Physics 4311: Thermal Physics - Homework 7

due date: Tuesday, March 14, 2023, please upload your solution as a pdf on Canvas

Problem 1: Additivity of the entropy (8 points)

A system consists of two subsystems A and B. Subsystem A can be in states $i = 1 \dots n$ and subsystem B can be states $j = 1 \dots m$. The Gibbs entropy of this system reads

$$S = -k_B \sum_{i=1}^{n} \sum_{j=1}^{m} p(i,j) \ln p(i,j)$$

where $p(i,j)$ are the joint probabilities for the states of the subsystems.

Show that if the two subsystems A and B are statistically independent, then the entropy S of the total system is the sum of the entropies of the two subsystems.

Problem 2: Maxima of entropy (16 points)

As system can be in N different states with probabilities p_i ($i = 1 \dots N$). Determine which p_i lead to the maximum (Gibbs) entropy under the following constraints:

a) Fixed normalization $\sum_i p_i = 1$

b) fixed normalization $\sum_i p_i = 1$ and fixed average energy $\langle E \rangle = \sum_i p_i E_i$.

Hint: Use Lagrange multipliers to enforce the constraints.

Problem 3: Air conditioner (16 points)

An ideal air conditioner consists of a Carnot cycle (running backwards). It absorbs heat from the inside of a house at the lower temperature T_l and discharges heat to the outside at the higher temperature T_h, consuming electric energy E. The heat leaking back into the house through the walls and windows is given by $\Delta Q = A(T_h - T_l)$ where A is a constant.

a) The air conditioner runs continuously, and the temperature in the house has reached a steady state. Derive a relation for the inside temperature T_l in terms of T_h, A, and E.

b) The system is designed such that it runs at 50% of the maximum electrical energy input if the outside temperature is 90 °F and the inside temperature is 70 °F. What is the highest outside temperature for which the system can maintain an inside temperature of 70 °F at full electrical input.