Physics 4311: Thermal Physics - Homework 12

due date: Tuesday, April 25, 2023, please upload your solution as a pdf on Canvas

Problem 3: Ideal gas in the gravitational field (20 points)

Consider a classical ideal gas of N particles at temperature T in a vessel of cross section A and height H. The particles of the gas are under the influence of a gravitational potential $E_{pot} = mgz$ where m is the mass of a particle, g is the free fall acceleration and z is the vertical coordinate. (Assume H to be large, $mgH \gg k_B T$)

a) Calculate the partition function and the free energy of the gas.

b) Determine the internal energy U and the specific heat C_V. Compare with the equipartition theorem.

c) Calculate how the particle density $n(z)$ changes with z. (Hint: the particle density $n(z)$ is a reduced probability density of the phase space density $p(\vec{r}, \vec{p})$.)

d) What is the approximate ratio between the air density at sea level and at an altitude of 8000m? Use the following approximate values: $g \approx 10\mathrm{ms}^{-2}$, $m \approx 4 * 10^{-26}\mathrm{kg}$ (mass of a nitrogen molecule), $k_B \approx 4/3 * 10^{-23}\mathrm{J/K}$, $T = 300\mathrm{K}$.

Problem 2: Ultra-relativistic classical ideal gas (20 points)

Consider a gas of N non-interacting, indistinguishable, classical particles at temperature T in a cubic box of linear size L. The energy-momentum relation is ultra-relativistic, $E = \mathrm{c}|\vec{\mathrm{p}}_\mathrm{i}|$, where c is the speed of light.

a) Calculate the partition function and the free energy of the gas.

b) Calculate the pressure as function of N, T, and V.

c) Find the internal energy U and the specific heat C_V at constant volume.

d) Also determine the specific heat at constant pressure, and compare the ratio of to that of the nonrelativistic case.