Profit-sharing between an open-source firm and application developers — Maximizing profits from applications and in-application advertisements

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ABSTRACT

More and more for-profit organizations are promoting their products using open-source strategies. In Google’s Android open-source project, the open-source firm and application developers share profits from the sales of paid applications and advertisements in free applications. Recently, the open-source strategy has received considerable attention in the literature. However, the profit-sharing model and in-application advertisements have not been well studied in the context of an open-source business. These are critical gaps in the literature, since the open-source firm may utilize a profit-sharing scheme to exercise non-coercive power and to grow the user network and advertising business. We propose a model to understand how the profit-share percentage and the percentage of paid applications, in relation to the size of the user network, affect the open-source firm’s profits from applications and in-application advertisements. Our study shows that growing the user network does not necessarily increase the open-source firm’s profit. Further, the study suggests that the optimal profit-share percentage maximizing the open-source firm’s profit from advertisements is lower than that maximizing the profit from applications. Additionally, our study illustrates a potential threat of application developers’ opportunistic behavior against the open-source firm.

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1. Introduction

Recognizing the great potential of open-source business strategies, more and more for-profit organizations are considering an open-source strategy, instead of a closed-source strategy, to promote their products (Kumar, Gordon, & Srinivasan, 2011). With a closed-source strategy, a firm maintains its control associated with the product (Casadesus-Masanell & Llanes, 2011). In contrast, with an open-source strategy, a firm makes its intellectual input for a product (e.g., a software’s source code) nonproprietary by allowing other organizations and individuals to access its intellectual input (Pitt, Watson, Berthon, Wynn, & Zinkhan, 2006). In this case, a firm delegates its control associated with the product to other organizations (Pitt et al., 2006). Delegating control subsequently leads to delegating power to other organizations (Belaya & Hanf, 2009; El-Ansary & Stern, 1972).

Delegating control and power to other organizations allows a group of organizations to create value jointly (Frels, Shervani, & Srivastava, 2003). For instance, within the Android open-source project, Google, an open-source firm, discloses the source code of the Android operating system to application developers. In return, application developers develop applications for the Android platform. Successfully utilizing an open-source strategy to promote its Android operating system, Google achieved a 79% share of the smartphone market worldwide (Al-Saleh & Forihat, 2013; Butler, 2011; Clark & Connors, 2013; Mallapragada, Grewal, & Lilien, 2012).

However, delegating control could result in opportunistic behavior by the application developers. For instance, application developers could copy the Android operating system and introduce their own operating systems, instead of developing applications for the Android platform. One way for the open-source firm to avoid partners’ opportunistic behavior is to build mutual relationships with partners (e.g., application developers) using monetary incentives (Wathne & Heide, 2000). As one of the monetary incentive strategies, the profit-sharing scheme has been commonly used in the Android open-source project, where the open-source firm takes a certain portion of the profits of application developers (Gandhewar & Sheikh, 2010). The profit-share percentage of the open-source firm, defined as the percentage that application developers pay to the open-source firm out of their profits, may significantly affect the developers’ motivation to join the network, the number of applications available for the platform, the size of the user network, and subsequently the success of the open-source project (Oh & Jeon, 2007; Roberts, Hann, & Slaughter, 2006). Here, the open-source firm uses the profit-sharing scheme in a positive manner to motivate its partners and to exercise its non-coercive power (Belaya & Hanf, 2009; Geyskens & Steenkamp, 2000; Wagner &
Lindemann, 2008). However, profit-sharing schemes have received inadequate attention in the literature on an open-source business. Specifically, the effect of the user network has not been well discussed in the context of profit-sharing. This is a critical gap in the literature.

In the open-source project, application developers create both paid applications and ad-supported free applications. While generating profits by selling advertising space in ad-supported free applications to advertisers, application developers commonly charge consumers for ad-free applications (Gandhewar & Sheikh, 2010; Gordon, 2013). Thus, application developers generate profits from both applications and in-application advertisements. The open-source firm and the application developers can strategically decrease (increase) the percentage of paid applications (free applications) to attract more users to the network (Manoogian, 2012). In the Android project, Google (an open-source firm) provides the developers with the platform (the Android operating system), an application store (Google Play), and an advertising platform (AdMob). AdMob, owned by Google, is an advertising platform for application developers to monetize their applications through in-application advertisements (Bavor, 2011). In-application advertisements, similar to banner ads, are displayed to smartphone users when they use applications on a smartphone. Although advertisers are dramatically increasing their spending on mobile advertisements, especially in-application advertisements (Gartner, 2013; Infiniti Research Limited, 2013), in-application advertising has received scant attention in the literature on open-source business. This is another critical gap, as advertising is a significant source of revenue for an open-source firm (Patel, 2011).

It is the general purpose of this paper to close these two critical gaps in the literature: the use of a profit-sharing scheme and the role of in-application advertising in an open-source business model. The profit-sharing mechanism may affect the size of the user network and subsequently the success of in-application advertising, as advertisers generally prefer a bigger network (Casadesus-Masanell & Zhu, 2010). Thus, it is worthy to investigate both the profit-sharing mechanism and in-application advertisements in the context of the open-source business to close these two gaps. In doing so, we investigate how the profit-sharing scheme between an open-source firm and application developers, as well as the percentage of paid applications, affects the size of the user network and the open-source firm’s profits from both applications and in-application advertisements.

In sum, our objectives are to address the following questions through analyzing our proposed model:

- Does a larger user network, achieved through lowering the profit-share percentage of the open-source firm and/or through lowering the percentage of paid applications, always benefit the open-source firm’s profits from applications and in-application advertisements?
- Is the user-network size equally important in maximizing the open-source firm’s profits from both advertising business and application business?
- Does maximizing the profit of the entire open-source community always lead to a win–win relationship between the open-source firm and application developers?

2. Literature review

2.1. Sources of profits and profit sharing in an open-source business

Open-source firms generate profits not only through applications, accessories, and support services for platform users (e.g., Casadesus-Masanell & Llanes, 2011; Kumar et al., 2011), but also through in-application advertisements (Patel, 2011). Through these activities, an open-source strategy, as opposed to a closed-source strategy, enhances a firm’s value creation, as it allows the firm to involve more organizations and individuals in the process of value creation (Casadesus-Masanell & Llanes, 2011; Jap, 1999). For instance, an open-source strategy enhances the variety of applications available to its platform users (Economides & Katsamakas, 2006). As the number of applications available in the Android market increases every year (Tibken, 2012), both the number of applications that users download and the amount of time they spend using these applications are on the rise (AFP Relaxnews, 2013; Nielsen, 2012). More time spent using the applications means more opportunities for in-application advertising. Consequently, application developers are providing more and more free ad-sponsored applications instead of charging for applications (Worrell, 2013).

The open-source firm uses a profit-sharing mechanism to distribute these profits from applications and advertisements. Profit-sharing mechanisms have been studied in various B2B contexts in the past, including profit-sharing between two organizations that are financially independent (Cachon & Lariviere, 2005; Chauhan & Proth, 2005; Jap, 2001), profit-sharing between two firms that form a joint venture (Du, Hu, & Liu, 2006; Wang & Zhu, 2005), profit-sharing between the franchisor and franchisee (Yan & Wang, 2012), multiple profit-sharing contracts within a supply chain (Giannoccaro & Pontrandolfo, 2004), and profit-sharing mechanisms in multi-channel contexts (Yan, 2011). The primary focus of these studies is to investigate how to utilize profit-sharing mechanisms to enhance overall profit among partners, and subsequently increase each partner’s shared profit to achieve win–win relationships (e.g., Chauhan & Proth, 2005; Du et al., 2006; Giannoccaro & Pontrandolfo, 2004; Yan & Wang, 2012).

Using the profit-sharing mechanism, these studies investigate the determinants of the overall profit among the partners, including information-sharing among the partners (Yan & Wang, 2012), a distributor’s wholesale price to a retailer (Giannoccaro & Pontrandolfo, 2004), a retailer’s selling price, and inventory levels of a distributor and a retailer (Chauhan & Proth, 2005). Unlike these B2B contexts studied in the past, the open-source firm’s success relies on the availability of applications for the open-source platform, as more applications lead to more users in the network (Frels et al., 2003; Tibken, 2012). Thus, it is critical to study how the open-source firm can utilize a profit-sharing mechanism to motivate more application developers to join the open-source platform and develop applications for the platform.

2.2. Power and control within an open-source community

The profit-sharing mechanism allows the open-source firm to exercise power over its partners through coercive action such as threats of negative consequences or through non-coercive action such as promises for positive consequences (Kumar, 2005). Coercive action includes threatening partners with the loss of any expected rewards and punishing partners (Belaya & Hanf, 2009). In contrast, non-coercive action includes the use of monetary incentives in a positive manner to motivate its partners (Belaya & Hanf, 2009). For instance, a manufacturer may promise more rewards to its suppliers to motivate them to improve their channel activities (Geyskens & Steenkamp, 2000; Wagner & Lindemann, 2008).

From an agency-theory perspective (Eisenhardt, 1989), the open-source movement offers a new type of governance structure, where an open-source firm or the principal delegates some work and authority to other firms or the agent (e.g., application developers) by making valuable information (e.g., the source code of the operating system) available to the agent. Delegating an open-source firm’s authority and control to its community members allows the open-source community to have a decentralized structure (Pitt et al., 2006). For instance, in the Android community, developers are not required to get approval from the open-source firm to make new applications available for users (Butler, 2011). Delegating more control and power to other community members sometimes facilitates the processing of the community members’ contributions (Hamm, 2005).

However, such delegation of control and power may lead to opportunistic behavior of the agents. For instance, the agent may copy the principal’s idea (e.g., the Android operating system). The principal can
reduce the opportunistic behavior of its agents with monetary incentives that increase the agents’ long-term benefits and encourage cooperative behavior (Mishra, Heide, & Cort, 1998; Watthe & Heide, 2000). Traditionally, imbalances in the amount of resources between the principal and the agent lead to imbalances in power (Bucklin & Sengupta, 1993). Setting a contractual agreement between parties, including a monetary incentive system, reduces power imbalances and conflicts between the principal and the agents, and avoids exploitive behavior from either party (Bucklin & Sengupta, 1993; Eisenhardt, 1989; Rindfleisch & Heide, 1997). Such an agreement further protects the rights and powers of the weaker party in the relationship (Bucklin & Sengupta, 1993) and restores balance in the relationship. Perceiving that they are receiving a fair share of the profit derived from the relationships with the principal, the agents also show trust and commitment toward the principal (Kumar, 2005; Scheer, Kumar, & Steenkamp, 2003). Especially when a powerful principal successfully creates the perception of fairness toward the distribution of outcome (distributive justice) among its agents, the principal can build trust and commitment among its agents (Kumar, 2005; Kumar, Scheer, & Steenkamp, 1995).

2.3. Network externality effects within an open-source business

The open-source strategy is believed to be effective in marketing products with significant network externalities (Bonaccorsi & Rossi, 2003; The Economist, 2004). Thus, incorporating network externalities is crucial in developing the framework of an open-source business model (Comino & Manenti, 2005). A network externality is described as the following characteristic of a product: the more users who adopt a product, the more utility the product has, resulting in even more new users adopting the product (Cheng, Liu, & Tang, 2011; Haruvy, Sethi, & Zhou, 2008). Particularly information goods, such as operating systems, are believed to rely on network externality in the diffusion process; a user can read, modify, and share files on the operating system relatively easily when more and more members with whom he or she communicates use the same operating system (Bonaccorsi & Rossi, 2003).

There are three sources of network externality: the user network, the complements network, and the producer network (Frels et al., 2003). Users receive benefits from these three different sources when network externality exists (Frels et al., 2003). The underlying notion is that users prefer to be part of a larger network (i.e., the user network), to have more complementary products and services (i.e., the complements network), and to have more providers of the products/services (i.e., the producer network) (Frels et al., 2003). For instance, Android dominates the worldwide smartphone market (i.e., the user network), with 800,000 applications available (McCracken, 2013), which results in a 75% market share of total application downloads for mobile devices (i.e., the complements network) (Bradley, 2013), and with multiple manufacturers of smartphones (e.g., Samsung, LG, and Motorola) (i.e., the producer network). In this paper, we specifically consider the role of the user network, in conjunction with the complements network, within the open-source community (Frels et al., 2003).

3. Model framework

In this section, we propose a model to study the profits of an open-source firm from both applications and in-application advertisements. We assume a monopoly market where there is no competition for the open-source firm. In the open-source project, the open-source firm may attempt to motivate the application developers to join the project and develop more applications with monetary incentives (Mishra et al., 1998; Watthe & Heide, 2000). For instance, to attract more application developers, the open-source firm may use its non-coercive power and promise a relatively high profit-share percentage for developers (Geyskens & Steenkamp, 2000; Wagner & Lindemann, 2008).

We assume that the open-source firm attempts to maximize its profits by controlling the profit-share percentage it takes from the sales of application developers and by influencing the percentage of paid (vs. ad-supported free) applications. We assume that the cost to the open-source firm for developing the platform (the Android operating system) is fixed and that there are no variable costs. In our model, we will ignore the fixed cost, because it has no role in determining the profit-maximizing share percentage and paid-application percentage (Lilien, Kotler, & Moorthy, 1992; p. 173).

3.1. Size of the network

Both the complements network (e.g., the number of applications available on the Android operating system) and the user network (e.g., the number of Android smartphone users) play an important role in the success of the open-source project. Thus, we will analyze how the profit-share percentage contributes to the growth of the complements and user networks. Let \( 0 \leq \theta \leq 1 \) denote the profit-share percentage of the open-source firm.

If the open-source firm allows the developers to keep a higher portion of their profit, more application developers are motivated to join the platform (Roberts et al., 2006), resulting in a larger complements network (Mishra et al., 1998; Watthe & Heide, 2000). We assume that the number of application developers increases linearly with the profit-share percentage they can keep. If their profit-share percentage is zero, no developers participate in the open-source project. In addition, we assume that the number of applications linearly increases with the number of developers. Denote \( N_a \) as the size of the complements network. Then, we have

\[
Q_a = K(1-\theta). \tag{1}
\]

Here, \( K > 0 \) represents the maximum number of applications the developers would create, if the application developers could keep all of the profit (i.e., \( \theta = 0 \)).

Next, we consider the effect of the complements network on the user network. Let \( 0 \leq \xi \leq 1 \) denote the percentage of paid applications, and \( 1 - \xi \) denote the percentage of ad-supported free applications on the platform. We assume that the percentage of paid applications is provided by the open-source firm to application developers as part of a strategic guidance and is not affected by the profit-share percentage. Generally, a platform with more applications can attract more users (Frels et al., 2003; Tibken, 2012). Furthermore, the user network grows relatively slowly (quickly), when all applications available on the platform are paid applications (ad-supported free applications) (Manoogian, 2012). Here, we assume that the user-network size increases linearly with respect to the percentage of free applications, and that the relationship between the user-network size and the complements-network size satisfies the constant elasticity model (Coffey, 1979; Huang, Leng, & Parlar, 2013; Lambin, 1972). Let \( N_u \) denote the size of the user network that is determined by:

\[
N_u = C(1 + \epsilon - \xi) Q_a^\theta. \tag{2}
\]

where \( C, \epsilon, \) and \( x \) are all positive constants. \( \epsilon \) is much smaller than 1, and \( C \) reflects to what extent increasing the size of the complements network contributes to the growth of the user-network size when \( \xi = 1 \). The constant \( \epsilon \) represents the elasticity of the user-network size to the complements-network size (Lambin, 1972). It may vary depending on the market-saturation level. For example, when the number of users is relatively small and the potential market is only partially covered, we expect that introducing new applications could lead to a dramatic increase in the number of new users; thus, we assume that \( \epsilon > 1 \). When the potential market is close to being saturated, we expect that introducing new applications leads to only a moderate increase in the number of new users; thus, we assume \( 0 \leq \epsilon < 1 \). Finally, when the market has been saturated, we expect that introducing new applications may not increase the number of new users; thus \( \epsilon = 0 \).
Substituting Eq. (1) into Eq. (2), we have:

\[ N_0 := N_0(\theta, \xi) = \alpha (1 + \varepsilon - \xi)(1 - \theta)^x \]  

(3)

with the constant \( \alpha = CK^\theta > 0 \). Eq. (3) implies that the size of the user network is maximized when the open-source firm does not share any profit from application developers (i.e., \( \theta = 0 \)) and all applications on the platform become free (i.e., \( \xi = 0 \)). Thus, to expand the user network, the open-source firm should promise a higher profit-share percentage to application developers and/or decrease the percentage of paid applications in the complements network.

### 3.2. Profits from applications

In this section, we study the profit of the open-source firm from paid applications. Let \( p_a \) denote the average price of paid applications, and \( M_i^f \) denote the average number of paid applications that a user downloads on his or her Android smartphone. Then, the profit of the open-source firm from applications \( \Pi_1 \) can be calculated by:

\[ \Pi_1 = \theta p_a M_i^f N_o. \]  

(4)

The number of applications downloaded per user depends on both the average selling price per application (Austin, 2013; Bradley, 2013) and the total number of paid applications available in the Android market. For example, an Android phone user may install fewer paid applications if the average selling price per application is higher. In contrast, if the average selling price stays constant, as more paid applications are available for the platform, a user may download more paid applications. Here, we assume that the relationship between the average number of paid applications per user and the total number of paid applications available satisfies the constant elasticity demand function (Coffey, 1979; Huang et al., 2013; Lambin, 1972).

Thus, the number of paid applications per user would decrease if the average price increases. If the average price \( p_a \) is too high, the function \( f(p_a) = 0 \), meaning that users do not download any paid applications.

Substituting Eqs. (1), (2), and (5) into Eq. (4), we obtain the profit of the open-source firm from selling paid applications:

\[ \Pi_1 := \Pi_1(p_a, \theta, \xi) = \mu p_a f(p_a) \left( \theta (1 - \theta)^x \xi (1 + \varepsilon - \xi) \right). \]  

(6)

where the constant \( \mu = CK^\theta \geq 0 \). Eq. (6) shows that the open-source firm can make profits from applications only when its profit-share percentage \( \theta \neq 0 \) and \( \theta \neq 1 \), and the percentage of paid applications \( \xi \neq 0 \).

### 3.3. Profits from in-application advertisements

In this section, we analyze the open-source firm’s profit from in-application advertisements. In the rest of this paper, unless noted otherwise, we will use “advertisements” to refer to in-application advertisements. In this paper, we assume that advertisers use the open-source network as an advertising media for in-application advertisements (Infiniti Research Limited, 2013), and that the advertisers are charged by the number of impressions. An ad impression is defined as a loading of a web page with an intended (banner) advertisement on a mobile device (Ahmed & Kwon, 2012). The advertising cost associated with an impression is generally described as CPM (cost per impression) (Ahmed & Kwon, 2012). Let \( p_i \) denote the average price per impression, \( N_o \) denote the total number of applications, and \( M_i^r \) denote the average number of impressions an advertiser wants to advertise on the platform. We assume that there are more advertising spaces available than advertisers demand (Kim, 2012; Meulen & Rivera, 2014). Then the open-source firm’s profit from advertisements \( \Pi_2 \) is calculated by:

\[ \Pi_2 = \theta p_i M_i^r N_o. \]  

(7)

In selecting advertising media, advertisers typically prefer a network with a larger audience; therefore, the larger the user network is, the more attractive it is to advertisers (Gabszewicz, Laussel, & Sonnac, 2004). In addition, advertisers are less likely to advertise in a small user network, and they may begin to advertise in the open-source network only as it reaches a certain size. Hence, the number of advertisers \( N_o \) is defined as (Coffey, 1979; Huang et al., 2013; Lambin, 1972):

\[ N_o = \begin{cases} \gamma N_o^e, & N_o \geq N_{0,\min}, \\ 0, & \text{otherwise}. \end{cases} \]  

(8)

where \( z > 0 \) denotes a constant elasticity, and the constant \( \gamma > 0 \). Here, \( N_{0,\min} \) denotes the minimum user–network size that is required to begin attracting advertisers. We assume that \( N_{0,\min} \) is much smaller than the user–network size that maximizes the profit of an open-source firm from advertisements.

Furthermore, the average number of impressions per advertiser \( M_i^r \) is affected by the average price per impression \( p_i \), the average number of free applications per user \( M_o \) and the size of the user network \( N_o \) (Fridegardsdottr & Asadollahi, 2013; Gabszewicz et al., 2004). A higher price per impression will lead to a higher advertising cost; thus, advertisers may be discouraged from advertising on the open-source network, resulting in a smaller number of impressions. Also, as the size of the user network increases, the network becomes more attractive to advertisers as an advertising media; thus, the number of impressions per advertiser becomes larger (Gabszewicz et al., 2004). Here, we assume that the advertisers always want to maintain the number of impressions per user. Then, we have

\[ M_i^r = g(p_i) M_o N_o = g(p_i) [(1 - \xi) Q_a y]^\gamma N_o. \]  

(9)

where the average number of free applications per user, \( M_o^f \) depends only on the total number of free applications available on the platform. Thus, following the same lines of Eq. (5), we have \( M_o^f = [1 - \xi] Q_a y \). The function \( g(p_i) \) describes the demand of impressions per advertiser with respect to the price \( p_i \). It satisfies \( \frac{\partial g(p_i)}{\partial p_i} \leq 0 \), indicating that the number of impressions per advertiser decreases if the average price per impression increases. If the average price \( p_i \) is too high, the function \( g(p_i) = 0 \).

Substituting Eqs. (1), (2), (8), and (9) into Eq. (7), we obtain the profit of the open-source firm from advertisements:

\[ \Pi_2 := \Pi_2(p_i, \theta, \xi) = \begin{cases} \lambda p_i g(p_i) (1 - \theta)^\gamma (1 - \xi) y (1 + \varepsilon - \xi)^{1 + z}, & (\theta, \xi) \in S, \\ 0, & (\theta, \xi) \notin S, \end{cases} \]  

(10)

where the constant \( \lambda = \gamma C^\theta + z K^\theta z + 1 + \gamma > 0 \). Here, \( S \) denotes the feasible set of \( \theta \) and \( \xi \), such that the resulting size of the user network is not smaller than \( N_{0,\min} \), i.e., \( S = \{ (\theta, \xi) | N_o(\theta, \xi) \geq N_{0,\min} \} \).
4. Propositions

In this section, we present four propositions to discuss the dependence of the open-source firm’s profits on the profit-share percentage, the percentage of paid applications, and the size of the user network. In the first two propositions, we will discuss the role of the size of the user network in generating profits from applications and advertisements.

Proposition 1. Increasing the size of the user network does not necessarily increase the open-source firm’s profit from applications. Excessively increasing the size of the user network by decreasing the profit-share percentage and/or the percentage of paid applications will lead to a decline in profit from applications.

(See Appendix B for the proof.)

Proposition 1 indicates that the size of the user network generally plays an important role in generating profits from applications. To increase the size of the user network, the open-source firm can decrease its profit-share percentage and/or reduce the percentage of paid applications. The former enhances the size of the user network by motivating more application developers to join the platform, consequently increasing the number of applications available on the platform. In other words, it expands the user network through increasing the size of the complements network. In contrast, the latter increases the size of the user network by attracting more users to join the network as more free applications become available. Proposition 1 suggests that although it is generally beneficial for the open-source firm to increase the size of the user network, growing the user network beyond a certain size could hurt the profit of the open-source firm from applications.

The proof of Proposition 1 provides the optimal profit-share percentage \( \theta_1^* \) and the optimal percentage of paid applications \( \xi_1^* \), maximizing the open-source firm’s profit from applications, as well as the corresponding network size \( N_{0,1}^* \). Decreasing the profit-share percentage of the open-source firm below \( \theta_1^* \) still encourages more application developers to participate in the open-source project, resulting in an increase in the size of both the complements network and the user network. However, when \( \theta < \theta_1^* \), the profit-share percentage of the open-source firm is too low to enhance its profits. In fact, the negative effect from the lower profit-share percentage outweighs the positive effect of the larger user network on the profit, leading to a decline in profit. Similarly, reducing the percentage of paid applications below \( \xi_1^* \) may attract more users as more free applications become available. However, without enough paid applications to sell, this strategy hurts the open-source firm’s profits from applications. In summary, the open-source firm may experience a decline in the profit from applications by excessively using non-coercive power (i.e., providing too much reward for application developers) and/or by lowering the percentage of paid applications too much. Next, we will discuss how the size of the user network affects the profits of the open-source firm from applications.

Proposition 2. Increasing the size of the user network does not necessarily increase the open-source firm’s profit from advertisements. Excessively increasing the size of the user network by decreasing the profit-share percentage will lead to a decline in profit from advertisements.

(See Appendix C for the proof.)

Proposition 2 suggests that maximizing the size of the user network does not maximize the open-source firm’s profit from advertisements. The proof of Proposition 2 provides the optimal profit-share percentage \( \theta_2^* \) and the optimal percentage of paid applications \( \xi_2^* = 0 \) that maximize the profit of the open-source firm from advertisements, as well as the corresponding network size \( N_{0,2}^* \). Here, one condition to maximize the profit of the open-source firm from advertisements is to make all applications on the platform free (i.e., \( \xi_2^* = 0 \)). Once all applications become free, to further grow the user network, the open-source firm must decrease its own profit-share percentage below \( \theta_2^* \). Decreasing the profit-share percentage of the open-source firm encourages more application developers to participate in the open-source project, increases the number of applications available, and subsequently increases the size of the user network. However, the negative effect from the lower profit-share percentage outweighs the positive effect of the larger user network, decreasing the profit from advertisements. In the following proposition, we will compare the effects of profit-share percentage and network size on profits between application business and advertising business.

Proposition 3. (a) The optimal profit-share percentage maximizing the open-source firm’s profit from advertisements is lower than that maximizing the profit from applications. (b) A larger user network is required for the open-source firm to maximize the profit from advertisements than to maximize the profit from applications.

(See Appendix D for the proof.)

Proposition 3 (a) indicates that the optimal profit-share percentage \( \theta_1^* \) that maximizes the profit of the open-source firm from applications does not maximize its profit from advertisements. In order to maximize the profit from advertisements, the open-source firm must lower its profit-share percentage below \( \theta_1^* \) to \( \theta_2^* \) and further enhance the size of the user network. In other words, the advertising business requires a larger network to maximize its profit than does the application business (Proposition 3 (b)). Furthermore, Proposition 3 (b) suggests that increasing the network size should be a more important strategic consideration for the success of advertising business than for the success of application business. In sum, Proposition 3 implies that the use of non-coercive power (e.g., rewards for application developers) is more important for advertising business than for application business. To maximize its profit from advertisements, the open-source firm needs to use non-coercive power to motivate more application developers to join the platform and further enhance the network size. Additionally, the choice of the profit-share percentage may also depend on the strategic priority of the open-source firm for application vs. advertising business. For example, prioritizing advertising (application) business over application (advertising) business, the open-source firm may choose an overall profit-share percentage \( \theta_2^* < \theta_1^* \) but close to \( \theta_1^* \) even by sacrificing the profit of its application (advertising) business.

Proposition 4. The open-source community can further increase the profit of the entire open-source community from applications (advertisements) by further decreasing the profit-share percentage of the open-source firm below \( \theta_1^* \) (\( \theta_2^* \)).

(See Appendix E for the proof.)

The open-source community that consists of the open-source firm and application developers can further enhance the community’s profit from applications (advertisements) by decreasing the profit-share percentage below \( \theta_1^* (\theta_2^*) \). In other words, sacrificing the profit of the open-source firm further enhances the joint profit of the open-source community in application and advertising businesses. Eventually, eliminating the profit of the open-source firm maximizes the profit of the open-source community. In reality, the open-source firm is unlikely to lower its profit-share percentage to zero and give up all of its profit. However, this insight may motivate a major application developer or a group of application developers to engage in opportunistic behavior against the open-source firm. A group of application developers may try to copy the open-source platform and create a similar open-source platform to keep all the profits and maximize their profits from applications and advertisements.

5. Numerical examples

To supplement our discussion in the propositions, we present numerical examples with \( x = y = z = 1 \), where we illustrate the effects of the profit-share percentage \( \theta \) and the percentage of paid applications
ξ on the profit of the open-source firm in relation to the size of the user network. We would like to note that the parameters x, y, and z may affect the value of profits, but they do not affect our conclusions on profit maximization. Fig. 1a (Fig. 2a) illustrates the effect of the profit-share percentage θ on the profit from applications Π1 (advertisements) Π2 for various percentages of paid applications ξ, while Fig. 1b (Fig. 2b) shows the relationship between the network size No and the profit from applications Π1 (advertisements) Π2.

First, we observe the effect of the profit-share percentage on the profit from applications (advertisements). As illustrated in Fig. 1a (Fig. 2a), for a fixed percentage of paid applications ξ, the profit Π1 (Π2) increases as the profit-share percentage θ decreases, and it reaches the maximum value when θ = θ1∗ (θ = θ2∗). Further decreasing the profit-share percentage θ below θ1∗ (θ2∗) leads to a decline of the profit Π1 (Π2). Comparing Figs. 1a and 2a shows that while the profit Π1 is maximized at (θ1∗, ξ1) = (\(\frac{1}{4}, \frac{1}{4}\)), the profit Π2 is maximized at (θ2∗, ξ2) = (\(\frac{1}{4}, 0\)), thus θ1∗ ≠ θ2∗ and ξ1 ≠ ξ2.

Next, we discuss the relationship between the size of the user network and the profit from applications (advertisements) in Fig. 1b (Fig. 2b). For a given percentage of paid applications ξ, the profit Π1 (Π2) continues to increase as the network grows until Π0(θ1∗, ξ) (Π0(θ2∗, ξ)). However, further growing the network decreases the profit Π1 (Π2). As illustrated in Fig. 1b, by maintaining a relatively high percentage of paid applications (e.g., ξ = 75%), the open-source firm can enjoy the high sensitivity of the profit Π1 with respect to the network size. Thus, the open-source firm can generate profits from applications relatively quickly even though the user network is relatively small. However, the relatively high percentage of paid applications may hinder the growth of the network and the profit from applications. Thus, the percentage of paid applications must be dropped (ξ = 50%) to further grow the network size and increase the profit. However, excessively decreasing the percentage of paid applications reduces the sensitivity of the profit Π1 with respect to network size. For example, when ξ = 25%, the profit Π1 increases much more slowly with respect to the size of the user network than when ξ = 50% or 75%.

Furthermore, as illustrated in Fig. 2b, when the percentage of paid applications is relatively high (e.g., ξ = 50%), the profit from advertisements Π2 increases relatively slowly as the user network expands. As the open-source firm decreases the percentage of paid applications, the profit from advertisements becomes more sensitive with respect to the network size. When all applications on the platform are free (i.e., ξ = 0), the profit from advertisements is maximized at the network size \(N_0(\theta_2^*, 0)\). In order to further expand the user network, the open-source firm must lower its profit-share percentage below \(\theta_2^*\). However, this strategy leads to a decrease in the profit from advertisements Π2.

Comparing Fig. 1b with Fig. 2b, we see that the size of the network that maximizes the profit from advertisements is much larger than that which maximizes the profit from applications. This means that application business allows a firm to maximize its profit even when the network is still relatively small. As the network grows, the open-source firm can take advantage of its larger network to maximize its profit from advertisements rather than its profit from applications.

6. General discussion and managerial implications

Since every firm has its functional specialization given limited resources, a firm is sometimes motivated to build alliances with other firms to exchange resources that benefit each other and to reduce environmental uncertainty (Bucklin & Sengupta, 1993; Frazier, 1983). Within the open-source project, the open-source firm allows application developers to access its platform (i.e., the Android operating system). In return, the application developers create new applications for the platform. This exchange of resources between the open-source firm and the application developers facilitates the creation of new resources (e.g., complements and user networks), for the platform that are difficult to imitate in a short period of time. Thus, all of the resources within the open-source community may lead to a sustainable competitive advantage for the open-source platform (Barney, 1991), which mutually benefits both the open-source firm and the application developers. Building this “network of mutual dependence” (Kumar, 2005) may allow the open-source firm to gain strong power in a market.

In this study, we analyze how the profit-sharing scheme, as well as the percentage of paid applications, affects the growth of the user network, and subsequently the profits from applications and advertisements. A profit-sharing model has been used to analyze how each member of a supply chain (e.g., a franchisor and a franchisee) optimizes the overall profit of the entire supply chain as well as its own profit by controlling various prices of a product (e.g., wholesale price, retail price) (e.g., Yan & Wang, 2012). These studies focus on how to enhance the profit of the entire supply chain as a product flows from upstream marketing channels to downstream marketing channels. In contrast, the open-source business model that we discuss in this paper reflects a rather complex network phenomenon, where products or ideas

![Fig. 1.](image-url)
The effects of the profit-share percentage $\theta$ and the size of the user network $N_o$ on the open-source firm’s profit from advertisements $\Pi_2$. Note: 1. The symbol "●" represents the point $(\theta_1, \xi, 1), \xi$ corresponds to $(N_o(\theta_1, \xi, 1), \Pi_2)$. 2. The y-axis represents the relative profit to the maximum profits from advertisements (e.g., $\Pi_2 = .5$ represents half of the maximum profit). 3. In panel 2b, for a given percentage of paid applications $\xi$, the increase of the size of the user network $N_o$ corresponds to the decrease of the profit-share percentage $\theta$. 4. The size of the user network in panel 2b represents the relative network size to the maximum network size (e.g., $N_o = .5$ represents half of the maximum network size).

First, our study provides some insights into the effects of the user-network size on the open-source firm’s profits from applications and advertisements. Our model implies that the open-source firm can earn significant profit from applications even with a relatively small user network by maintaining a relatively high percentage of paid applications on the platform. This analysis sheds light on an alternative open-source business model to Google’s Android project. An open-source firm may be able to implement a niche marketing strategy by promoting a unique platform and applications targeted toward a specific customer segment. Such a niche marketing strategy should focus more on generating profits from application business than from advertising business, which requires a larger network.

Second, our study provides insights into the relative importance of user–network size for application business and advertising business. The analysis of our model shows that the size of a user network that maximizes the profit from advertisements is larger than that which maximizes the profit from applications. This finding implies that expanding the user network may be more crucial for advertising business than for application business. Thus, the open-source firm’s use of non-coercive power in expanding the network may be more essential for advertising business than for application business. The importance of network size for advertising business is explained by the multiple benefits of a larger network size for advertisers. Advertisers prefer a larger user network, as it allows them to reach more customers (Casadesus-Masanell & Zhu, 2010). Thus, the larger the user network, the greater the number of advertisers who may advertise on the platform. Additionally, as the user network grows, each advertiser may increase the amount of its advertising on the platform.

Third, our study shows that the entire open-source community’s profits from applications (advertisements) can be enhanced by lowering the profit-share percentage of the open-source firm below $\theta_1(\xi)$, and by sacrificing the profit of the open-source firm. The open-source firm is unlikely to reduce its profit-share percentage below $\theta_1(\xi)$, as this strategy reduces its profits from applications (advertisements). However, it is still important to understand that this excessive use of non-coercive power further enhances the profits of application developers, and subsequently the profits of the entire open-source community. Thus, a major application developer or a group of application developers may engage in opportunistic behavior by creating a similar open-source platform to enhance their own profits. To avoid such opportunistic behavior, the open-source firm could create a long-term relationship with application developers (Voeth & Herbst, 2006).

7. Limitation and future research

First, our model assumes a monopolistic environment, where a single open-source firm attempts to maximize its profits from applications and advertisements. Researchers are encouraged to extend the model to a duopolistic environment, where an open-source firm competes with a closed-source firm. Similarly, we assume that a single open-source firm controls the application market (Google’s Play Store). In practice, Android users can obtain applications for Android devices from other places (e.g., Amazon Appstore for Android). Thus, more studies are needed to analyze the competitive environment in selling applications for the same open-source platform.

Second, recent work (e.g., Hingley, 2005; Hingley & Lindgreen, 2002) suggests that imbalances of power among channel members may not necessarily result in unstable B2B relationships. As Android devices continue to dominate the smartphone market, the market power of Google may increase, resulting in an imbalance of power. The profit-sharing model may help an open-source firm to reduce such an imbalance of power and maintain stable relationships among its channel members. More research is needed to analyze benefits of the imbalance of power in the context of open-source business.

Third, generating profits from applications and advertisements may not be the only motivation for an open-source firm to build an open-source platform. For instance, the Android project allows Google to collect big data, including users’ location data, which is useful to Google and its partners (e.g., advertisers) for various marketing activities (Angwin & Valentino-DeVries, 2011). Thus, the open-source firm may be motivated to expand the user network simply to have access to more consumer big data and provide advertisers with more customer insights even without earning any profit from the application business. Researchers are encouraged to investigate the role of big data in the context of open-source business.

Finally, we believe that the open-source movement facilitates the idea generation (e.g., applications) in the platform by utilizing the resources of a set of organizations and individuals in the open-source
community. In order to create this open-source community, Google made their proprietary idea available to other organizations. In one way, this reflects the shift in the source of competitive advantage and power from the idea itself (the Android operating system) to the speed of idea generation ("imaginative intensity") (Erevelles, Horton, & Fukawa, 2007). More research is needed to understand the role of the open-source movement in facilitating "imaginative intensity" and gaining power in a market.

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Appendix A. Main notations

We list the main notations used in our model as follows.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>Profit-share percentage of the open-source firm</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Percentage of paid applications on the platform</td>
</tr>
<tr>
<td>$p_o$</td>
<td>Average selling price per paid application</td>
</tr>
<tr>
<td>$p_a$</td>
<td>Average price per impression for advertisers</td>
</tr>
<tr>
<td>$M_p$</td>
<td>The number of paid applications downloaded per user</td>
</tr>
<tr>
<td>$M_a$</td>
<td>The number of free applications downloaded per user</td>
</tr>
<tr>
<td>$M_i$</td>
<td>The number of impressions per advertiser</td>
</tr>
<tr>
<td>$Q_o$</td>
<td>Size of the complements network, also referred to as the number of applications available on the open-source platform</td>
</tr>
<tr>
<td>$N_o$</td>
<td>The number of advertisers on the open-source platform</td>
</tr>
<tr>
<td>$N_1$</td>
<td>Size of the user network</td>
</tr>
<tr>
<td>$\Pi_1$</td>
<td>Profit of the open-source firm from applications</td>
</tr>
<tr>
<td>$\Pi_2$</td>
<td>Profit of the open-source firm from in-application advertisements</td>
</tr>
</tbody>
</table>

Appendix B. Proof of Proposition 1

Here, we present our proof in three steps.

1. We seek the local maximum points of the profit function $\Pi_1$, when $\theta \in (0, 1)$ and $\xi \in (0, 1)$. Differentiating $\Pi_1$ with respect to $\theta$ or $\xi$, we obtain

$$\frac{\partial \Pi_1}{\partial \theta} = \mu p_a f(p_o) (1-\theta)^{-y-x-1} \xi^{-1} (1+\epsilon-\xi) \left[1-(1+x+y)\theta\right],$$

$$\frac{\partial \Pi_1}{\partial \xi} = \mu p_a f(p_o) \theta (1-\theta)^{-y-x} \xi^{-1} \epsilon \left[1+y\left(1+\epsilon-\xi\right)\right].$$

Setting the equations above equal to zero, we obtain

$$1-(1+x+y)\theta = 0 \Rightarrow \theta^*_1 := \frac{1}{1+\epsilon+\xi},$$

$$y(1+\epsilon-\xi) = 0 \Rightarrow \xi^*_1 := \frac{y(1+\epsilon)}{y+1}.$$

That is, $(\theta^*_1, \xi^*_1)$ is one critical point of the profit function $\Pi_1$. To determine whether it is the minimum or maximum point, we further compute the second derivatives:

$$\frac{\partial^2 \Pi_1}{\partial \theta^2} = \mu p_a f(p_o) (1+\epsilon-\xi) \left[1-(1+x+y)\theta\right].$$

$$\frac{\partial^2 \Pi_1}{\partial \xi^2} = \mu p_a f(p_o) \theta (1-\theta)^{-y-x} \xi^{-2} \left(1+y\epsilon\right).$$

$$\frac{\partial^2 \Pi_1}{\partial \theta \partial \xi} = \mu p_a f(p_o) \theta (1-\theta)^{-y-x} \xi^{-2} \left[1+(1+x+y)\theta\right].$$

The second derivative test shows that

$$\frac{\partial^2 \Pi_1}{\partial \theta^2} |_{(\theta^*_1, \xi^*_1)} = -\mu p_a f(p_o) (1+\epsilon-\xi) (1+x+y) < 0,$$

$$\frac{\partial^2 \Pi_1}{\partial \xi^2} |_{(\theta^*_1, \xi^*_1)} = \mu p_a f(p_o) \theta (1-\theta)^{-y-x} \xi^{-2} \left(1+y\epsilon\right) > 0,$$

$$\frac{\partial^2 \Pi_1}{\partial \theta \partial \xi} |_{(\theta^*_1, \xi^*_1)} = \mu p_a f(p_o) \theta (1-\theta)^{-y-x} \xi^{-2} \left[1+(1+x+y)\theta\right] > 0.$$

This implies that the point $(\theta^*_1, \xi^*_1)$ is one local maximum point of the profit function $\Pi_1$ for $\theta \in (0, 1)$ and $\xi \in (0, 1)$.

2. We prove that the point $(\theta^*_1, \xi^*_1)$ is also the global maximum point of $\Pi_1$, for any $\theta \in [0, 1]$ and $\xi \in [0, 1]$. When $\theta = 0$ or $\theta = 1$, the profit $\Pi_1(p_o, 0, \xi) = 1(p_o, 1, \xi) = 0$ for any $\xi \in [0, 1]$. Additionally, when $\xi = 0$, we have $\Pi_1(p_o, \theta, 0) = 0$ for any $\theta \in [0, 1]$. While $\xi = 1$, following the previous discussion, we can easily obtain that the maximum value of the profit $\Pi_1$ is reached when $\theta = \theta^*_1$, and it is

$$\Pi_1(p_o, \theta^*_1, 1) = \mu p_a f(p_o) (1-\theta^*_1)^{y-x} \xi^*_1.$$

To prove that $(\theta^*_1, \xi^*_1)$ is the global maximum point of $\Pi_1$, in the following we will show that $\Pi_1(p_o, \theta^*_1, \xi^*_1) > \Pi_1(p_o, \theta, 1)$

$$= \mu p_a f(p_o) (1-\theta^*_1)^{y-x} \xi^*_1 \left(1+y\left(1+\epsilon-\xi^*_1\right)\right).$$

$$= \mu p_a f(p_o) (1-\theta^*_1)^{y-x} \xi^*_1 \left(1+y\left(1+\epsilon-\xi^*_1\right)\right) \left[1+y\left(1+\epsilon-\xi^*_1\right)\right] > 0.$$

Since for any $x$ and $y$, the term $\mu p_a f(p_o) (1-\theta^*_1)^{y-x} > 0$, we will focus our discussion on

$$H(y, \epsilon) := \left(\frac{y+y\epsilon}{y+1}\right)^{y} \left(1+y\right) - \epsilon.$$

Noticing that $y \leq 1$ and $\epsilon \ll 1$ (i.e., $\epsilon$ is much smaller than 1), we obtain

$$\frac{\partial H}{\partial y} = (H+y\epsilon) \ln \left(\frac{y+y\epsilon}{y+1}\right) - 0.$$

This equation implies that when $y$ increases, the function $H(y, \epsilon)$ decreases. Hence, for any $y \leq 1$ and $\epsilon \ll 1$, there is

$$H(y, \epsilon) > H(1, \epsilon) = \frac{(1-y)^2}{4} > 0.$$

Consequently, we have for any $y \leq 1$ and $\epsilon \ll 1$, $\Pi_1(p_o, \theta^*_1, \xi^*_1) > \Pi_1(p_o, \theta, 1)$. Hence, the point $(\theta^*_1, \xi^*_1)$ is the global maximum point of the profit function $\Pi_1$ for any $\theta \in [0, 1]$ and $\xi \in [0, 1]$. The corresponding user-network size is

$$N_a^*: = N_a(\theta^*_1, \xi^*_1) = \alpha(1+\epsilon-\xi^*_1)(1-\theta^*_1)^y.$$

3. We study the dependence of the size of the user network $N_o$ on the profit-share percentage $\theta$ and the percentage of paid applications $\xi$. For $\theta \in (0, 1)$ and $\xi \in (0, 1)$, we can compute

$$\frac{\partial N_o}{\partial \theta} = -\alpha(1+\epsilon-\xi^*_1)(1-\theta)^{-y-1} < 0,$$

$$\frac{\partial N_o}{\partial \xi} = -\alpha(1-\theta)^{-y} < 0.$$

That is, the size of the user network $N_o$ always increases by decreasing the profit-share percentage $\theta$ and/or the percentage of paid applications $\xi$. However, when $\theta < \theta^*_1$ or $\xi < \xi^*_1$, the resulting user-network size is larger than $N_a^*$, but the profit is smaller than $\Pi_1(p_o, \theta^*_1, \xi^*_1)$. In other words, excessively increasing the size of the user network by either decreasing the profit-share percentage below $\theta^*_1$ or reducing the percentage of paid applications below $\xi^*_1$ will lead to a decline in the profit $\Pi_1$. 
Appendix C. Proof of Proposition 2

When \( N_0 < N_{\text{min}} \), the profit \( I_2 = 0 \). Thus, in the following we will restrict our discussion for \( N_0 \geq N_{\text{min}} \). For \( \xi \in (0, 1) \) and \( \theta \in (0, 1) \), we can compute
\[
\frac{\partial I_2}{\partial \xi} = -\lambda \rho g(p)(1-\theta)^{(z-2)(z-1)}(1-\xi)^{-1}(1-\xi)/(1+\theta+\xi) > 0,
\]
that is, the profit function \( I_2 \) has no maximum point when \( \xi \in (0, 1) \) and \( \theta \in (0, 1) \). When \( \xi = 1 \), the function \( I_2(p_0, \theta) = 0 \) for \( \theta \in [0, 1] \). Hence, the possible maximum value of the function \( I_2 \) can be achieved only when \( \xi = 0 \). When \( \xi = 0 \) and \( \theta \in (0, 1) \), we have
\[
\frac{\partial I_2(p_0, \theta, 0)}{\partial \theta} = \lambda \rho g(p)(1-\theta)^{(z-2)(z-1)}(1+\theta+\xi)/(1+\theta+\xi) > 0.
\]

Setting it equal to zero, we obtain
\[
1 - [1 + y + x(z + 1)] \theta = 0 \Rightarrow \theta^*_2 := \theta = \frac{1}{1 + y + x(z + 1)}.
\]
Furthermore, we find
\[
\frac{\partial^2 I_2}{\partial \theta^2}(p_0, \theta^*_2, 0) = -\lambda \rho g(p)(y + x(z + 1))\left[\frac{y + x(z + 1)}{(1 + y + x(z + 1))^{y+z(x+1)-2}}(1+\theta+\xi)/(1+\theta+\xi) > 0.
\]
Hence, when \( \theta = \theta^*_2 \) and \( \xi = \xi^*_2 := 0 \), the profit function \( I_2 \) reaches its maximum value, and it is the global maximum point because when \( \theta = 0 \) or \( \theta = 1 \), the profit \( I_2 = 0 \). The corresponding user-network size is computed by:
\[
N_{\theta^*_2} := N_0(\theta^*_2, \xi^*_2) = \alpha(1+\epsilon)(1-\theta^*_2)^{y}.
\]

As discussed in Appendix B, the size of the user network \( N_0 \) always increases by decreasing the profit-share percentage \( \theta \). Hence, when \( \theta < \theta^*_2 \), the resulting user-network size \( N_0(\theta, \xi^*_2) \) is larger than \( N_{\theta^*_2} \), but the profit is smaller than \( I_2(p_0, \theta^*_2, \xi^*_2) \). In other words, excessively increasing the size of the user network by decreasing the profit-share percentage below \( \theta^*_2 \) will lead to a decline of the profit \( I_2 \).

Appendix D. Proof of Proposition 3

To simplify the expression, we will denote \( \theta_1^* \) (\( \theta_2^* \)) as the optimal profit-share percentage maximizing the profit of the open-source applications (advertisements) and denote \( N_{\theta_1^*}(N_{\theta_2^*}) \) as the size of the user network when the profit of applications (advertisements) is maximized.

(a). From the proof of Proposition 1 in Appendix B, we obtain
\[
\theta_1^* = \frac{1}{1 + x + y}.
\]
From the proof of Proposition 2 in Appendix C, we have
\[
\theta_2^* = \frac{1}{1 + y + x(z + 1)}.
\]
Since \( x, y \) and \( z \) are all positive constants, we obtain \( 1 + y + x(z + 1) > 1 + x + y \), implying the optimal profit-share percentage \( \theta_2^* < \theta_1^* \).

(b). When the profit of the open-source firm applications (advertisements) is maximized, we can compute the corresponding size of the user network as
\[
N_{\theta_1} = N_0(\theta_1, \xi_1) = \alpha(1+\epsilon)(1-\theta_1^*)^y. \quad N_{\theta_2} = N_0(\theta_2, \xi_2) = \alpha(1+\epsilon)(1-\theta_2^*)^y.
\]
Since \( \theta_2^* < \theta_1^* \), we have \((1-\theta_2^*)/(1-\theta_1^*) > 1 \). Hence,
\[
\frac{N_{\theta_2}}{N_{\theta_1}} = (y + 1)(1-\theta_2^*)^y > 1.
\]
This implies that the user-network size maximizing the profit from advertisements is larger than that maximizing the profit from applications.

Appendix E. Proof of Proposition 4

1. Denote \( T_1 \) as the profit of the entire open-source community from applications. We have
\[
T_1 := T_1(p, \theta, \xi) = \mu S(p)(1-\theta)^{x+y}(1+\epsilon-\xi).
\]
Our focus here is to study how the profit-share percentage of the open-source firm \( \theta \) affects the profit function \( T_1 \). Hence, we assume that \( \xi = \xi^*_2 \), which is the optimal percentage of paid applications maximizing the profit of the open-source firm from applications.

For \( \theta \in (0, 1) \), we obtain
\[
\frac{\partial T_1}{\partial \theta}(p, \theta, \xi^*_2) = -\mu S(p)(x + y)(1-\theta)^{x+y-1}(1+\epsilon-\xi^*_2) < 0.
\]
That is, the profit of the entire open-source community from applications always increases as the profit-share percentage \( \theta \) decreases. However, as discussed in Appendix B, the profit of the open-source firm from applications is maximized only when \( \theta = \theta_1^* \). In other words, lowering the profit-share percentage below \( \theta_1^* \) decreases the joint profit of the entire community from applications but decreases the profit of the open-source firm from applications.

2. Denote \( T_2 \) as the profit of the entire open-source community from advertisements. Here, we will only focus on the case with \( N_0 \geq N_{\text{min}} \). Then we have
\[
T_2 := T_2(p, \theta, \xi) = \lambda \rho g(p)(1-\theta)^{(z-2)(z-1)}(1-\xi)^{y}(1+\epsilon-\xi)^{z}.
\]
Similarly, we assume that \( \xi = \xi^*_2 = 0 \), which is the optimal percentage of paid applications maximizing the profit of the open-source firm from advertisements. For \( \theta \in (0, 1) \) we obtain
\[
\frac{\partial T_2}{\partial \theta}(p, \theta, \xi^*_2) = -\lambda \rho g(p)(y + x(z + 1))(1-\theta)^{(z-2)(z-1)-1}(1+\epsilon-\xi^*_2)^{z} < 0.
\]
That is, the joint profit of the entire community from advertisements always increases as the profit-share percentage \( \theta \) decreases. However, as discussed in Appendix C, the profit of the open-source firm from advertisements is maximized only when \( \theta = \theta_2^* \). In other words, lowering the profit-share percentage below \( \theta_2^* \) increases the joint profit of the entire community from advertisements but decreases the profit of the open-source firm from advertisements.

References


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