Physics 2135 Exam 3

November 15, 2016

Exam Total

200 / 200

Printed Name: ___**Key**___

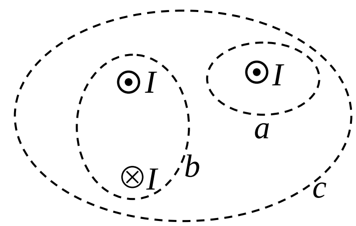
Rec. Sec. Letter: **N/A**

Five multiple choice questions, 8 points each. Choose the **best** or **most nearly correct** answer.

__B__ 1. The three wires shown carry identical currents I in the directions indicated. For which of the three paths a, b, and c is the line integral $\oint \vec{B}\cdot d\vec{s}$ equal to zero?

[A] *a* only [B] *b* only

[C] *b* and *c* [D] *a*, *b*, and *c*

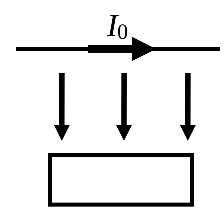


__C__ 2. Two solenoids both have n turns per meter. Solenoid 1 has radius R and carries current I, producing a magnetic field B_1. Solenoid 2 has radius $R/2$ and carries current $2I$, producing magnetic field B_2. What is the ratio B_2/B_1?

[A] ½ [B] 1 [C] 2 [D] 4

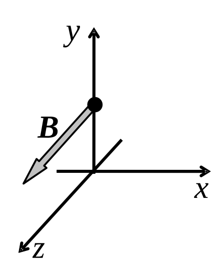
__A__ 3. A long straight wire carries a constant current I_0. A conducting rectangular loop is pushed away from the wire as shown. The induced current in the loop is

[A] ↻ [B] ↺ [C] ⊙ [D] ⊗



__C__ 4. The figure to the right shows the magnetic field of an electromagnetic wave at a certain point in space and a certain instant in time. If the wave transports energy in the $+x$ direction what is the direction of the electric field at this point and instant?

[A] $+z$ [B] $-z$ [C] $+y$ [D] $-y$



__ABCD__ 5. It is well-known that a dropped cat always lands feet first and a peanut butter sandwich always lands peanut butter side down. If a peanut butter sandwich is strapped to the back of a cat (peanut butter facing away from cat) and both are dropped into a 10 T vertical magnetic field, the cat-peanut butter system will

[A] levitate [B] rotate [C] float [D] spiral down

Note: no cats were harmed in the making of this exam.

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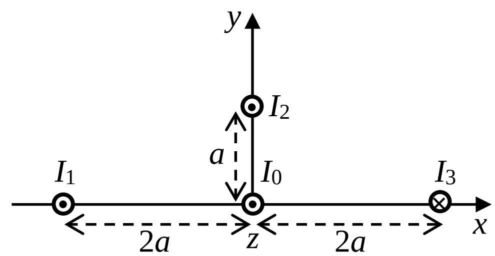

6. (40 points total) Four long, parallel wires are arranged as shown in the diagram. Wire 0, along the z axis, carries current I_0 out of the page (+z direction). Wire1 carries current I_1 out of the page. Wire 2, carries current I_2 out of the page. Wire 3 carries current I_3 into the page. All currents are equal in magnitude, $I_0 = I_1 = I_2 = I_3 = I$. Show starting equations the first time you use them.

(a) (5 points) Find the magnetic field at point (0,0,0) due to wire 1. Express your answer in unit vector notation.

$$B = \frac{\mu_0 I}{2\pi r} \qquad \vec{B}_1 = \frac{\mu_0 I_1}{2\pi (2a)}\hat{j} = \boxed{\frac{\mu_0 I}{4\pi a}\hat{j}}$$

(b) (5 points) Find the magnetic field at point (0,0,0) due to wire 2. Express your answer in unit vector notation.

$$\vec{B}_2 = \frac{\mu_0 I_2}{2\pi (a)}\hat{i} = \boxed{\frac{\mu_0 I}{2\pi a}\hat{i}}$$

(c) (5 points) Find the magnetic field at point (0,0,0) due to wire 3. Express your answer in unit vector notation.

$$\vec{B}_3 = \frac{\mu_0 I_3}{2\pi (2a)}\hat{j} = \boxed{\frac{\mu_0 I}{4\pi a}\hat{j}}$$

(d) (5 points) Find the total magnetic field at point (0,0,0) due to wires 1, 2, and 3. Express your answer in unit vector notation.

$$\vec{B}_{123} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = \frac{\mu_0 I}{4\pi a}\hat{j} + \frac{\mu_0 I}{2\pi a}\hat{i} + \frac{\mu_0 I}{4\pi a}\hat{j} = \boxed{\frac{\mu_0 I}{2\pi a}(\hat{i} + \hat{j})}$$

(e) (5 points) Circle the arrow below that best describes the direction of the total magnetic field at point (0,0,0).

↑ ↓ ← → ↗ ↘ ↙ ↖

(f) (10 points) Find the magnetic force per unit length on wire 0. Express your answer in unit vector notation.

$$\vec{F}_0 = I_0 \vec{L}_0 \times \vec{B}_{123} = IL\hat{k} \times \frac{\mu_0 I}{2\pi a}(\hat{i} + \hat{j})$$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

$$\boxed{\frac{F_0}{L} = \frac{\mu_0 I^2}{2\pi a}(\hat{j} - \hat{i})}$$

(g) (5 points) Circle the arrow below that best describes the direction of the magnetic force on wire 0.

↑ ↓ ← → ↗ ↘ ↙ ↖

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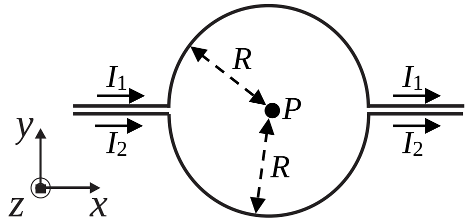
7. (20 points total) Two wires insulated from each other carry different currents I_1 and I_2 as shown in the figure. The small distance between the parallel segments can be neglected. Use the Biot-Savart Law to calculate magnetic fields, and express your answers in unit vector notation.



(a) (5 points) What is the magnetic field at point P due to the straight sections of the wires?

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{s}\times\hat{r}}{r^2} \qquad dB = \frac{\mu_0}{4\pi}\frac{I\, ds\sin\theta}{r^2} = \boxed{0}$$

because angle between $d\vec{s}$ and $\hat{r}$ is $0°$ or $180°$

(b) (15 points) What is the magnetic field at point P due to the curved sections of the wires?

$$d\vec{B}_1 = \frac{\mu_0 I_1}{4\pi}\frac{d\vec{s}\times\hat{r}}{R^2} = -\frac{\mu_0 I_1}{4\pi R^2}\, ds\, \hat{k}$$

$$\vec{B}_1 = -\frac{\mu_0 I_1}{4\pi R^2}\int ds\, \hat{k} = -\frac{\mu_0 I_1}{4\pi R^2}\frac{1}{2}(2\pi R)\,\hat{k}$$

$$= -\frac{\mu_0 I_1}{4R}\hat{k}$$

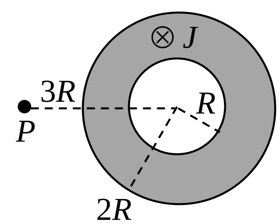
$$d\vec{B}_2 = \frac{\mu_0 I_2}{4\pi R^2}\, ds\, \hat{k}$$

$$\vec{B}_2 = \frac{\mu_0 I_2}{4\pi R^2}\int ds\, \hat{k} = \frac{\mu_0 I_2}{4\pi R^2}\frac{1}{2}(2\pi R)\,\hat{k}$$

$$= \frac{\mu_0 I_2}{4R}\hat{k}$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \boxed{\frac{\mu_0 (I_2 - I_1)}{4R}\hat{k}}$$

8. (20 points total) A long straight cylindrical conductor of radius $2R$ with a hole inside of radius R carries a uniformly distributed current density J directed into the page. The diagram shows a cross section of the conductor.



(a) (5 points) What is the direction of the magnetic field at point P, which is a distance of $3R$ from the center? (circle one below)

↑ ↓ ← →

(b) (15 points) Use Ampere's Law to calculate the magnitude of the magnetic field at a distance of $3R$ from the center.

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}$$

$$B(2\pi)(3R) = \mu_0 J A_{encl} = \mu_0 J \pi\left[(2R)^2 - R^2\right] = \mu_0 J 3\pi R^2$$

$$B = \frac{\mu_0 J 3\pi R^2}{6\pi R} = \boxed{\frac{\mu_0 J R}{2}}$$

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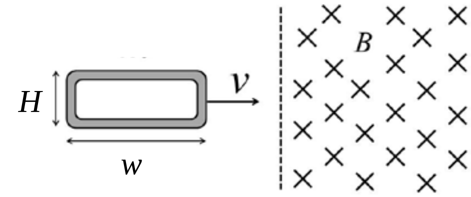
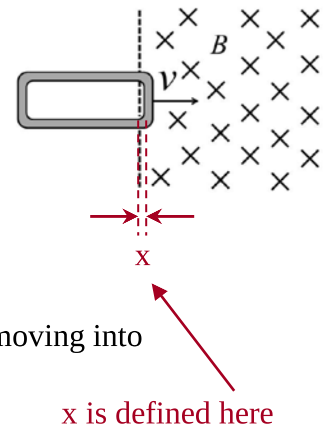
9. (40 points total) A rectangular loop of wire of width w, height H, and resistance R travels at constant speed v into a uniform magnetic field B. The plane of the rectangular loop is perpendicular to the magnetic field, which is directed into the page. (All solutions MUST start with OSE's and MUST be expressed in terms of the given parameters.)



(a) (5 points) At the moment the rectangular loop enters the magnetic field region, an emf is induced which causes a current to circulate around it. What is the direction of the induced current while it is moving into the magnetic field (i.e., while the right side is immersed in the field and the left side is not)? (Circle one of the following four options)



$\odot$ ↻ $\otimes$ ↺

(b) (15 points) Calculate the magnitude of the current induced in the loop while it is moving into the magnetic field.

$$|\varepsilon| = \left|-\frac{d\Phi_B}{dt}\right| = \left|\frac{d(BA)}{dt}\right| = B\left|\frac{d(Hx)}{dt}\right| = BH\left|\frac{dx}{dt}\right| = BHv$$

$$\varepsilon = IR$$

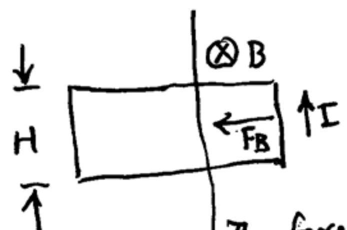
$$I = \frac{\varepsilon}{R} = \boxed{\frac{BHv}{R}}$$

(c) (15 points) What are the magnitude and direction of the total magnetic force exerted on the loop while it is entering the magnetic field?

$$\vec{F} = I\vec{L} \times \vec{B}$$

$$F_B = IHB$$

$$= \left(\frac{BHv}{R}\right)HB = \boxed{\frac{B^2H^2v}{R}}$$

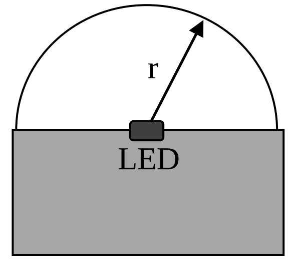


The forces on the two horizontal segments cancel.

(d) (5 points) Once the loop is entirely in the region of uniform magnetic field, what is the magnitude of the current I induced in it.

no flux change $\Rightarrow \varepsilon = 0 \Rightarrow \boxed{I = 0}$

10. (40 points total) A particular string of holiday lights uses LEDs covered by transparent hemispherical domes, as shown in the diagram. The dome has a radius of 7.50 mm and the LEDs convert electrical energy to light at a rate of 0.096 W. The LEDs radiate uniformly into the hemisphere of the dome.



(a) (10 points) Find the wavenumber and angular frequency if the LEDs are emitting red light with a wavelength of 650 nm.

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{650\times10^{-9}} = \boxed{9.67\times10^{6}\ m^{-1}}$$

$$\omega = 2\pi f = 2\pi\frac{c}{\lambda} = 2\pi\frac{3\times10^{8}}{650\times10^{-9}} = \boxed{2.9\times10^{15}\ rad/s}$$ or Hz

(b) (10 points) What is the intensity of the radiation at the surface of the dome?

$$I = \frac{P}{A} = \frac{P}{\frac{1}{2}(4\pi R^2)} = \frac{P}{2\pi R^2} = \frac{0.096}{2\pi(7.5\times10^{-3})^2}$$

$$\boxed{I = 272\ \frac{W}{m^2}}$$

(c) (10 points) What are the amplitudes of the electric and magnetic fields at the surface of the dome?

$$I = \frac{1}{2}c\epsilon_0 E_{max}^2$$

$$E_{max} = \sqrt{\frac{2I}{c\epsilon_0}} = \sqrt{\frac{2(272)}{(3\times10^{8})(8.85\times10^{-12})}} = \boxed{453\ \frac{V}{m}}$$

$$B_{max} = \frac{E_{max}}{c} = \frac{453}{3\times10^{8}} = 151\times10^{-8} = \boxed{1.51\times10^{-6}\ T}$$

(d) (10 points) What average pressure does the light exert on a perfectly absorbing speck of dust resting on the dome?

$$\langle P_{rad}\rangle = \frac{I}{c} = \frac{272}{3\times10^{8}} = \boxed{9.07\times10^{-7}\ Pa}$$

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