Effect of local pumping on random laser modes in one dimension

Xiaohua Wu,[1] Jonathan Andreasen,[1] Hui Cao,[1,*] and Alexey Yamilov[2]

[1]*Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208, USA*
[2]*Department of Physics, University of Missouri–Rolla, Rolla, Missouri 65409, USA*
**Corresponding author: h-cao@northwestern.edu*

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We have developed a numerical method based on the transfer matrix to calculate the quasi modes and lasing modes in one-dimensional random systems. Depending on the relative magnitude of the localization length versus the system size, there are two regimes in which the quasi modes are distinct in spatial profile and frequency distribution. In the presence of uniform gain, the lasing modes have one-to-one correspondence to the quasi modes in both regimes. Local excitation may enhance the weight of a mode within the gain region due to local amplification, especially in a weakly scattering system. © 2007 Optical Society of America

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1. INTRODUCTION

The random laser, in which optical feedback is provided by scattering of light due to spatial inhomogeneity of the medium rather than by well-defined mirrors, has recently attracted much attention [1]. One important topic of research is the nature of random laser modes. For a random laser with nonresonant feedback, the lasing modes are the diffusive modes, i.e, the eigenmodes of the diffusion equation [2]. For a random laser with resonant feedback, the lasing modes are believed to be the quasi modes, i.e, the eigenmodes of the Maxwell equations [3]. This belief implies the quasi modes of a passive random system are not modified by the presence of gain. Such an assumption is confirmed by the numerical studies of lasing modes in the localization regime [4,5]. With the introduction of gain, the localized modes of a passive random system are preserved and serve as the lasing modes. This conclusion is extended to the random systems far from the localization regime without direct confirmation. The lasing modes are regarded as the quasi modes with a small decay rate, in particular, the anomalously localized states [6,7]. However, a recent theoretical study [8] reveals that the quasi modes of a passive random system are not the genuine normal modes of the same system with gain. This is because the spatial inhomogeneity of the dielectric constant introduces a linear coupling between the quasi modes, mediated by the polarization of the active medium. The latest development of semiclassical laser theory for open complex or random media leads to the speculation that the lasing mode in a weakly scattering system may be a composite of many quasi modes with a low-quality factor [9,10]. Moreover, under local excitation the reabsorption outside the local gain region suppresses the feedback from the unpumped part of the random sample and effectively reduces the system size [11]. The lasing modes are therefore completely different from the quasi modes and confined in the vicinity of the pumped region. All these studies prompt us to investigate carefully the relation between the lasing modes and the quasi modes in both global pumping and local pumping. In this paper, we address the question whether the lasing modes are the quasi modes of passive random systems. The answer to this question determines whether the statistical distribution of the decay rates of quasi modes can be used to predict the lasing threshold and the number of lasing modes for the random laser [12–19].

We conduct detailed numerical studies of quasi modes and lasing modes in one-dimensional (1D) random systems. A numerical method based on the transfer matrix is developed to calculate the quasi modes as well as the lasing modes in the presence of global or local gain. The main advantage of this method as compared with the finite-difference time-domain method is that it can calculate the quasi modes of weakly scattering systems that overlap spectrally and have short lifetimes. In our numerical simulation, the scattering strength is varied over a wide range. The quasi modes, as well as the lasing modes, are formed by distributed feedback in the random system. The conventional distributed feedback laser, made of periodic structures, operates in either the overcoupling regime or the undercoupling regime [20]. The random laser, which can be considered a randomly distributed feedback laser, also has these two regimes of operation. In the undercoupling regime the system size L is much less than the localization length ξ, whereas in the overcoupling regime $L > \xi$. The dominant mechanism for the mode formation differs in these two regimes, leading to distinct characteristics of mode profile and frequency distribution. With the introduction of uniform gain, the lasing modes have one-to-one correspondence to the quasi modes in both regimes. However, local pumping can make the lasing modes significantly different from the quasi modes, especially in the undercoupling systems. Some quasi modes even fail to lase, no matter how high the pumping level is. The results we obtain help in understanding the random lasing with resonant feedback in the weakly scattering systems [21], especially the recent observations of periodic lasing peaks in frequency [22,23].

2. NUMERICAL METHOD

We have developed a numerical method based on the transfer matrix to compute the quasi modes of 1D passive systems. This time-independent method is also applied to the calculation of lasing modes at the threshold under global or local excitation. The random system is a 1D layered structure. It is composed of N dielectric layers with air gaps in between. The refractive index of the dielectric layers is n_d, and that of the air gaps is 1. Both the thickness d_1 of the dielectric layers and the thickness of air gaps d_2 are randomized. $d_{1,2}=\bar{d}_{1,2}(1+\sigma\eta)$, where $0<\sigma<1$ represents the degree of randomness, η is a random number in $[-1,1]$, and $\bar{d}_1$ ($\bar{d}_2$) is the average thickness of the dielectric layers (air gaps). Outside the random system the refractive index is constant, and its value is equal to the average refractive index n_{eff} of the random system to eliminate the boundary reflection.

According to the transfer-matrix formula:

$$\begin{pmatrix} p_1 \\ q_1 \end{pmatrix} = M \begin{pmatrix} p_0 \\ q_0 \end{pmatrix}, \quad (1)$$

where p_0 and q_0 represent the forward- and backward-propagating waves on one side of the random system, p_1 and q_1 represent those on the other side, and M is a 2×2 transfer matrix that characterizes wave propagation through the random system. The eigenmode of such an open system can be defined as a natural mode or quasi mode, which generalizes the concept of an eigenmode of a closed system [24]. It satisfies the boundary condition that there are no incoming waves but only outgoing waves through the boundary of a random system, namely, $p_0=0$ and $q_1=0$. In a passive system (without gain or absorption, the refractive indices being real numbers), such boundary condition requires the vacuum wave vector to be a complex number, $k_0=k_{0r}+ik_{0i}$. Substituting the boundary condition into Eq. (1), we get $M_{22}=0$. Since M_{22} is a complex number, both the real part and the imaginary part of M_{22} are equal to 0. These two equations are solved to find k_{0r} and k_{0i}. $k_{0r}=\omega/c$ tells the frequency ω of a quasi mode, and $k_{0i}=-\gamma/c$ gives the decay rate γ of a quasi mode.

After finding k_0 of a quasi mode, one can obtain the corresponding wave function by calculating the electric field distribution $E(x)$ throughout the random system with the transfer matrix $M(k_0)$. The wave function inside the random system can be written as $E(x)=E_+(x)e^{in(x)k_0x}+E_-(x)e^{-in(x)k_0x}$, where $n(x)$ is the (real part of) refractive index at position x, $E_+(x)e^{in(x)k_0x}$ represents the forward-propagating field, and $E_-(x)e^{-in(x)k_0x}$ represents the backward-propagating field. Since k_0 is a complex number, the amplitudes of forward- and backward-propagating fields are $E_+(x)e^{-n(x)k_{0i}x}$ and $E_-(x)e^{n(x)k_{0i}x}$ ($k_{0i}<0$). These expressions show that there are two factors determining the wave function. The first is $E_\pm(x)$, which originates from the interference of multiply scattered waves. The second is $e^{\pm n(x)k_{0i}x}$, which leads to exponential growth of the wave function toward the system boundary. Outside the random system, the wave function grows exponentially to infinity due to the negative k_{0i}. This is clearly unphysical. Thus we disregard the wave function outside the random system and normalize the wave function within the random system to unity.

We introduce optical gain into the random system by adding an imaginary part n_i (negative number) to the refractive index, whose value at the lasing threshold is to be determined later. In the case of uniform gain, n_i is constant everywhere inside the system. Outside the random system, n_i is set to zero. Different from the quasi mode of a passive system, the vacuum wave vector k_0 of a lasing mode is a real number. The wave vector inside the random system is a complex number, $k=k_r+ik_i=k_0[n(x)+in_i]$. Its imaginary part $k_i=k_0n_i$ is inversely proportional to the gain length l_g. The onset of lasing oscillation corresponds to the condition that there be only outgoing waves through the boundary of the random system. The absence of incoming waves requires $M_{22}=0$ in Eq. (1). Again, since M_{22} is a complex number, both its real part and its imaginary part are zero. These two equations are solved to find k_0 and n_i. Each set of solution (k_0,n_i) represents a lasing mode. $k_0=\omega/c$ sets the lasing frequency ω, and $n_ik_0=k_i=1/l_g$ gives the gain length l_g at the lasing threshold. We then obtain the spatial profile of the lasing mode by calculating the field distribution throughout the random system with the transfer matrix $M(k_0,n_i)$. Since our method is based on the time-independent wave equation, it holds only up to the lasing threshold [25]. In the absence of gain saturation, the amplitude of a lasing mode would grow in time without bound. Thus we can only get the spatially normalized profile of a lasing mode at the threshold. The lasing mode is normalized in the same way as the quasi mode for comparison. The amplitudes of forward- and backward-propagating fields of a lasing mode are $E_+(x)e^{-n_ik_0x}$ and $E_-(x)e^{n_ik_0x}$ ($n_i<0$). The exponential growth factors $e^{\pm n_ik_0x}$ depend on the gain value $|n_ik_0|$.

Local pumping is commonly used in the random laser experiment. To simulate such a situation, we introduce gain into a local region of the random system. Our method can be used to find the lasing modes with arbitrary spatial distribution of gain. The imaginary part of the refractive index $n_i(x)=\tilde{n}_if(x)$, where $f(x)$ describes the spatial profile of gain and its maximum is set to 1 and $\tilde{n}_i$ represents the gain magnitude. The lasing modes can be found in a way similar to the case of uniform gain. The solution to $M_{22}=0$ gives the lasing frequency k_0 and threshold gain $\tilde{n}_ik_0$. The normalized spatial profile of a lasing mode is then computed with $M(k_0,\tilde{n}_i)$.

3. RESULTS AND DISCUSSION

Using the method described in the previous section, we calculate the quasi modes of 1D random systems. The quasi modes are formed by distributed feedback from the randomly positioned dielectric layers. We investigate many random structures with different scattering strengths. Depending on the relative values of the localization length ξ and the system length L, there are two distinct regimes in which the quasi modes are dramatically different: (i) overcoupling regime $L>\xi$ and (ii) undercoupling regime $L\ll\xi$.

As an example, we consider the random structure with $\bar{d}_1=100$ nm and $\bar{d}_2=200$ nm. $\sigma=0.9$ for both d_1 and d_2. To change from the undercoupling regime to the overcoupling regime, we increase the refractive index n_d of the dielectric layers. In particular, we take $n_d=1.05$ and 2.0. The larger n_d leads to stronger scattering and shorter localization length ξ. To obtain the value of ξ, we calculate the transmission T as a function of system length L. $\langle \ln T \rangle$ is obtained from averaging over 10,000 configurations with the same L and σ. When $L>\xi$, $\langle \ln T(L) \rangle$ decays linearly with L, and $\xi^{-1}=-\mathrm{d}\langle \ln T(L)\rangle/\mathrm{d}L$. In the wavelength ($\lambda$) range of 500 to 750 nm, ξ exhibits slight variation with λ due to the residual photonic bandgap effect. For $n_d=1.05$, $\xi \sim 200-240$ μm, whereas, for $n_d=2.0$, $\xi\sim 1.2-1.5$ μm. In the calculation of quasi modes, we fix the number of dielectric layers $N=81$ and $\langle L \rangle=24.1$ μm. For $n=1.05$, $\xi \gg L$ in the wavelength range of interest; thus the random system is in the undercoupling regime. In contrast, for $n=2.0$, $\xi \ll L$, and the system is in the overcoupling regime.

To illustrate the difference between the overcoupling regime and the undercoupling regime, we compare the quasi modes of the same random structure with different n_d, namely, $n_d=2.0$ or 1.05. Figures 1(a) and 1(b) are the typical transmission spectra of these two systems. For the system with $n_d=2.0$, most transmission peaks are narrow and well separated in frequency, whereas for $n_d=1.05$, the transmission peaks are typically broad and overlapped. We find $k_0=k_{0r}+ik_{0i}$ of the quasi modes in the wavelength range of 500–750 nm. Figure 1(c) shows the values of k_{0r} and $k_{0i}/\langle k_{0i}\rangle$ of these modes; ($\langle k_{0i}\rangle$ is the average over all the quasi modes in the wavelength range of 500–750 nm). In the system with $n_d=2.0$, most quasi modes are well separated spectrally, and they match the transmission peaks. k_{0r} corresponds to the frequency of a transmission peak, and k_{0i} corresponds to the linewidth of a transmission peak. However, some quasi modes are located close to the system boundary, thus having relatively large k_{0i}. They are usually invisible in the transmission spectrum owing to spectral overlap with neighboring transmission peaks, which causes the number of transmission peaks [Fig. 1(a)] to be slightly less than the number of quasi modes [solid squares in Fig. 1(c)]. In the system with $n_d=1.05$, however, the number of peaks or maxima in the transmission spectrum [Fig. 1(b)] is significantly less than the number of quasi modes [open circles in Fig. 1(c)]. This is because in the undercoupling regime the decay rates of the quasi modes often exceed the frequency spac-

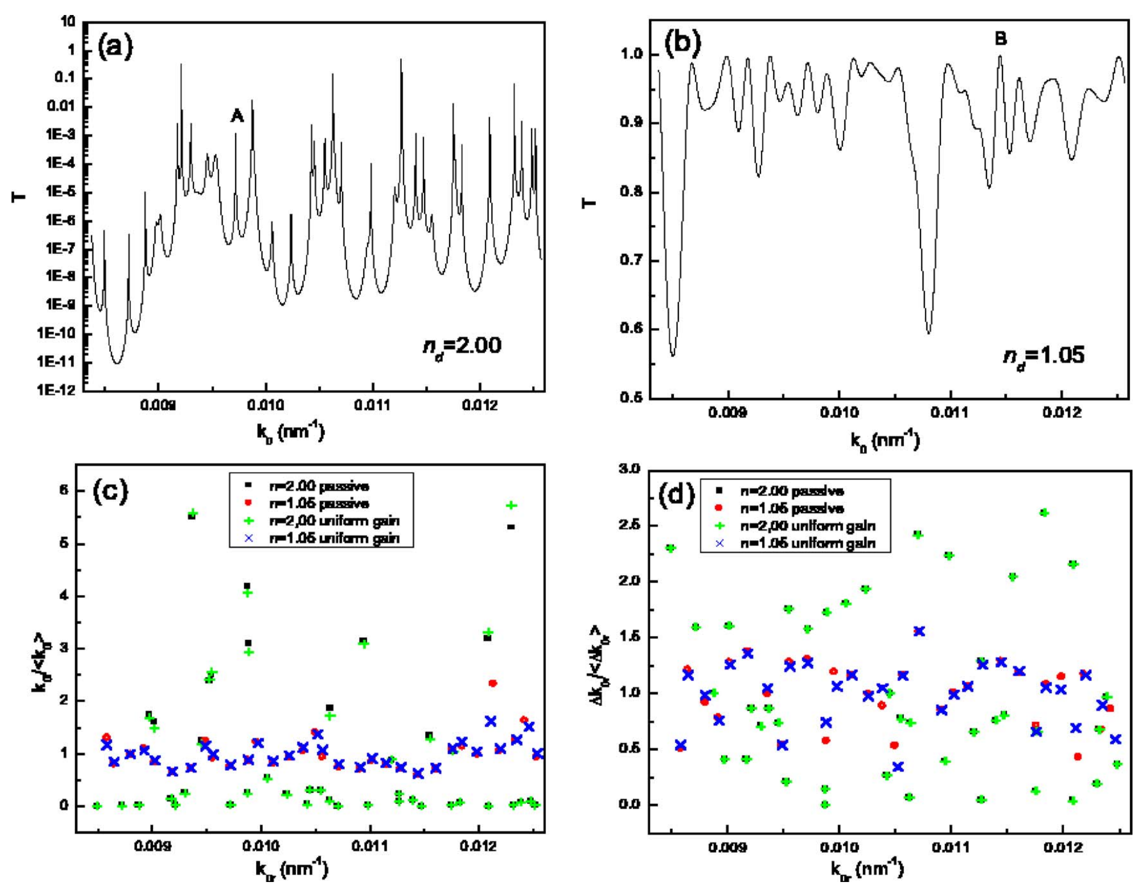

Fig. 1. (Color online) (a), (b): Transmission T through a 1D random structure with $n_d=2.0, 1.05$ as a function of vacuum wave vector k_0. (c) Frequencies k_{0r} and normalized decay rates $k_{0i}/\langle k_{0i}\rangle$ of the quasi modes in the random systems with $n_d=2.0$ (solid squares) and $n_d=1.05$ (open circles), compared with the frequencies k_0 and normalized threshold gain $k_i/\langle k_i\rangle$ of lasing modes in the same systems with $n_d=2.0$ (plus signs) and $n_d=1.05$ (crosses) under uniform excitation. (d) Normalized frequency spacing $\Delta k_{0r}/\langle \Delta k_{0r}\rangle$ of neighboring quasi modes in the random systems with $n_d=2.0$ (solid squares) and $n_d=1.05$ (open circles), compared with the normalized frequency spacing $\Delta k_0/\langle \Delta k_0\rangle$ of neighboring lasing modes in the same systems with $n_d=2.0$ (plus signs) and $n_d=1.05$ (crosses) under uniform excitation.

ing to neighboring modes. The spectral overlap of the quasi modes makes the transmission peaks less evident and some even buried by the neighboring ones.

It is clear in Fig. 1(c) that the decay rate fluctuation is much stronger in the random system with $n_d=2.0$ (solid squares) than that with $n_d=1.05$ (open circles). This is consistent with the broadening of quasi-mode decay rate distribution as a system approaches the localization regime with increasing scattering strength. Figure 1(d) plots the frequency spacing Δk_{0r} between adjacent quasi modes normalized to the average value $\langle \Delta k_{0r} \rangle$. The quasi modes of the random system with $n_d=1.05$ are more regularly spaced in frequency than those in the system with $n_d=2.0$. The average mode spacing is inversely proportional to the system length L.

To interpret this phenomenon, we investigate the wave functions of the quasi modes. Figure 2(a) [Fig. 2(b)] shows the spatial distribution of intensity $I(x)=|E(x)|^2$ for a typical quasi mode of the random system with $n_d=2.0$ ($n_d=1.05$). $I(x)$ is normalized such that the spatial integration of $I(x)$ within the random system is equal to unity. The expression of $E(x)$ given in the previous section reveals the two factors determining the envelope of the wave function, i.e., the interference term $E^{\pm}(x)$ and the exponential growth term $e^{\pm n(x)k_{0i}x}$. Depending on which term is dominant, the spatial profile of the quasi modes can be drastically different. In the overcoupling regime, strong scattering makes the interference term dominant, and $I(x)$ exhibits strong spatial modulation. Most quasi modes are localized inside the random system, similar to the mode in Fig. 2(a). Their decay rates are low as a result of the interference-induced localization. In the undercoupling regime, the interference effect is weak owing to the small amount of scattering. The exponential growth term $e^{\pm n(x)k_{0i}x}$ dominates $E(x)$, making $I(x)$ increase exponentially toward the boundaries. The interference term causes only weak and irregular intensity modulation. A typical example of such mode profile is exhibited in Fig. 2(b). Since the quasi modes in the undercoupling system are spatially extended across the entire random system, the rates of light leakage through the boundaries are much higher than those of the localized modes in the overcoupling system.

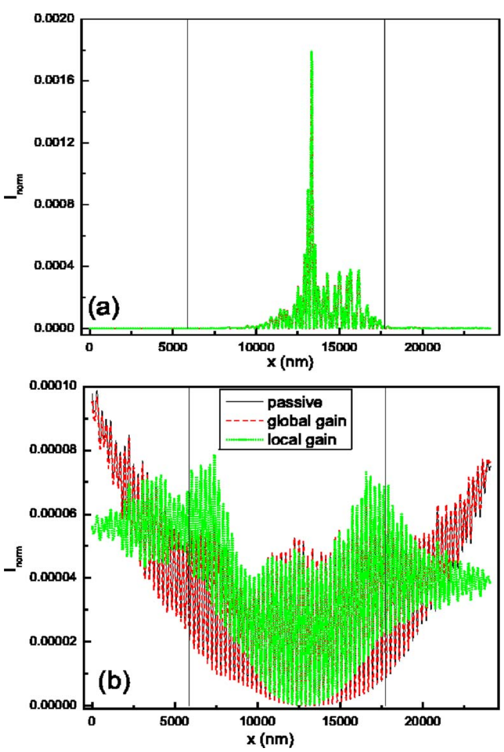

Fig. 2. (Color online) Spatial intensity distributions of quasi modes (black solid curve) and the corresponding lasing modes in the presence of global gain (red dashed curve) or local gain (blue dotted curve). The pumped region is between the two vertical lines, $L_p=11.87$ μm. (a) For the mode marked A in Fig. 1(a), $\lambda=646$ nm, $n_d=2.0$. (b) For the mode marked B in Fig. 1(b), $\lambda=549$ nm, $n_d=1.05$.

We repeat the above calculations with many random systems and find the two different types of quasi mode are rather typical for the systems in the overcoupling and undercoupling regimes. The mode profiles and frequency spacings in the undercoupling systems reveal that the feedbacks from the dielectric layers close to the boundaries are dominant over those from the interior. Thus the quasi modes in the undercoupling systems are formed mainly by the feedbacks from the scatterers near the system boundaries. However, the feedbacks from the scatterers in the interior of the system are weak but not negligible; e.g., they induce small fluctuations in the frequency spacings and the decay rates. Note that a random system in the undercoupling regime cannot be approximated as a uniform slab with the average refractive index n_{eff}, even though its quasi modes exhibit features similar to the Fabry–Perot modes formed by the reflections from the slab boundaries. Since in our calculation the refractive index outside the random system is set to n_{eff}, there would be no quasi modes if the random system were replaced by a dielectric slab of n_{eff}. Hence, the quasi modes in the undercoupling regime are not formed by the boundary reflection. In the overcoupling regime, the feedback from the scatterers deep inside the system becomes dominant, and the interference of multiply scattered waves leads to spatial localization of the quasi modes.

Next, we study the lasing modes in the random system with uniform gain and compare them with the quasi modes. n_i is constant everywhere within the random system, so that the gain length $l_g=1/k_i=1/k_0 n_i$ in the dielectric layers is equal to that in the air gaps. Using the method described in the previous section, we find the frequency and threshold gain of each lasing mode. We calculate the lasing modes in the same random systems as in Fig. 1 within the same wavelength range (500–750 nm). The frequency k_0 and normalized threshold $k_i/\langle k_i \rangle$ of each lasing mode are plotted in Fig. 1(c) for comparison with the quasi modes. It is clear that there exists one-to-one correspondence between the lasing modes and the quasi modes for the random systems in both overcoupling and undercoupling regimes. For the system with $n_d=2.0$, the lasing modes match well the quasi modes, with only a slight difference between $k_i/\langle k_i \rangle$ and $k_{0i}/\langle k_{0i} \rangle$ for the relatively leaky modes. For the system with $n_d=1.05$, the deviation of the lasing modes from the quasi modes is more evident, especially for those modes with large decay rates. Such deviation can be explained by the modification of transfer matrix M. In the passive system, k_{0i} is constant, but $k_i = k_{0i}n(x)$ varies spatially. With the introduction of uniform gain, k_i becomes constant within the random system, and the feedback inside the random system is caused only by the contrast in the real part of the wave vector $k_r = k_0 n(x)$ between the dielectric layers and the air gaps. With a decrease in the scattering strength, k_{0i} in the passive system gets larger, and the ratio of the feedback caused by the contrast in k_i to that in k_r increases. The addition of uniform gain results in a bigger change of M, as it removes the feedback due to the inhomogeneity of k_i. Moreover, since there is no gain outside the random system, k_i suddenly drops to zero at the system boundary. This discontinuity of k_i generates additional feedback for the lasing modes. In the weakly scattering system, the threshold gain is high. The larger drop of k_i at the system boundary makes the additional feedback stronger. To check its contribution to lasing, we replace the random system with a uniform slab of n_{eff} while keeping the same gain profile. Since the real part of the refractive index or k_r is homogeneous throughout the entire space, the feedback comes only from the discontinuity of k_i at the slab boundaries. We find the lasing threshold in the uniform slab is significantly higher than that in the random system with $n_d = 1.05$. This result confirms that, for the random systems in Fig. 1, the additional feedback caused by the k_i discontinuity at the system boundary is weaker than the feedback due to the inhomogeneity of k_r inside the random system. However, if we further reduce n_d or L, the threshold gain increases, and the feedback from the system boundary due to gain discontinuity eventually plays a dominant role in the formation of lasing modes.

We also compute the intensity distribution $I(x)$ of each lasing mode at the threshold. $I(x)$ is normalized such that its integration across the random system is equal to 1. Such normalization facilitates the comparison of the lasing mode profile with the quasi-mode profile. In Fig. 2(a) [Fig. 2(b)], $I(x)$ of the lasing mode is plotted together with that of the corresponding quasi mode. Although the lasing mode profiles in Figs. 2(a) and 2(b) are quite different, they are nearly identical to those of the quasi modes. For the localized mode in the random system with $n_d = 2.0$, $I(x)$ of the lasing mode does not exhibit any visible difference from that of the quasi mode in Fig. 2(a). For the extended mode in the system with $n_d = 1.05$, the lasing mode profile deviates slightly from the quasi-mode profile, especially near the system boundaries. This deviation results from the modification of the transfer matrix M by the introduction of uniform gain across the random system. The modification is bigger in the undercoupling system, leading to a larger difference in the mode profile.

Finally, we investigate the lasing modes under local excitation. In particular, $f(x)=1$ for $|x-x_c| \leq L_1/2$, $f(x) = \exp[-|x-x_c|/L_2]$ for $L_1/2 < |x-x_c| \leq L_1/2 + 2L_2$, and $f(x) = 0$ elsewhere. The lasing mode frequency k_0, the threshold gain $k_i = k_0 \tilde{n}_i$, and the spatial profile $I(x)$ are calculated with the method described in the previous section. $I(x)$ is normalized in the same way as that of the quasi mode for comparison. As an example, we consider the same random structures as in Fig. 1 and introduce gain into the central region $x_c = L/2$ of length $L_p = L_1 + 4L_2 = 8.84 + 3.03 = 11.87\ \mu$m (marked by two vertical lines in Fig. 2). Figure 3(a) plots k_0 and $k_i/\langle k_i\rangle$ for all the lasing modes within the wavelength range of 500–750 nm. Comparing Fig. 3 with Fig. 1, we find some quasi modes fail to lase under local pumping, no matter how high the pumping level is. The other modes lase, but their wave functions can be significantly modified by the particular local excitation. Both of the two modes shown in Fig. 2 lase under the local pumping configuration we consider. Their intensity distributions are plotted in Fig. 2. The mode in Fig. 2(a) is localized within the pumped region, and its spatial profile is barely modified by the local gain. In contrast, the mode in Fig. 2(b) is spatially extended and has less overlap with the central gain region. The intensity distribution of the lasing mode differs notably from that of the quasi mode. The exponential growth of $I(x)$ toward the system boundaries is suppressed outside the gain region, wheres inside the gain region $I(x)$ grows exponentially toward the ends of the gain region at a rate higher than that of the quasi mode. These behaviors can be explained by the spatial variation of gain. Outside the pumped region, there is no optical amplification; thus light intensity does not increase exponentially. Within the pumped region, the faster intensity growth results from the higher threshold gain for lasing with local pumping than that with global pumping. Nevertheless, the close match in the number and spatial position of intensity maxima justifies the correspondence of the lasing mode to the quasi mode.

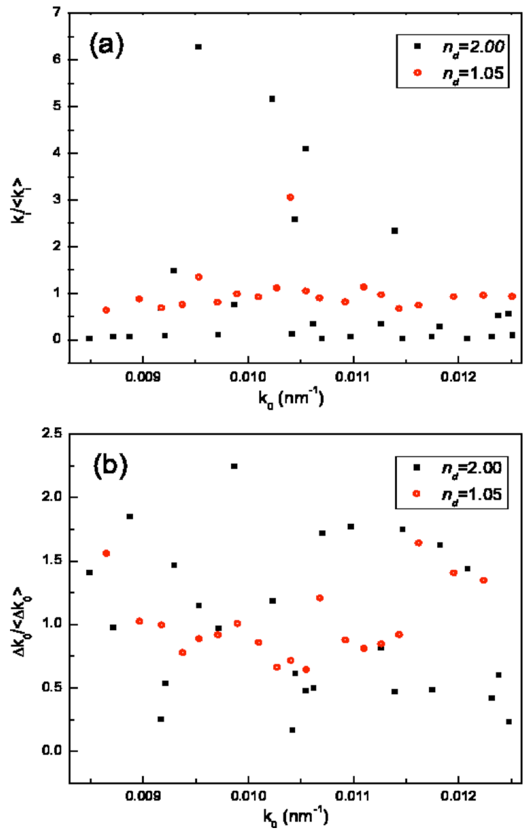

Fig. 3. (Color online) (a) Normalized threshold gain $k_i/\langle k_i\rangle$ versus the frequency k_0 of lasing modes in the random systems with $n_d = 2.0$ (solid squares) and $n_d = 1.05$ (open circles) under local excitation (between the two vertical lines in Fig. 2). (b) Normalized frequency spacing $\Delta k_0/\langle\Delta k_0\rangle$ of neighboring lasing modes in the systems with $n_d = 2.0$ (solid squares) and $n_d = 1.05$ (open circles) under local excitation.

We repeat the calculation with many modes under the same pumping configuration and find the weight of a mode within the gain region is often enhanced. To quantify such enhancement, we introduce a parameter δ, which is equal to the ratio of $I(x)$ integrated over the pumped region to that over the entire random system. We compare the values of δ for the lasing modes under local excitation with those of the corresponding quasi modes. For the mode in Fig. 2(b), δ is increased from 0.33 for the quasi mode to 0.41 for the lasing mode, whereas for the mode in Fig. 2(a) δ remains at 0.98. Thus the effect of local pumping is stronger for the modes in the weakly scattering system. This is because when scattering is weak the local gain required for lasing is high. The feedback within the pumped region is greatly enhanced, leading to the modification of mode profile.

We also investigate the fluctuations in threshold gain and frequency spacing of lasing modes under local excitation. Figure 3(a) shows that the lasing threshold fluctuation for the random system with $n_d=1.05$ is smaller than that with $n_d=2.0$. Since the number of lasing modes under local pumping is usually less than that of quasi modes, the average mode spacing $\langle \Delta k_0 \rangle$ is increased. Figure 3(b) plots the frequency spacing Δk_0 of adjacent lasing modes normalized to the average value $\langle \Delta k_0 \rangle$. There is more fluctuation in the mode spacing for the random system with $n_d=2.0$ than that with $n_d=1.05$. Hence, with local gain the frequency spacing of lasing modes is more regular in the undercoupling regime than in the overcoupling regime. This result is similar to that with uniform gain.

Although the local pumping enhances the feedback within the pumped region, the feedback outside the pumped region cannot be neglected. To demonstrate this, we calculate the lasing modes in the reduced systems of length L_p by replacing the random structures outside the gain region with a homogeneous medium of n_{eff}. The reduced system has uniform gain instead of the gain profile $f(x)$ in the original system. The results are shown in Fig. 4(a) for the system with $n_d=2.0$ and in Fig. 4(b) for the system with $n_d=1.05$. The number of lasing modes in the reduced system is less than that in the original system under local pumping. In fact, the lasing modes are generally different, with the only exception being a few modes localized within the gain region in the system with $n_d=2.0$. Moreover, the lasing threshold in the reduced system is higher than that in the original system with local gain. These differences are attributed to the feedbacks from the random structure outside the pumped region of the original system. It demonstrates that the scatterers in the unpumped region also provide feedback for lasing. By comparing Figs. 4(a) and 4(b), we find the difference in the lasing threshold between the original system under local pumping and the reduced system is smaller for the system with $n_d=1.05$ than that with $n_d=2.0$. It indicates the contribution from the scatterers outside the gain region to lasing is reduced as the system moves further into the undercoupling regime.

We note that local pumping introduces inhomogeneity into the imaginary part of the refractive index, which generates additional feedback for lasing. To check its effect, we simulate lasing in a homogeneous medium with the average refractive index n_{eff}. The local gain profile $f(x)$ remains the same. Only the spatial variation of $k_i(x)=k_0\tilde{n}_i f(x)$ provides feedback for lasing. As shown in Figs. 4(a) and 4(b), the lasing thresholds are much higher than those in the random systems, even for the system with $n_d=1.05$. This result demonstrates that for the random systems in Figs. 3 and 4 the feedbacks for lasing under local pumping are predominately caused by the inhomogeneities in the real part of the refractive index $n(x)$ or the wave vector $k_r(x)=k_0 n(x)$. However, a further reduction in n_d or L_p could make the feedback due to the inhomogeneity of $k_i(x)$ significant.

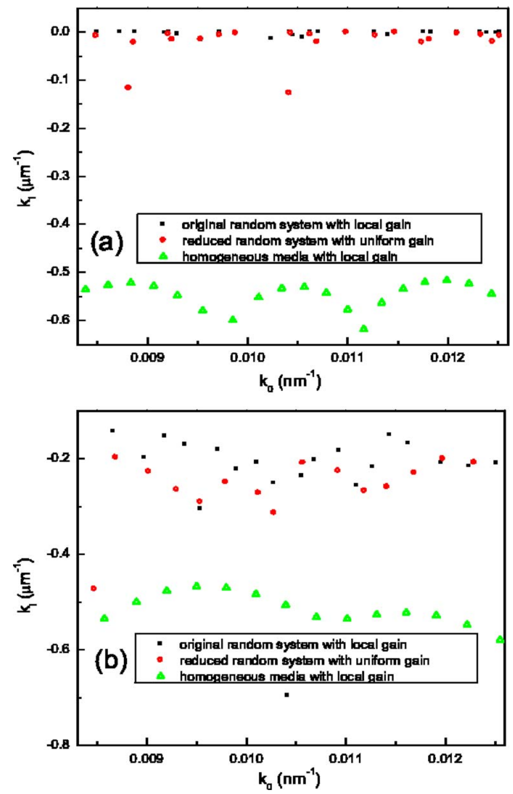

Fig. 4. (Color online) Threshold gain $k_i/\langle k_i \rangle$ of lasing modes in the (original) random system of length 24.1 μm with local excitation in the central region of length 11.87 μm (solid squares), compared with the threshold gain of lasing modes in the reduced system of length 11.87 μm under uniform excitation (open circles) and the threshold gain of lasing modes in the homogeneous medium with n_{eff} under local excitation in the region of length 11.87 μm. (a) $n_d=2.0$, $n_{eff}=1.3361$; (b) $n_d=1.05$, $n_{eff}=1.0168$.

4. CONCLUSION

We have developed a numerical method to calculate the quasi modes of 1D passive random systems and the lasing modes at the threshold with either global or local pumping. We identified two regimes for the quasi modes: the overcoupling regime ($L>\xi$) and undercoupling regime ($L \ll \xi$). In the undercoupling regime the electric field of a quasi mode grows exponentially toward the system boundaries, whereas in the overcoupling regime the field maxima are located inside the random system. The frequency spacing of adjacent modes is more regular in the undercoupling regime, and there is less fluctuation in the decay rate. The distinct characteristic of the quasi modes in the two regimes results from the different mechanisms of mode formation. In an overcoupling system, the quasi modes are formed mainly by the interference of multiply scattered waves by the particles in the interior of the random system. In contrast, the feedbacks from the scatterers close to the system boundaries play a dominant role in the formation of quasi modes in an undercoupling system. The contributions from the scatterers in the interior of the random system to the mode formation are weak but not negligible. They induce small fluctuations in mode spacing and decay rate. As the scattering strength is increased, the feedbacks from those scatterers in the interior of the system get stronger, and the frequency spacing of the quasi modes becomes more random.

In the presence of uniform gain across the random system, the lasing modes (at the threshold) have one-to-one correspondence with the quasi modes in both overcoupling and undercoupling systems. However, the lasing modes may differ slightly from the corresponding quasi modes in frequency and spatial profile, especially in the undercoupling systems. This is because the introduction of uniform gain removes the feedback caused by spatial inhomogeneity of the imaginary part of the wave vector within the random system and creates additional feedback by the discontinuity of the imaginary part of the wave vector at the system boundaries. As long as the scattering is not too weak, the quasi modes are only slightly modified by the introduction of uniform gain to a random system, and they serve as the lasing modes. This conclusion is consistent with that drawn from the time-dependent calculations [4,5,26]. Hence, with the knowledge of the decay rates of the quasi modes, in conjunction with the gain spectrum, the first lasing mode can be predicted. Because of the correspondence between the lasing modes and the quasi modes, the frequency spacing of adjacent lasing modes is more regular in the undercoupling systems with smaller mode-to-mode variations in the lasing threshold.

When optical gain is introduced into a local region of the random system, some quasi modes cannot lase, no matter how high the gain is. The other modes can lase, but their spatial profiles may be significantly modified. Such modifications originate from strong enhancement of feedbacks from the scatterers within the pumped region. This enhancement increases the weight of a lasing mode within the gain region. Nevertheless, the feedbacks from the scatterers outside the pumped region are not negligible. Moreover, the spatial variation in the imaginary part of the refractive index generates additional feedback for lasing. As the pumped region becomes smaller, the number of lasing modes is reduced, and the frequency spacing of lasing modes is increased. In an undercoupling system, the regularity in the lasing mode spacing remains under local excitation. Our calculation results will help to interpret the latest experimental observations [22,23] of spectral periodicity of lasing peaks in weakly scattered random systems under local pumping. We note that the effect of local excitation can be significant in an overcoupling system if the size of the pumped region is much smaller than the spatial extent of a localized mode or the spatial overlap between the pumped region and the localized mode is extremely small. Hence, caution must be exerted in using the decay rates of quasi modes to predict the lasing threshold or the number of lasing modes under local excitation. Finally, we comment that the increase in the mode concentration in the gain region by local pumping has a physical mechanism distinct from the absorption-induced localization of lasing modes in the pumped region [11]. The former is based on selective enhancement of feedback within the gain region, whereas the latter is based on the suppression of the feedback outside the pumped region by reabsorption.

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