Physics 4311: Thermal Physics - Homework 11

due date: Tuesday, April 18, 2023, please upload your solution as a pdf on Canvas

Problem 1: Model of DNA (10 points)

A simple model of the DNA double helix molecule is analogous to a zipper: a chain of N links each of which can be open or closed. A closed link has energy ϵ_0, and an open link has energy $\epsilon_1 > \epsilon_0$. Replication of the DNA starts with the opening of the "zipper". Assume that it can only open from one end (say the left), i.e., a link can only be open if all links left of it are also open.

a) Calculate the partition function for this DNA model.
b) Find the average number n of open links as a function of N and the temperature T .
c) Discuss the behavior of n in the limits of high and low temperatures.

Problem 2: Spin-1 paramagnet (20 points)

A paramagnetic material contains N non-interacting spins with quantum numbers $S = 1$ and $S^{(z)} = -1, 0, +1$. In an external magnetic field B, the energy of the system reads

$$E = -\sum_{i=1}^{N} \mu B S_i^{(z)}$$

where the constant μ is the magnetic moment associated with the spin.

a) Calculate the partition function and the free energy.
b) Determine the magnetization as a function of temperature and discuss its behavior in the limits $T \to 0$ and $T \to \infty$.
c) Compute the magnetic susceptibility χ.
d) Find the leading low-field (high-temperature) behavior of χ and identify the Curie law. Compare with the spin-1/2 case discussed in class.

Problem 3: Generalized equipartition theorem (10 points)

Consider a classical degree of freedom q that makes a contribution to the Hamiltonian of the form $\frac{1}{2}A|q|^n$ where n and A are positive constants. Find the average internal energy stored in this degree of freedom as a function of temperature.