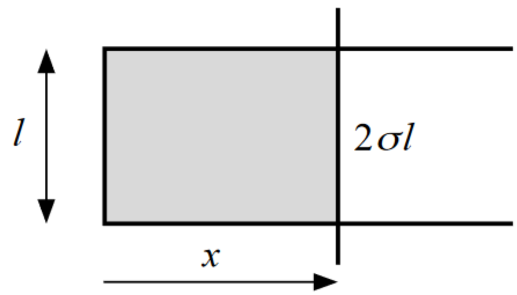
Physics 4311: Thermal Physics - Homework 9

due date: Tuesday, April 4, 2023, please upload your solution as a pdf on Canvas

Problem 1: Maxwell relations for an elastic rod (9 points)

An elastic rod of length L can be stretched or compressed by changing the applied tension force f. The work differential reads $\delta W = f\, dL$. Using the thermodynamic potentials that you found in homework 8.3, derive the four Maxwell relations for this system.

Problem 2: Soap film (15 points)



The figure illustrates a soap film (shown in gray) supported by a wire frame. Because of surface tension γ, the film exerts a force $2\gamma l$ on the cross wire, in the direction that decreases the area of the film. The factor 2 occurs because the film has two surfaces. The temperature dependence of γ is given by $\gamma = \gamma_0 - aT$ where γ_0 and a are constants.

a) Write a relation expressing the change dU in internal energy of the film in terms of the heat TdS absorbed by it and the work done on it in an infinitesimal quasi-static process in which the distance x is changed by an amount dx.

b) Calculate the change in internal energy $\Delta U = U(x) - U(0)$ of the film when it is stretched at a constant temperature T_0 from a length 0 to a length x.

c) Calculate the work done on the film in order to stretch it at this constant temperature from a length 0 to a length x.

Problem 3: Thermodynamic and caloric equations of state (16 points)

For a gas or liquid described in terms of pressure p, volume V, and temperature TT, show that the thermodynamic equation of state (the relation between p, V, and T) and the caloric equation of state (the dependence of the internal energy U on the other variables) are not independent.

a) Specifically show that $(\partial U/\partial V)_T = -p + T(\partial p/\partial T)_V$

b) Apply the above relation to the ideal gas and show that the internal energy must be volume-independent if $pV = Nk_BT$.

This problems requires a bit of creativity working with partial derivatives. Start from the differential of the entropy as function of U and V. Express the energy differential in terms of T and V. Now use the equality of the mixed second derivatives of the entropy with respect to T and V.