# Physics 2135 Exam 3

April 21, 2015

**Exam Total**

**200 / 200**

Printed Name: _____ **Key** _____

Rec. Sec. Letter: __**N/A**__

Five multiple choice questions, 8 points each. Choose the **best** or **most nearly correct** answer.

__C__1. Two long straight wires are parallel and a distance $D$ apart. Point $P$ is located a distance $D$ to the right of the right-hand wire. Both wires carry constant currents in opposite directions, as shown in the diagram. If the **magnitude** of the magnetic field at point $P$ is zero, what is the ratio $I_1/I_2$?
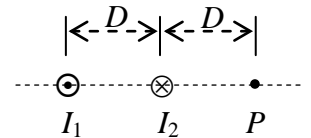
[A] ¼　　　　[B] ½　　　　[C] 2　　　　[D] 4

__D__ 2. The three wires shown carry identical currents $I$ in the directions indicated. For which of the three paths a, b, and c is the line integral $\oint \vec{B} \cdot d\vec{s}$ equal to zero?
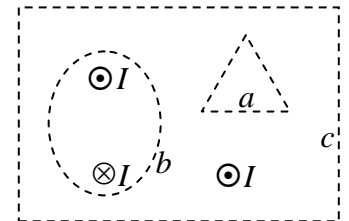
[A] $a$ only　　　　　　[B] $a$, $b$, and $c$
[C] $b$ and $c$　　　　　[D] $a$ and $b$

__A__ 3. A conducting loop is held stationary a fixed distance from a long straight wire carrying a changing current $I$ as shown. The changing current $I$ induces a counterclockwise current $I_{ind}$ in the loop. The current in the straight wire is
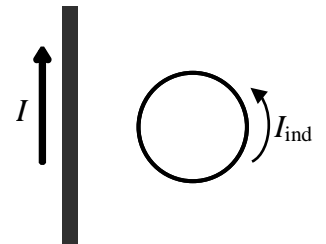
[A] increasing　　　　　　[B] constant
[C] decreasing　　　　　　[D]zero.

__C__ 4. At a certain point in space and time, the magnetic field and Poynting vector of an electromagnetic wave are given by $\vec{B} = (10^{-5}\,\text{T})\hat{k}$ and $\vec{S} = -(2.38 \times 10^4\,\text{W/m}^2)\hat{j}$. What is the magnitude $E$ of the wave's electric field vector at that position and time?

[A] 7.93x10$^{-5}$ V/m　　　　[B] 3.33x10$^{-4}$ V/m
[C] 3x10$^3$ V/m　　　　　　　[D] 10$^5$ V/m

ABCD 5. What is this?
[A] Laser pig invasion.
[B] Electromagnetic Wave Pigs™.
[C] The logo of a pizzeria near Melbourne, Australia.
[D] Dr. Pringle's worst nightmare.

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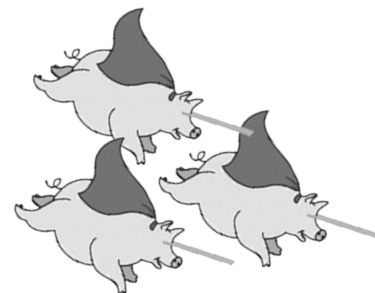
6. (40 points total) A long, straight conducting wire carries a 10 A current. At some instant in time an electron is moving parallel to this wire with a velocity of 220 km/s in the same direction as the current. At this instant the electron is 2 cm from the wire.
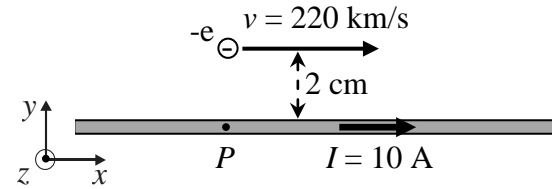
(a) (10 points) Find the magnitude and direction of the magnetic field generated by the current-carrying wire at the location of the electron.

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 2\times 10^{-2}} = \frac{2\times 10^{-6}}{2\times 10^{-2}}$$

$$B = 10^{-4} T$$

direction is $\odot$ by right-hand rule

or $\boxed{B = 10^{-4} T \; \hat{k}}$

$v = 220$ km/s

-e

2 cm

$y$

$z$ $x$

$P$ $I = 10$ A

(b) (15 points) Find the magnitude and the direction of the electron's acceleration at the instant shown.

magnetic force on electron: $\vec{F} = q\vec{v} \times \vec{B} = (-e)\, v\hat{i} \times B\hat{k} = +e\,v\,B\,\hat{j}$

$$\vec{a} = \frac{\vec{F}}{m_e} = \frac{e\,v\,B}{m_e}\hat{j} = \frac{(1.6\times 10^{-19})(220\times 10^3)(10^{-4})}{9.11\times 10^{-31}}\hat{j}$$

you can also use the right-hand rule to get direction of $\vec{F}$

$$\boxed{\vec{a} = 3.86 \times 10^{12} \; \frac{m}{s^2} \; \hat{j}} \quad \text{big!}$$

(c) (15 points) Find the magnetic field generated by the electron at point $P$ at this instant. Point $P$ is 2 cm from the electron and directly below it.
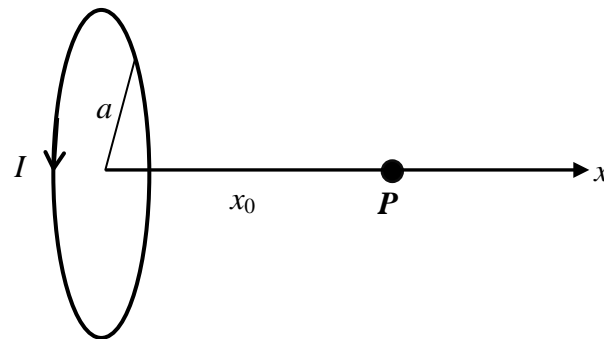
$$\vec{B_e} = \frac{\mu_0}{4\pi} \frac{q\vec{v}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{(-e)(v\hat{i})\times(-\hat{j})}{r^2} = \frac{\mu_0}{4\pi}\frac{ev}{r^2}(\hat{i}\times\hat{j}) = \frac{\mu_0}{4\pi}\frac{ev}{r^2}\hat{k}$$

$$\vec{B_e} = \frac{4\pi\times 10^{-7}}{4\pi}\frac{(1.6\times 10^{-19})(220\times 10^3)}{(2\times 10^{-2})^2}\hat{k}$$

you can also use the right-hand rule to get direction of $\vec{B_e}$

$$\boxed{\vec{B} = 8.8\times 10^{-18} \; T \; \hat{k}} \quad \text{small!}$$

7. (40 points total) A conducting loop of radius $a$ carries current $I$. Point $P$ is on the loop's axis a distance $x_0$ from the center of the loop.



(a) (5 points) On the *side view* in the figure below, draw an arrow indicating the direction of the magnetic field $dB$ at $P$ produced only by the current segment $I\,ds$ going into the page ($\otimes$) at the very bottom of the current loop.



*side view*
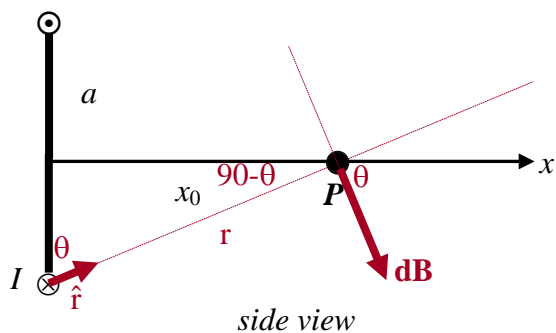
(b) (30 points) Use the Biot-Savart Law to compute $B_x$, the $x$-component of the magnetic field at $P$, due to the entire current loop.

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

$d\vec{s} \parallel (-\hat{k})$ and $\hat{r}$ is in $x$-$y$ plane $\Rightarrow d\vec{s}$ and $\hat{r}$ are $\perp$

$\Rightarrow d\vec{s} \times \hat{r} = ds \sin 90° = ds$

$$dB_x = dB \cos\theta = \frac{\mu_0}{4\pi} \frac{I\, ds}{r^2} \cos\theta \qquad \text{from diagram: } r = \sqrt{x_0^2 + a^2} \quad \& \quad \cos\theta = \frac{a}{r}$$

$$= \frac{\mu_0}{4\pi} I\, ds \frac{a}{r^3} = \frac{\mu_0 I\, a}{4\pi (x_0^2 + a^2)^{3/2}}\, ds$$

$$B_x = \int_{ring} dB_x = \frac{\mu_0 I\, a}{4\pi (x_0^2 + a^2)^{3/2}} \int_{ring} ds = \frac{\mu_0 I\, a}{4\pi (x_0^2 + a^2)^{3/2}} \cdot 2\pi a$$

$$\boxed{B_x = \frac{\mu_0 I\, a^2}{2(x_0^2 + a^2)^{3/2}}}$$

(c) (5 points) The total magnitude $B$ of the magnetic field at $P$ is (circle one below)

    (i) greater than $|B_x|$      (ii) less than $|B_x|$      (iii) equal to $|B_x|$

                                                    (circled)

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8. (40 points total) A rectangular loop of wire of width $L$, height $H$, and resistance $R$ travels at constant speed $v$ into a uniform magnetic field $B$. The plane of the rectangular loop is perpendicular to the magnetic field which is pointing into the page. (All solutions MUST start with OSE's and MUST be expressed in terms of the given parameters.)
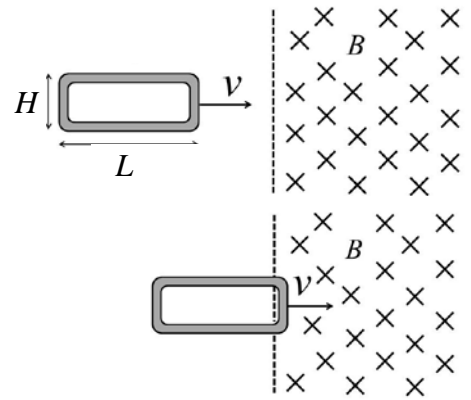
(a) (15 points) At the moment the rectangular loop enters the magnetic field region, an emf is induced which causes a current to circulate around the loop. Calculate the magnitude of the power dissipated in the loop while it is moving into the magnetic field.

$$|\varepsilon| = \left|-N\frac{d\Phi_B}{dt}\right| = \left|\frac{d(BA)}{dt}\right| = B\left|\frac{dA}{dt}\right|$$

$$= B\left|\frac{dHx}{dt}\right| = BH\left|\frac{dx}{dt}\right| = BHv$$

$$|P| = \frac{|\varepsilon|^2}{R} = \boxed{\frac{B^2H^2v^2}{R}}$$

or you can use $I = \frac{\varepsilon}{R} = \frac{BHv}{R} \Rightarrow P = J^2R = \frac{B^2H^2v^2}{R}$

(b) (5 points) What is the direction of the induced current (circle one)?   ↰ Ⓐ(↻)  ⊙   ⊗

(c) (15 points) While the rectangular loop is entering the magnetic field region, the right side of the loop experiences a magnetic force due to the current $I$ induced in it. What is the magnitude of the magnetic force exerted on the right side of the loop?

$$\vec{F_B} = I\vec{L} \times \vec{B}$$

$$F_B = IHB \quad \text{because } IH \text{ is } \perp \text{ to } \vec{B}$$

$$I = \frac{\varepsilon}{R} = \frac{BHv}{R} \qquad F_B = \frac{BHv}{R}HB = \boxed{\frac{B^2H^2v}{R}}$$

(d) (5 points) Once the loop is entirely in the region of uniform magnetic field, what is the magnitude of the current $I$ induced in it.

entirely inside $\Rightarrow$ no flux change $\Rightarrow$ no emf

$$\Rightarrow \boxed{I = 0}$$

9. (40 points total) A particular Argon ion laser has an output power of 1W and emits a cylindrical beam with a radius of 0.95 mm. The beam strikes a flat surface 75 cm away and is completely absorbed. All solutions must start with OSEs, and all answers must include proper units.

(a) (10 points) The beam contains light of many wavelengths, one of which is 488 nm. What is the angular frequency of this wavelength?

Note: in part c you can use $E = pt$, where t is the time it takes a photon to travel 0.75 m.

$$\omega = 2\pi f$$

$$c = f\lambda = \frac{\omega}{2\pi}\lambda$$

$$\omega = \frac{2\pi c}{\lambda} = \frac{2\pi(3\times10^8)}{488\times10^{-9}} = \boxed{3.86\times10^{15}\,Hz}$$

(b) (10 points) What is the magnetic field amplitude of the beam?

$$I = \frac{P}{A} = \frac{1}{2}\frac{cB_{max}^2}{\mu_o} \quad\text{and}\quad A = \pi r^2 \quad\text{where}\quad r = 0.95\times10^{-3}\,m$$

$$B_{max} = \sqrt{\frac{2\mu_o P}{\pi r^2 c}} = \sqrt{\frac{2(4\pi\times10^{-7})(1)}{\pi(0.95\times10^{-3})^2(3\times10^8)}} = \boxed{5.44\times10^{-5}\,T}$$

(c) (10 points) How much energy is contained in the beam between the laser and the absorbing surface?

You can also use $\langle u\rangle = \frac{1}{2}\frac{B_{max}^2}{\mu_o}$

$$I = c\langle u\rangle \implies \langle u\rangle = \frac{I}{c} = \frac{P}{Ac} = \frac{P}{\pi r^2 c}$$

$$E_{beam} = \langle u\rangle\,\text{Volume of beam} = \langle u\rangle\pi r^2 L \quad\text{where}\quad L = 0.75\,m$$

$$E_{beam} = \langle u\rangle\pi r^2 L = \frac{P}{\pi r^2 c}\pi r^2 L = \frac{PL}{c} = \frac{(1)(0.75)}{3\times10^8} = \boxed{2.5\times10^{-9}\,J}$$

(d) (10 points) What average pressure does the beam exert on the completely absorbing surface?

$$\langle P_{rad}\rangle = \frac{I}{c} = \frac{P}{Ac} = \frac{P}{\pi r^2 c} \qquad \left(\text{beam is "completely absorbed"} \implies A = \pi r^2\right)$$

$$\langle P_{rad}\rangle = \frac{1}{\pi(0.95\times10^{-3})^2(3\times10^8)}$$

$$\boxed{\langle P_{rad}\rangle = 1.18\times10^{-3}\,Pa} \quad \frac{N}{m^2}\text{ is acceptable for units}$$

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