Physics 24 Exam 2

March 18, 2014

Exam Total

200 / 200

Printed Name: **Key**

Rec. Sec. Letter: **N/A**

Five multiple choice questions, 8 points each. Choose the **best** or **most nearly correct** answer.

C 1. You need to store electrical energy with a simple parallel plate capacitor of fixed plate area A_0 and separation d_0. Which of the following configurations stores the most energy?

[A] capacitor voltage equals V and dielectric with $\kappa = 3$ between the plates
[B] capacitor voltage equals $2V$ and dielectric with $\kappa = 1.5$ between the plates
[C] capacitor voltage equals $3V$ and dielectric with $\kappa = 1$ between the plates
[D] capacitor voltage equals $2V$ and dielectric with $\kappa = 2$ between the plates

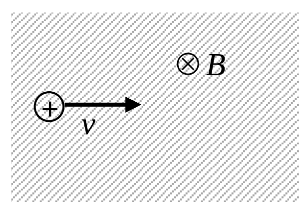
D 2. If the current through an ohmic resistor is doubled, which of the following is true?

[A] The power dissipated by the resistor doubles and the resistance doubles.
[B] The power dissipated by the resistor quadruples and the resistance doubles.
[C] The power dissipated by the resistor doubles and the resistance is unchanged.
[D] The power dissipated by the resistor quadruples and the resistance is unchanged.

B 3. Your car's intermittent windshield wipers are based on an RC timing circuit. It has a fixed capacitor and a variable resistor, and executes one wiper sweep every τ seconds, where τ is the RC time constant. If you want to decrease the time between wiper sweeps you should

[A] increase the resistance [B] decrease the resistance.

A 4. A proton moves through a region of space that has both a uniform electric field and a uniform magnetic field. The direction of the magnetic field is shown in the diagram. In order for the proton to move through this region with a constant velocity, the direction of the electric field must be



[A] ↓ [B] ↑ [C] ⊗ [D] ⊙

ABCD 5. What would you name the movie that this picture was taken from?

[A] Attack of the Flying Dogs
[B] Dawn of the Flying Dogs
[C] Flight of the Living Dogs
[D] Flying Dogs Versus Zombies

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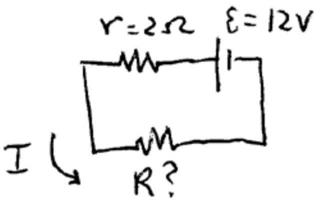
6. (10 points total) A battery has an emf of 12 V and an internal resistance of 2 Ω. You connect an external resistor of resistance *R* across the terminals of the battery and observe that the total power dissipated in the circuit is 18 W. Find *R*.



$$\varepsilon = I(r+R)$$

$$I = \frac{\varepsilon}{r+R}$$

$$P_{diss} = I^2(r+R)$$

$$18 = \frac{\varepsilon^2}{(r+R)^2}(r+R)$$

$$18 = \frac{\varepsilon^2}{r+R} = \frac{144}{R+2}$$

$$R+2 = \frac{144}{18} = 8$$

$$\boxed{R = 6\Omega}$$

There are other correct ways to solve this problem!

7. (30 points total) An air-filled parallel-plate capacitor consists of two circular plates of radius R_0 and plate separation d_0. It is charged to an initial voltage V_0.

(a) (10 points) Calculate the energy U_0 stored in the capacitor in terms of R_0, d_0, V_0, and constants.

$$U_0 = \frac{1}{2}C_0V_0^2 = \frac{1}{2}\frac{\varepsilon_0\pi R_0^2}{d_0}V_0^2 = \boxed{\frac{\varepsilon_0\pi R_0^2 V_0^2}{2d_0}}$$

(b) (20 points) The capacitor is disconnected from the battery. After that, the plates are slowly pulled apart until their distance is $5d_0$. Find the work done by the external force that pulled the plates apart. You may express your answer in terms of R_0, d_0, and V_0, and/or the initial energy U_0.

$$W_{ext} = \Delta U = U_f - U_0 \qquad C_f = \frac{\varepsilon_0\pi R_0^2}{5d_0} = \frac{C_0}{5}$$

$$U_f = \frac{Q_f^2}{2C_f} = \frac{Q_0^2}{2(C_0/5)} = 5\frac{Q_0^2}{2C_0} = 5U_0$$

$$W_{ext} = 5U_0 - U_0$$

$$\boxed{W_{ext} = 4U_0}$$

There are other ways to solve this.

$$W_{ext} = 4(C_0V_0^2) = 2\frac{\varepsilon_0\pi R^2}{d_0}V_0^2$$

is also correct

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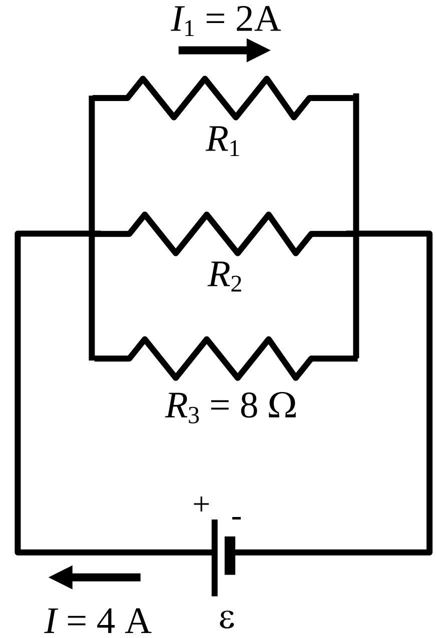
8. (40 points total) In the circuit shown, a total current of 4A comes out of the battery. The current through resistor R_1 is 2A. The electrical power dissipated by resistor R_1 is 16.0 W. Find R_1, R_2, and the emf ε of the battery. Make sure to clearly draw a box around your three answers.



$P_1 = I_1^2 R_1$

$R_1 = \frac{P_1}{I_1^2} = \frac{16}{4}$

$\boxed{R_1 = 4\Omega}$

$V_1 = V_2 = V_3 = \mathcal{E} = I_1 R_1 = (2)(4) = 8$

$\boxed{\mathcal{E} = 8V}$

$\mathcal{E} = I R_{eq} \Rightarrow R_{eq} = \frac{\mathcal{E}}{I} = \frac{8}{4} = 2$

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

$\frac{1}{R_2} = \frac{1}{R_{eq}} - \frac{1}{R_1} - \frac{1}{R_3} = \frac{1}{2} - \frac{1}{4} - \frac{1}{8} = \frac{4-2-1}{8} = \frac{1}{8}$

$\boxed{R_2 = 8\Omega}$

As is usually the case for circuit problems, there are other correct ways to solve this.

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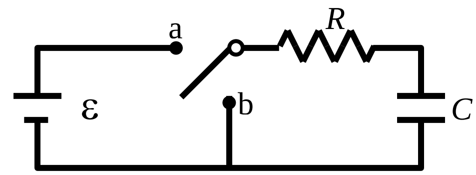

9. (40 points total) In the circuit shown with a resistance *R*, capacitance *C*, and voltage source ε, the capacitor is initially uncharged.

(a) (15 points) The switch is set to position "a" at time $t = 0$. Derive the expression for the current, $I(t)$, through the resistor *R*, at time *t*, in terms of *R*, *C*, ε, and *t*.

Charging $Q(t) = Q_f(1-e^{-t/RC})$ where $Q_f = C\mathcal{E}$

$$I(t) = \frac{dQ}{dt} = Q_f(-e^{-t/RC})\left(-\frac{1}{RC}\right) = \frac{Q_f}{RC}e^{-t/RC} = \frac{C\mathcal{E}}{RC}e^{-t/RC}$$

$$\boxed{I(t) = \frac{\mathcal{E}}{R}e^{-t/RC}}$$

(b) (15 points) If ε = 10V what is the voltage across the capacitor when $t = 2\,RC$?

$$Q(t) = Q_f(1-e^{-t/RC})$$
$$CV(t) = C\mathcal{E}(1-e^{-t/RC})$$
$$V(t) = \mathcal{E}(1-e^{-t/RC})$$
$$V(2RC) = \mathcal{E}(1-e^{-2RC/RC}) = \mathcal{E}(1-e^{-2}) = 10(1-e^{-2})$$
$$\boxed{V = 8.65\text{V}}$$

(c) (10 points) After a long time, the capacitor fully charges to a value of 5 µC. Then the switch is set to position "b." What is the charge on the capacitor when the voltage across the capacitor drops to 2V?

Discharging $Q(t) = Q_0 e^{-t/RC}$

$CV(t) = C\mathcal{E}e^{-t/RC}$ $Q_0 = C\mathcal{E}$ because "fully charged"

$V(t) = \mathcal{E}e^{-t/RC}$

$2 = 10e^{-t/RC} \Rightarrow e^{-t/RC} = \frac{2}{10} = \frac{1}{5}$

See note about this problem on page 6.

When $e^{-t/RC} = \frac{1}{5}$

$$Q(t) = Q_0 e^{-t/RC} = 5\left(\frac{1}{2}\right) \quad \left(\frac{1}{5}\right)$$

$$\boxed{Q(t) = 1\,\mu\text{C}}$$

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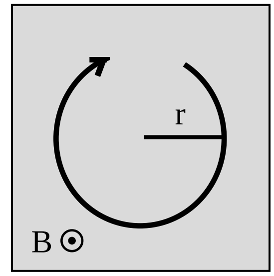
10. (20 points) A beam of positive ions moves in a clockwise orbit with a 10.0 cm radius inside a uniform 0.500 T magnetic field pointing out of the page, as shown in the figure below. The ions have a mass of 6.64×10^{-27} kg and a speed of 4.82×10^{6} m/s.

(a) (15 points) What is the charge on each ion?



$$F_B = |q\vec{v} \times \vec{B}| = qvB = ma = m\frac{v^2}{R}$$

$\vec{v}$ and $\vec{B}$ are perpendicular

We don't need absolute value signs around q because we are told it is positive.

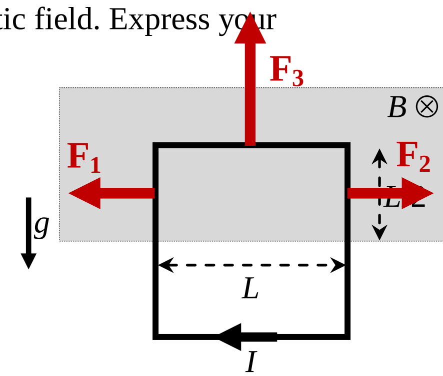
$$qvB = \frac{mv^2}{R}$$

$$q = \frac{mv}{RB} = \frac{(6.64 \times 10^{-27})(4.82 \times 10^{-6})}{(0.1)(0.5)} = \boxed{6.4 \times 10^{-19}\ C}$$

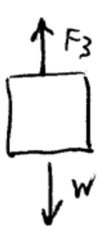
(b) (5 points) According to your answer for part (a), how many electrons were removed from each neutral atom to produce an ion?

$$N = \frac{q}{|e|} = \frac{6.4 \times 10^{-19}}{1.6 \times 10^{-19}} = \boxed{4 \text{ electrons removed}}$$

11. (20 points) A square loop made of a single turn of current-carrying wire is oriented vertically with its top half in a uniform magnetic field and is found to hover in midair. The loop has a total mass *m*, side length *L*, and carries a current *I*. Find the magnitude *B* of the magnetic field. Express your answer in terms of *m*, *L*, *I*, and any necessary constants.



$\vec{F}_1 = -\vec{F}_2$ magnetic forces on vertical segments are equal magnitude, opposite direction



$$\sum \vec{F} = m\vec{a} \quad (a = 0)$$

$$F_3 - W = 0$$

$$ILB = W = mg$$

$$\boxed{B = \frac{mg}{IL}}$$

$$|\vec{F}_3| = |I\vec{L} \times \vec{B}| = ILB$$

$\vec{L}$ and $\vec{B}$ are perpendicular

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Here is another way to do part c. Very simple, and correct.

(c) (10 points) After a long time, the capacitor fully charges to a value of 5 µC. Then the switch is set to position “b.” What is the charge on the capacitor when the voltage across the capacitor drops to 2V?

The capacitance doesn’t change. Let Q_0 be the full charge (5 µC) when the capacitor is fully charged to V_0 (10 volts). Let Q_1 be the charge when the capacitor voltage is V_1 = 2 volts. Then

$$C = \frac{Q_0}{V_0} = \frac{Q_1}{V_1}$$

$$\frac{5}{10} = \frac{Q_1}{2}$$

$$Q_1 = 1\mu C$$