \text{ m}^2)(2\pi \times 60 \text{ s}^{-1})}$$
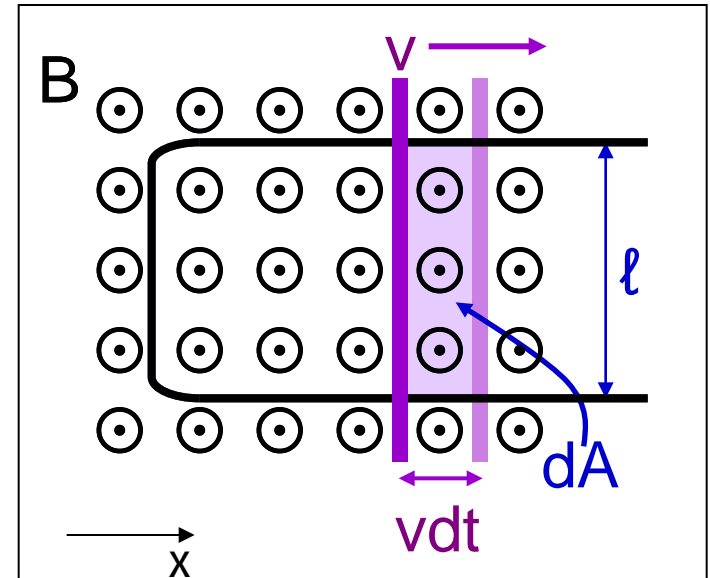
$$\boxed{N = 150 \text{ (turns)}}$$

Let's look in detail at each of these four ways of using flux change or motion to produce an emf.

Method 2...

- a U-shaped fixed conductor and a moveable conducting rod are placed in a magnetic field

- rod moves to the right with constant speed v

- during time dt, rod moves distance v dt

- area of loop increases by dA = ℓ v dt

- loop is perpendicular to the magnetic field, thus

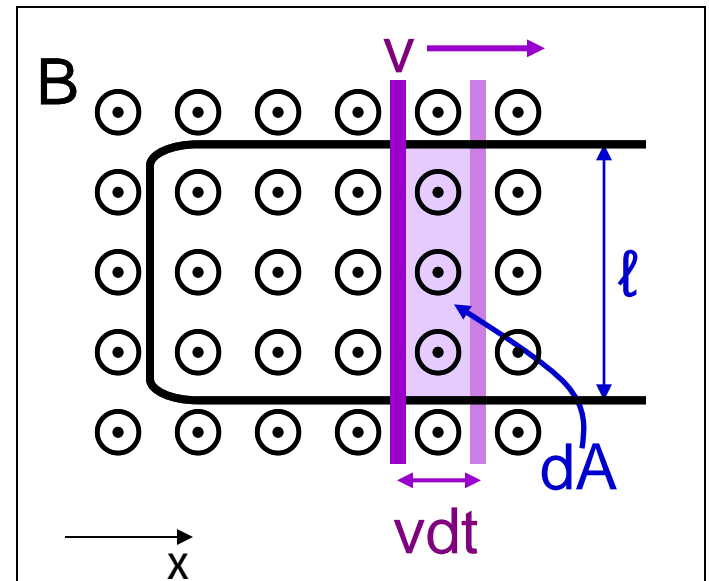$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = BA$$

Faraday's law:

$$|\boldsymbol{\varepsilon}| = \left| -N \frac{d\Phi_B}{dt} \right|$$

$$|\boldsymbol{\varepsilon}| = \left| 1 \frac{d(BA)}{dt} \right|$$

$$|\boldsymbol{\varepsilon}| = \left| \frac{B\, dA}{dt} \right|$$

$$|\boldsymbol{\varepsilon}| = \left| B\ell \frac{dx}{dt} \right|$$
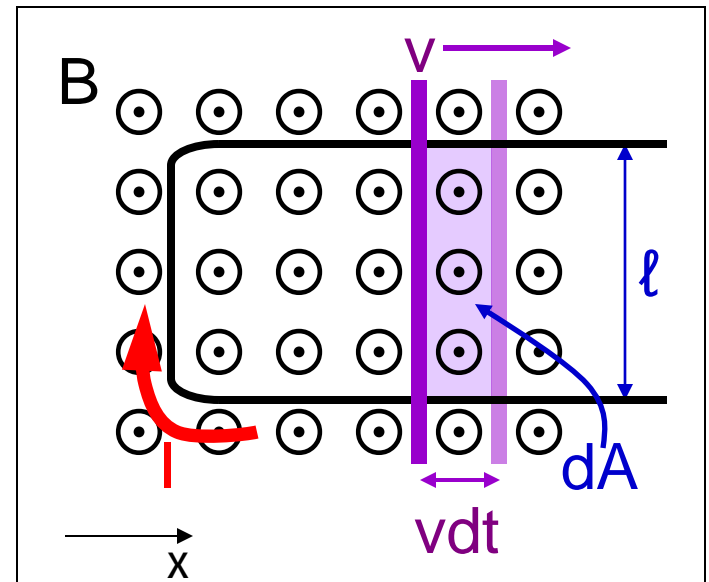
$$|\boldsymbol{\varepsilon}| = B\,\ell\, v.$$

Direction of current?

- magnetic flux inside the loop increases (area increases)

- induced emf causes current to flow in the loop.

Lenz's law:
- system opposes flux increase, so induced magnetic field must be into the page

- induced current is **clockwise**

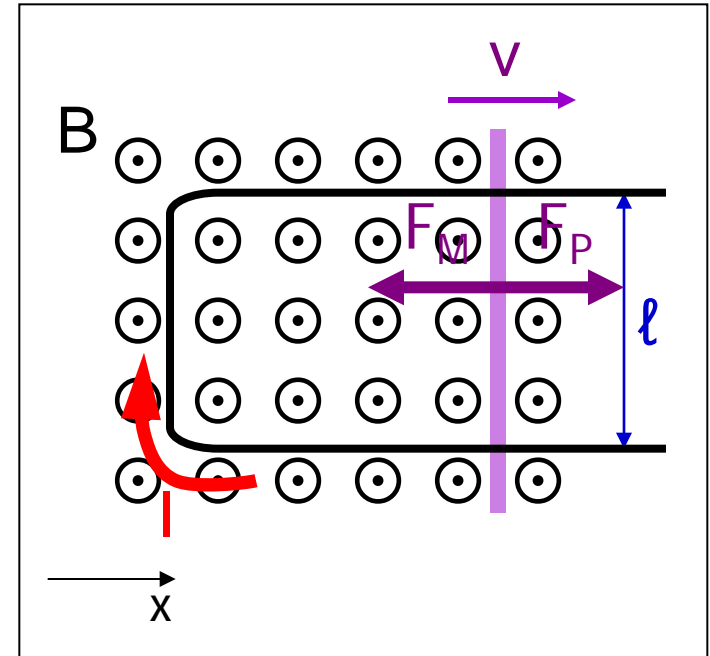What would happen if the bar were moved to the left?

Power and current

- if loop has resistance R, current is $I = \dfrac{\varepsilon}{R} = \dfrac{B\ell v}{R}$.

- electric power dissipated in loop:

$$P = I^2 R = \frac{B^2 \ell^2 v^2}{R}$$

Where does the energy come from?

- magnetic force on bar $\vec{F}_M = I\vec{\ell} \times \vec{B}$

- pulling force $\vec{F}_P = -\vec{F}_M$ necessary to keep bar moving

- power of the pulling force $P = \vec{F}_P \cdot \vec{v} = I\ell B v = \dfrac{B^2 \ell^2 v^2}{R}$

Mechanical energy is converted into electrical energy, electrical energy is dissipated by the resistance of the wire.

Let's look in detail at each of these four ways of using flux change or motion to produce an emf.
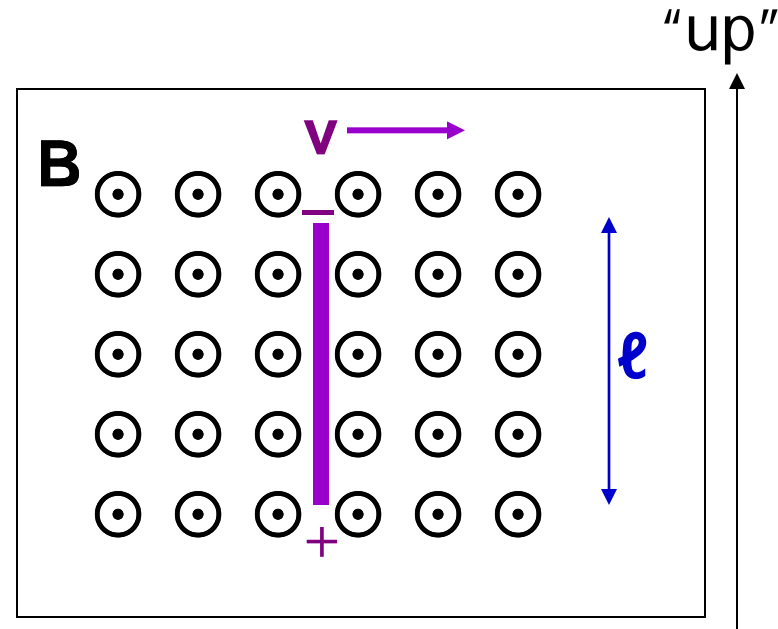
Method 3...

Example 3 of motional emf: moving conductor in $\vec{B}$ field.

Motional emf is emf induced in a conductor moving in a magnetic field.

"up"

- conductor (purple bar) moves with speed v in a magnetic field
- electrons in bar experience force

$$\vec{F}_M = q\vec{v} \times \vec{B} = -e\vec{v} \times \vec{B}$$

B

v

ℓ

+

Charge separation:

- force on the electrons is "up,"
- "top" end of the bar acquires a net – charge
- "bottom" end of the bar acquires a net + charge.

- separated charges produce an electric field E pointing "up"

- voltage difference between ends: $\varepsilon = E\ell$
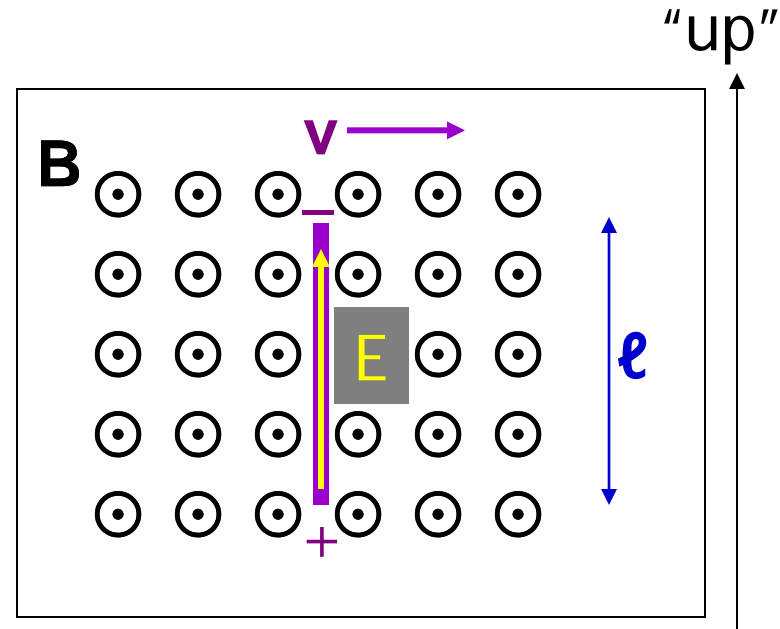
- electric field exerts "downward" force on the electrons:

$$\vec{F}_E = q\vec{E} = -e\vec{E}$$

Equilibrium:
magnetic and electric forces are equal and opposite:

"up"

$$evB = eE = e\frac{\varepsilon}{\ell}$$

$$\Rightarrow \varepsilon = B\ell v$$

Let's look in detail at each of these four ways of using flux change or motion to produce an emf.

Method 4...

# Example 4 of induced emf: flux change through conducting loop. (Entire loop is moving.)

I'll include some numbers with this example.

Remember, it's the **flux change** that induces the emf. Flux has no direction associated with it. However, the presence of flux is due to the presence of a magnetic field, which **does** have a direction, and allows us to use Lenz's law to determine the "direction" of current and emf.

A square coil of side 5 cm contains 100 turns and is positioned perpendicular to a uniform 0.6 T magnetic field. It is quickly and uniformly pulled from the field (moving ⊥ to B) to a region where the field drops abruptly to zero. It takes 0.10 s to remove the coil, whose resistance is 100 Ω.

B = 0.6 T

5 cm
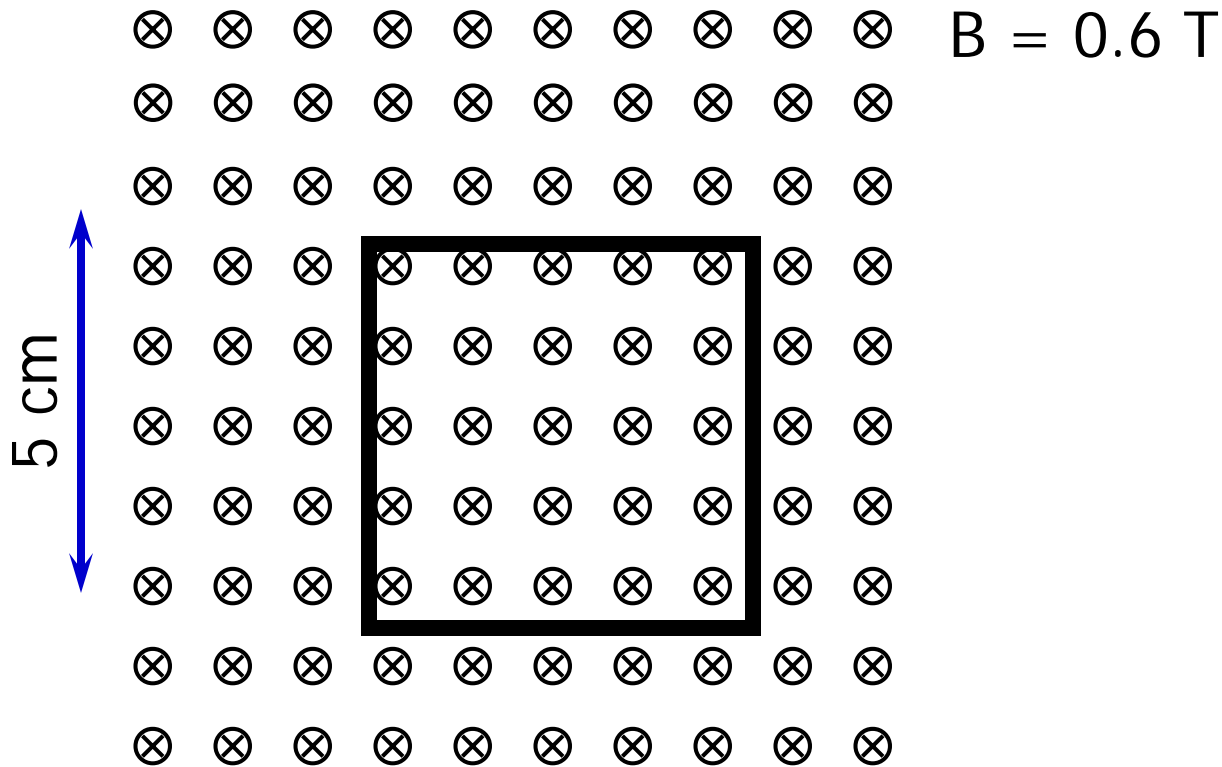
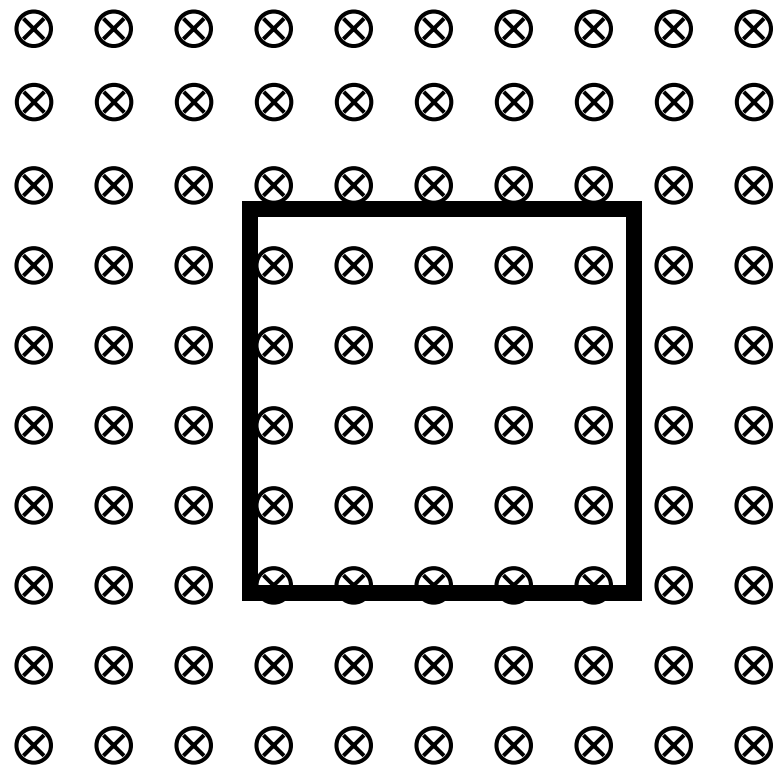# (a) Find the change in flux through the coil.
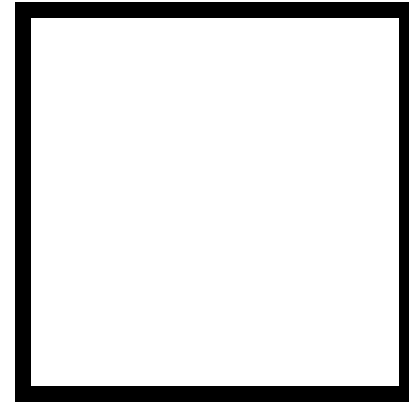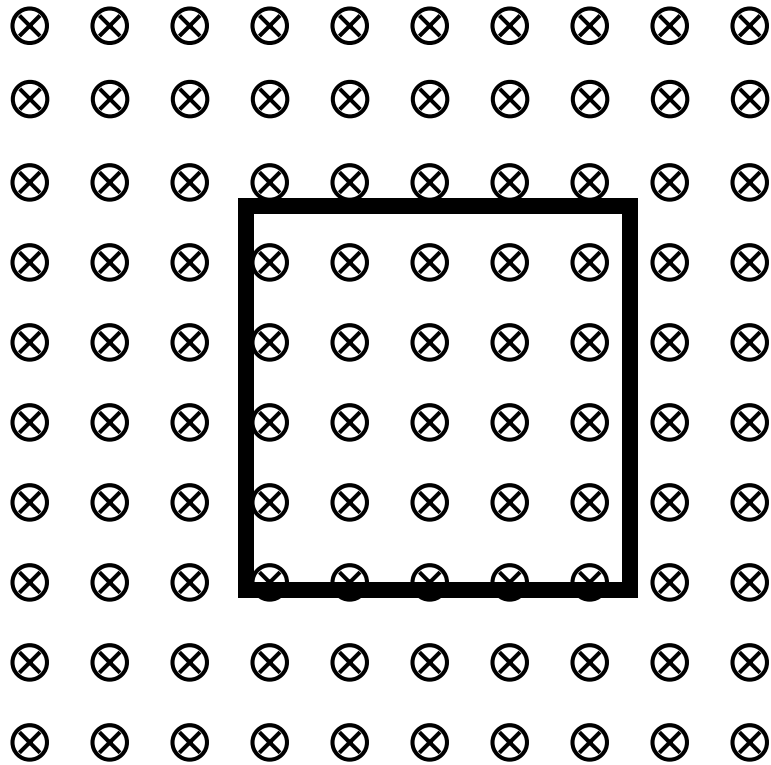
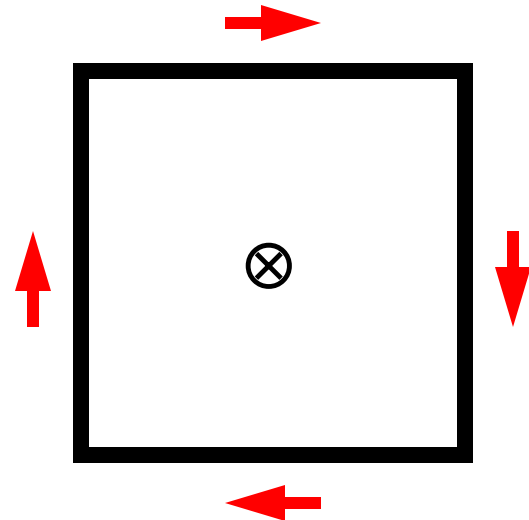Initial: $\Phi_{Bi} = \oint \vec{B} \cdot d\vec{A} = BA$.

Final: $\Phi_{Bf} = 0$.

$\Delta\Phi_B = \Phi_{Bf} - \Phi_{Bi} = 0 - BA = -(0.6 \text{ T})(0.05 \text{ m})^2 = -1.5 \times 10^{-3} \text{ Wb}$.

(b) Find the current and emf induced.

Current will begin to flow when the coil starts to exit the magnetic field. Because of the resistance of the coil, the current will eventually stop flowing after the coil has left the magnetic field.
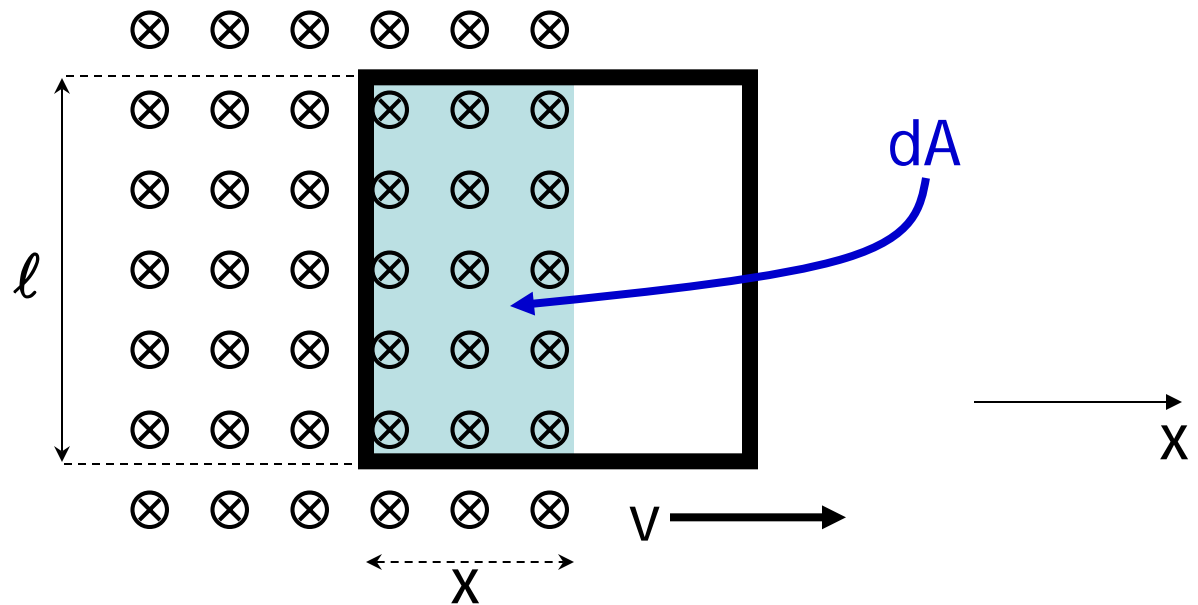
initial

final

The current must flow clockwise to induce an "inward" magnetic field (which replaces the "removed" magnetic field).

The induced emf is

$$|\varepsilon| = \left| -N\frac{d\Phi_B}{dt} \right| = \left| -N\frac{d(BA)}{dt} \right| = \left| -NB\frac{dA}{dt} \right|$$

$$\left| \frac{dA}{dt} \right| = \left| \frac{d(\ell x)}{dt} \right| = \left| \ell\frac{dx}{dt} \right| = |\ell v|$$

$$|\varepsilon| = |-N B \ell v|$$

"uniformly" pulled

$$v = \frac{\Delta x}{\Delta t} = \frac{5 \text{ cm}}{0.1 \text{ s}} = 0.5 \frac{m}{s}$$

$$|\varepsilon| = \left|(100)(0.6 \text{ T})(0.05 \text{ m})\left(0.5 \frac{m}{s}\right)\right|$$

$$\boxed{|\varepsilon| = 1.5 \text{ V}}$$

The induced current is

$$\boxed{I = \frac{\varepsilon}{R} = \frac{1.5 \text{ V}}{100 \text{ } \Omega} = 15 \text{ mA} .}$$

Current flows during the time flux changes.

$$E = P\,t = I^2R\,t = (1.5 \times 10^{-2}\ A)^2\,(100\ \Omega)\,(0.1\ s) = 2.25 \times 10^{-3}\ J$$

The loop has to be "pulled" out of the magnetic field, so there is a pulling force, which does work.

The "pulling" force is opposed by a magnetic force on the current flowing in the wire. If the loop is pulled "uniformly" out of the magnetic field (no acceleration) the pulling and magnetic forces are equal in magnitude and opposite in direction.

The flux change occurs only when the coil is in the process of leaving the region of magnetic field.
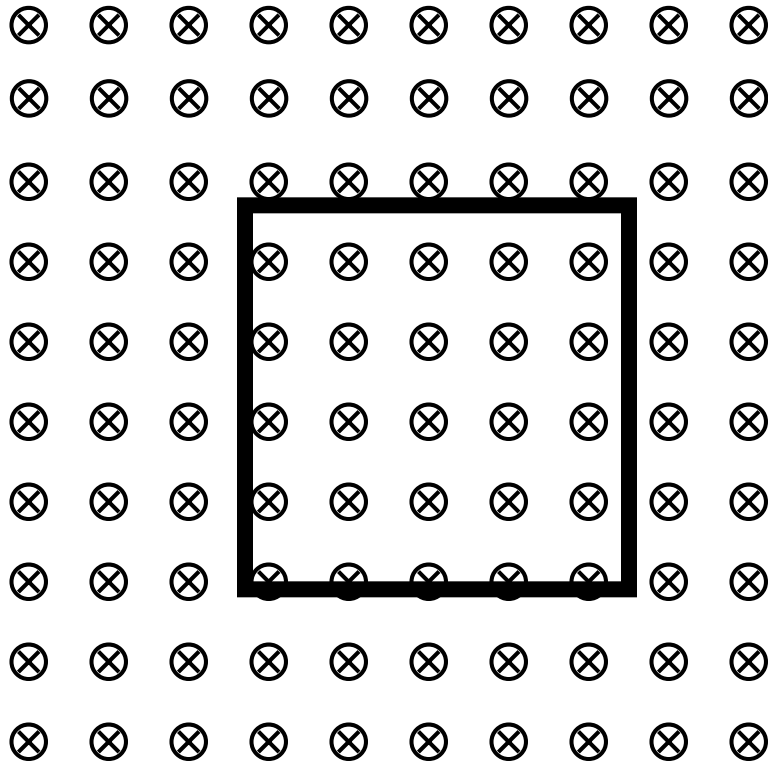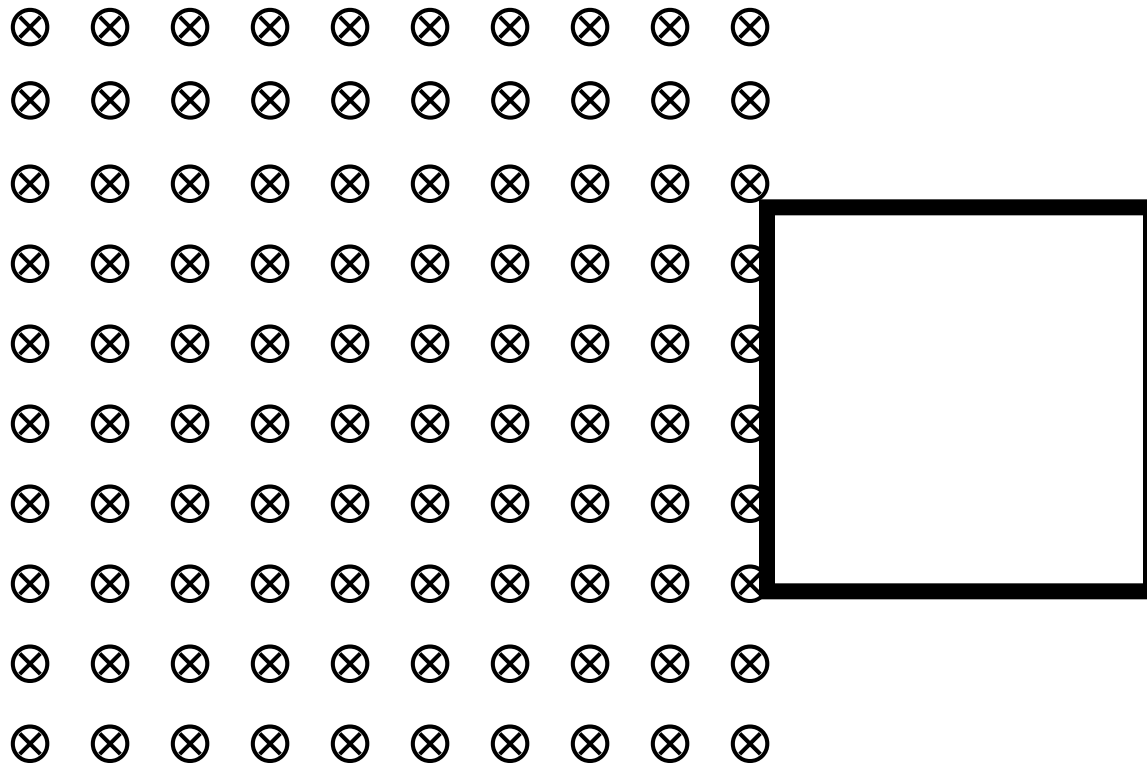
No flux change.
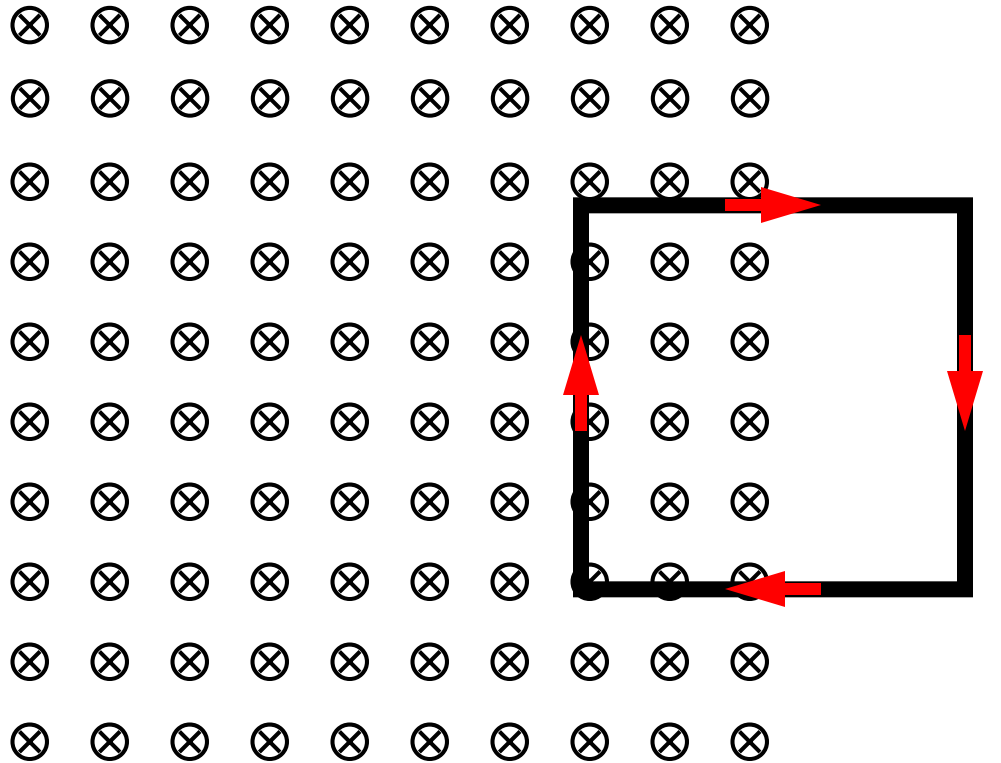No emf.
No current.
No work (why?).
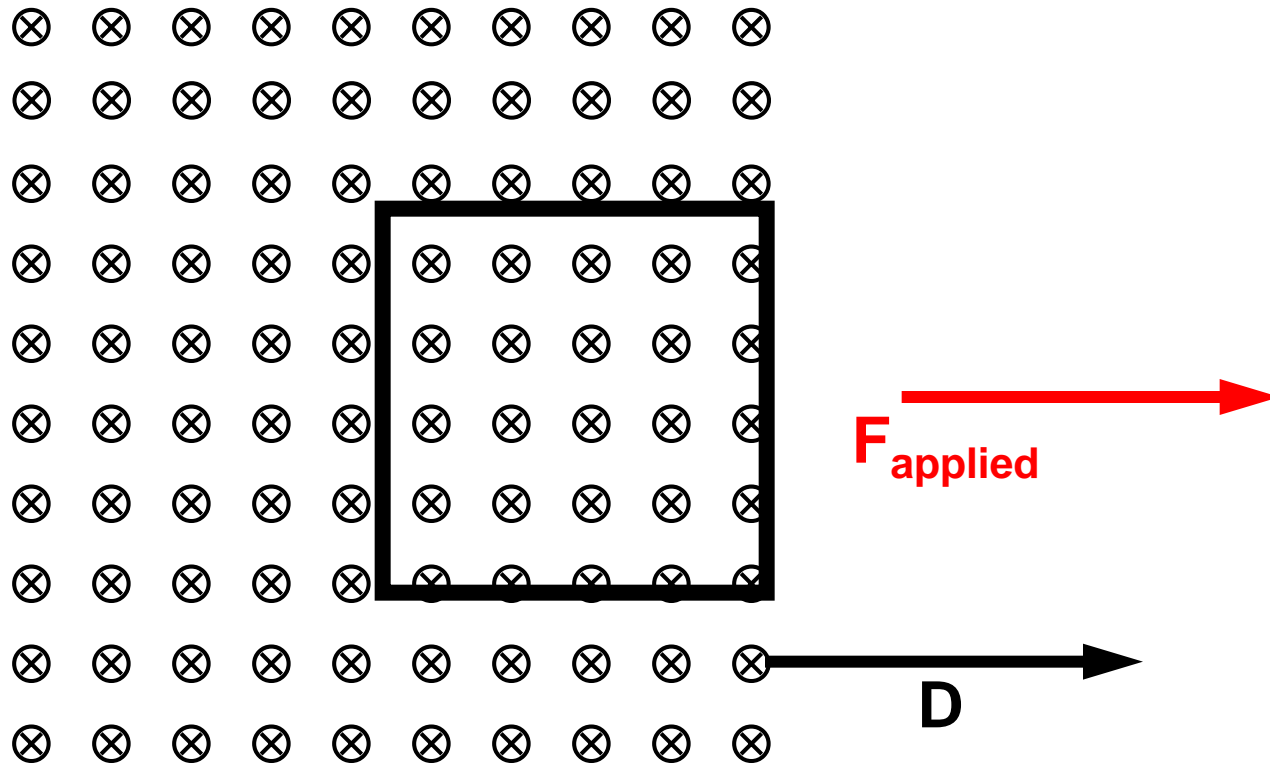
Flux changes. emf induced. Current flows. Work (equal to FD) is done.
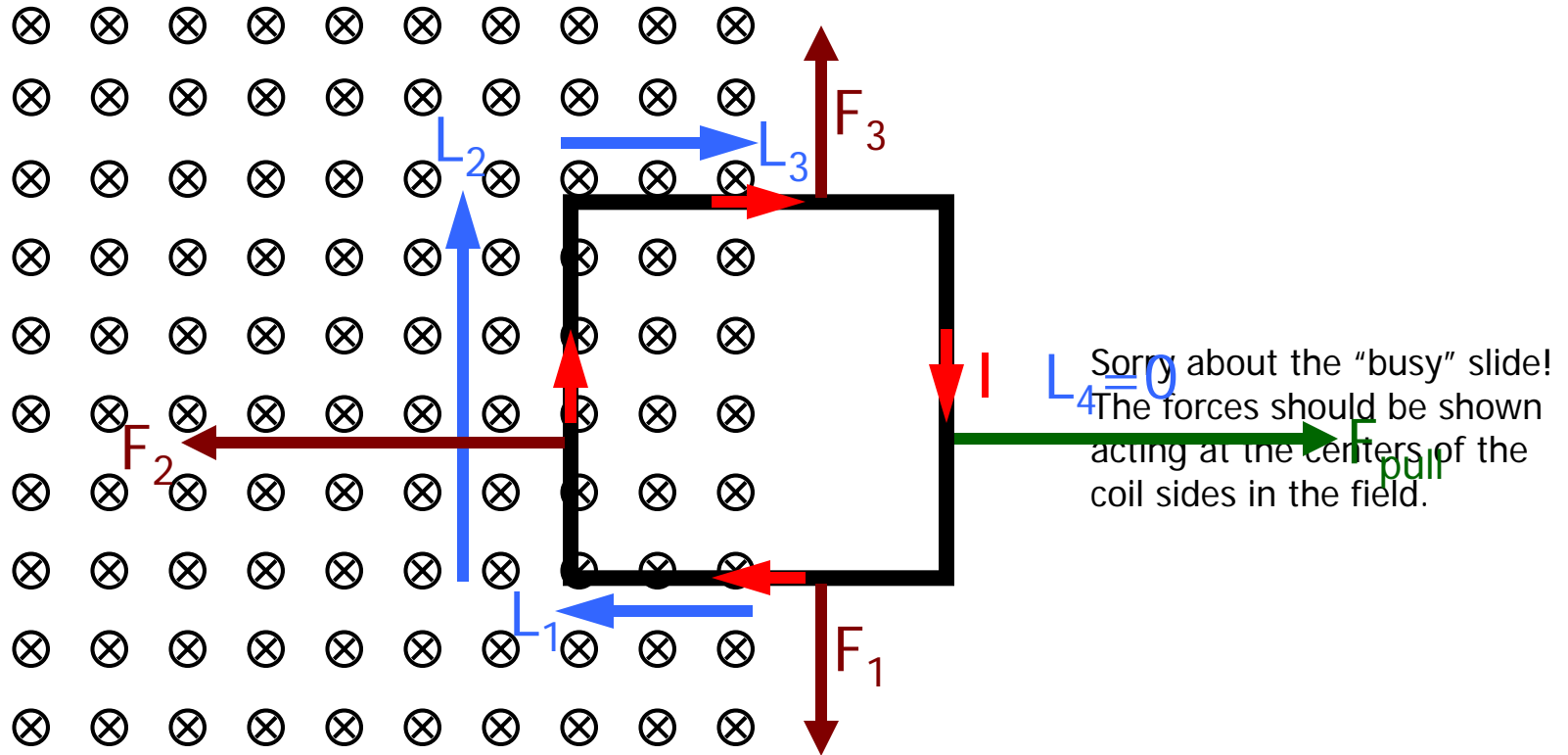
No flux change. No emf. No current. (No work.)

Remember, a force is needed only when the coil is partly in the field region.

$$\vec{F}_{mag} = N\,I\vec{L} \times \vec{B}$$

Multiply by N because there are N loops in the coil.

where L is a vector in the direction of I having a magnitude equal to the length of the wire inside the field region.

F_3

L_2 L_3

I

$L_4 = 0$

Sorry about the "busy" slide! The forces should be shown acting at the centers of the coil sides in the field.

F_2

F_{pull}

L_1

F_1

There must be a pulling force to the right to overcome the net magnetic force to the left.

Magnitudes only (direction shown in diagram):

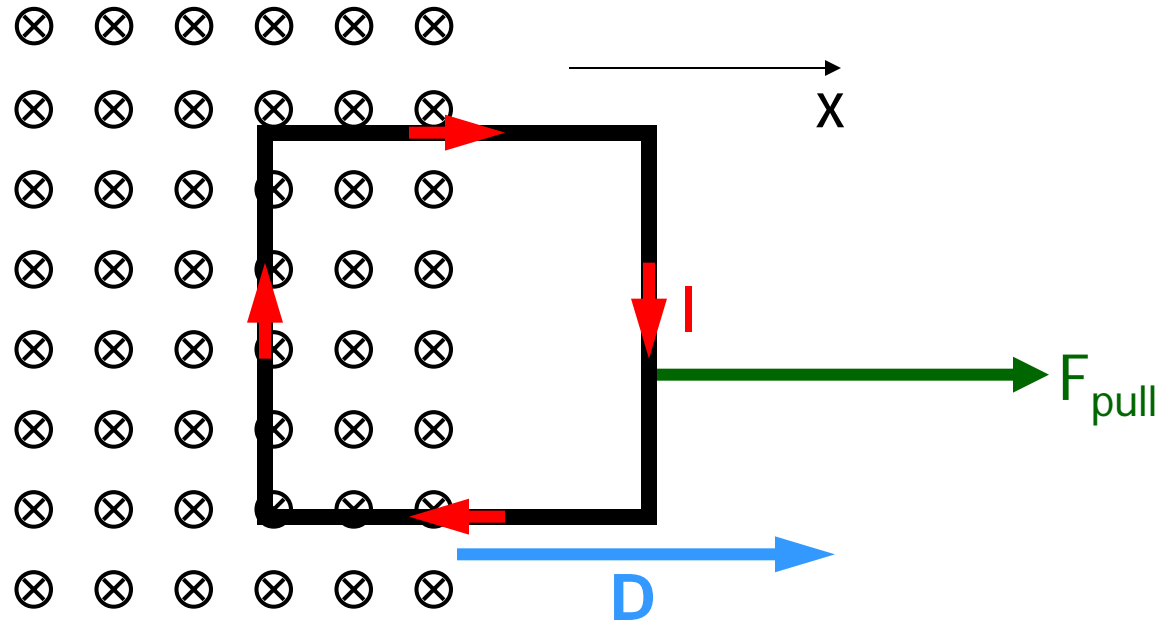$$F_{mag} = NILB = (100)(1.5 \times 10^{-2})(5 \times 10^{-2})(0.6) = 4.5 \times 10^{-2} N = F_{pull}$$
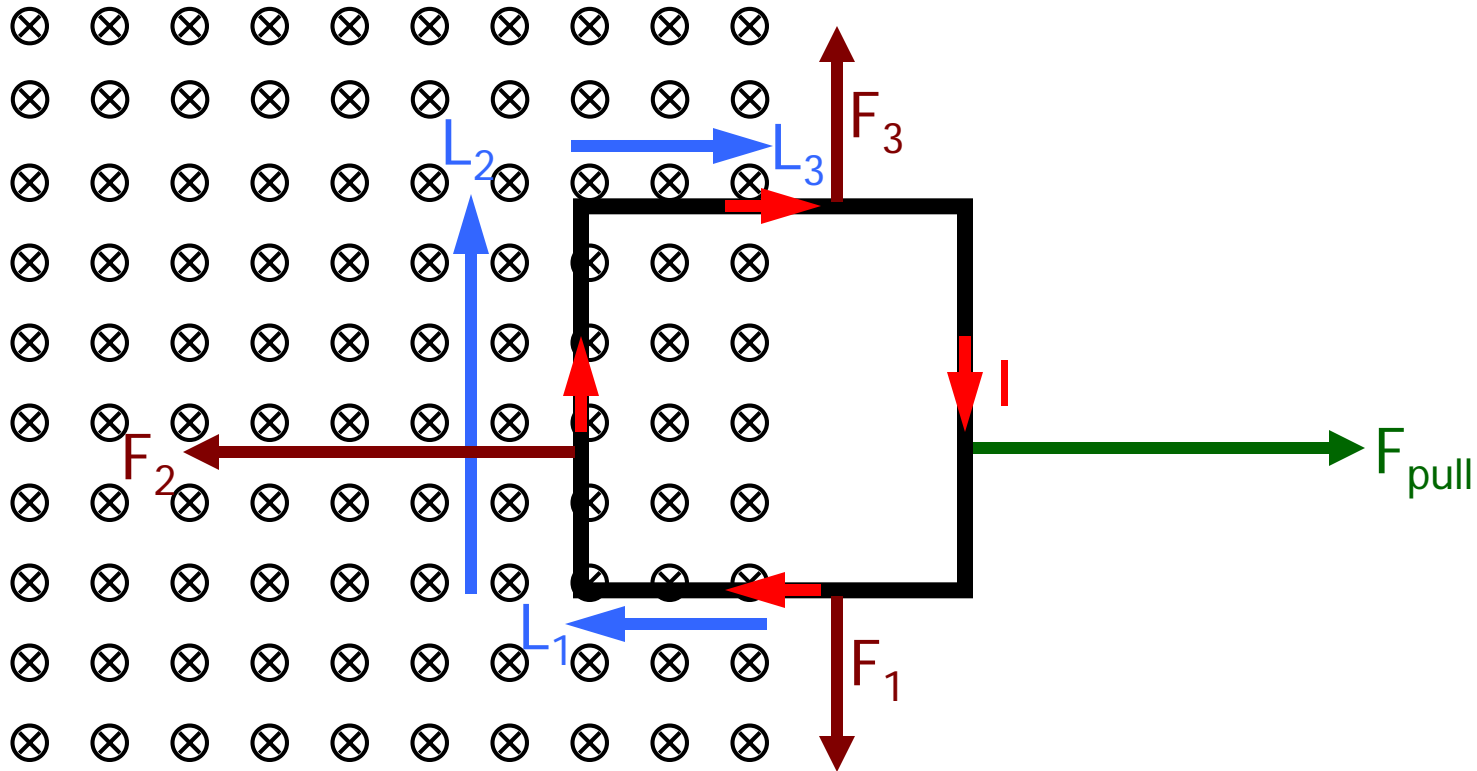
This calculation assumes the coil is pulled out "uniformly;" i.e., no acceleration, so $F_{pull} = F_{mag}$.

Work done by pulling force:

$$W_{pull} = \vec{F}_{pull} \cdot \vec{D} = \left(NILB\ \hat{i}\right) \cdot \left(L\ \hat{i}\right)$$

$$W_{pull} = (100)\left(1.5 \times 10^{-2}\right)\left(5 \times 10^{-2}\right)(0.6)\left(5 \times 10^{-2}\right) = 2.25 \times 10^{-3}\,J$$

The work done by the pulling force is equal to the electrical energy provided to (and dissipated in) the coil.